Processing direct measurement results in mechanical engineering technology

Обробка результатів прямих вимірювань у технології машинобудування

Обработка результатов прямых измерений в технологиях машиностроения

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У статті розроблена методика оцінки похибки прямих вимірювань, виконаних при однакових умовах. Представлений приклад оцінки виміряного значення сили різання з урахуванням числа виконаних вимірювань, заданого рівня довірчої ймовірності, класу точності вимірювального приладу і межі його вимірів.

Ключові слова: *загальна похибка вимірювання, виміряна величина, інструментальна похибка, рівень довіри, вимірювальний прилад, клас точності приладу.*

В статье разработана методика оценки погрешности прямых измерений, выполненных при одинаковых условиях. Представлен пример оценки измеренного значения силы резания с учетом числа выполненных измерений, заданного уровня доверительной вероятности, класса точности измерительного прибора и предела его измерений.

Ключевые слова: *полная погрешность измерения, измеряемая величина, инструментальная погрешность, доверительный интервал, измеритель, класс точности инструмента.*

A methodology for evaluation the direct measurements' error is given in paper when the measurements are performed under the same conditions. An example of the evaluation for the measured value of the cutting force is presented taking into account the number of measurements performed, the specified confidence level, the accuracy class of the measuring instrument and the limit of its measurements.

Key words: total measurement error, measured quantity, instrumental error, confidence level, measuring instrument, instrument accuracy class.

In direct measurements, the value of the measured quantity is found directly from the experimental data. In such case, the total measurement error of the result is determined by the geometric sum of the three components [1-6]:

$$\Delta = \sqrt{\Delta_{ran}^2 + \Delta_{in}^2 + \Delta_{rou}^2}\,, \qquad (1)$$

where $\Delta_{ran}, \Delta_{in}$, and Δ_{rou} are the absolute values of random, instrumental, and rounded error components, respectively.

Although the instrumental error is systematic, it is summed up with other errors as a random value, i.e., according to the geometric summation rule. That is so because the sign of the instrumental error is usually unknown.

The random error component may be found as follows [7-10]:

$$\Delta_{ran} = \frac{t_{\alpha;n} \cdot S}{\sqrt{n}}, \tag{2}$$

where $t_{\alpha;n}$ is the Student's t-test value (criterion value) which depends on the pre-accepted significance level α and the number of performed experiments n; S is the mean squared deviation or mean squared error, i.e., the average squared difference between the estimated values and the actual value instead of which the sample average $\bar{x}$ is frequently used [11-14].

The following formula is used to find the S value:

$$S = \sqrt{\frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n-1}}. \tag{3}$$

The significance level α determines the acceptable value of a statistical error. It is related to confidence level (not confidence interval) p by the formula $\alpha = 1 - p$. As a rule, for technical applications is assumed that $\alpha = 0.05$, i.e., $p = 0.95$ (i.e., 95 %). Once again, a confidence level refers to the percentage of all possible samples that can be expected to include the true population parameter. For example, suppose all possible samples were selected from the same population, and a confidence interval were computed for each sample. A 95% confidence level implies that 95% of the confidence intervals would include the true population parameter [15].

The instrumental error component can be found according to the formula [16]:

$$\Delta_{inst} = \frac{\lambda_p}{3} \cdot \frac{K \cdot x_{\max}}{100}, \tag{4}$$

where λ_p is the Laplace inverse distribution quantile depending on the pre-selected confidence level p; K is the measuring instrument (MI) accuracy class; $x_{\max}$ is the measurement limit for the MI mentioned.

The MI accuracy class shows the value of its absolute error in percentage from the limit of its measurement (the size of the MI scale). Mostly used measurement instruments have the following accuracy classes: 0.1; 0.2; 0.5; 1.0; 1.5; 2.5; etc. Sometimes the MI limit error value is noted in its passport. For example, the Laplace inverse distribution quantile for $p = 0.95$ has a value $\lambda_p = 1.96$ [16].

For MI with the scale, the rounding error component Δ_{rou} is equal to either one division value or to half of the division value if the count is taken with the accuracy up to one or half of the division of MI scale, respectively. Sometimes this error is even less (up to one third or even one fourth of the division value of MI scale) when the MI is equipped with a mirror sector to eliminate the MI scale parallax. The rounding error component of digital MI is equal to half of its junior bit.

For example, direct measurements of cutting force were performed five times ($n = 5$) by using the "UDM -100" dynamometer. Measurement results are the following: 650; 670; 660; 680; 650 *N*.

Determine the amount of the cutting force and its error with a confidence level p = 95 %, if the MI accuracy class is 2, the MI limit is 1000 *N*, and one division value of MI scale is 5 *N*.

Solution

1. The cutting force mean value is $\bar{x} = 662$ *N*.

2. The mean squared deviation (MSD) by the formula (3):

$$S = \sqrt{\frac{(650-662)^2 + (670-662)^2 + (660-662)^2 + (680-662)^2 + (650-662)^2}{4}} =$$

$$= \sqrt{\frac{144+64+4+324+144}{4}} = 13.04\, N.$$

3. The Student's t-test value (Student's criterion) is $t_{0.05;5} = 2.57$ [10]

4. The random error component by the formula (2):

$$\Delta_{ran} = \frac{2.57 \cdot 13.07}{\sqrt{5}} = 14.99\, N.$$

5. The instrumental error component by the formula (4):

$$\Delta_{in} = \frac{1.96}{3} \cdot \frac{2 \cdot 1000}{100} = 13.07\ N.$$

6. The rounding error component (with accuracy to one division value): $\Delta_{rou} = 5$ *N*.

7. Then, the total measurement error by the formula (1) will be the following

$$\Delta = \sqrt{14.99^2 + 13.07^2 + 5^2} = 20.91\ N.$$

8. Therefore, the cutting force measuring result for the confidence level $p_{дов} = 0.95$ will be

$$P = 662 \pm 20{,}9\, N.$$

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