UDC 62-503.57
DOI: 10.15587/1729-4061.2020.220326

CONSTRUCTION OF A METHOD FOR REPRESENTING AN APPROXIMATION MODEL OF AN OBJECT AS A SET OF LINEAR DIFFERENTIAL MODELS

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This paper has demonstrated the need to use models not only at the stage of theoretical research and design operations but also when studying existing objects. The techniques to build them on the basis of identification methods have been analyzed. The identification methods have been shown when determining the parameters of processes and objects. The difficulty of defining the models' structures has been emphasized.

A method has been proposed to determine the structure of an arbitrary object's model as the approximating set of linear differential models. The data on the object's response to external impact have been used as source data. Demonstrating the method's feasibility employed a set of standard links and a standard external influence in the form of a stepped function as a model. This approach helps assess the adequacy of the obtained approximation results based on the precise solutions available. In a general case, there are no specific requirements for the form of an external influence and an object's reaction.

The data that reflect the object's response should allow their approximation using a polynomial. That makes it possible to represent them following a Laplace transform in the form of a truncated power series in the image domain. The transfer function is written in a general form as a rational fraction. It underlies a Padé approximant of the truncated power series.

The comparison of the available accurate calculation results and those derived on the basis of the built model has shown good agreement. In the cases under consideration, the computation error did not exceed the 5 % value permissible for engineering calculations. This is also the case when using the approximation of original data over a limited period.

The response of the resulting model to the external influence that simulates a real pulse was investigated. The comparison with precise results showed a discrepancy not exceeding the value permissible for engineering calculations (<5 %)

Keywords: object approximation model, structural identification, Padé approximant, linear differential models

Received date 23.10.2020
Accepted date 02.12.2020
Published date 25.12.2020

1. Introduction

At the stage of theoretical research or design work, models of different forms can be built when constructing automatic control systems. However, the range of acceptable models is limited when studying existing objects and processes. The forms and parameters of an acceptable model are to be determined. In the scientific literature, this process is termed a model identification (from Latin *Identifico,* to identify). Identification implies the identity of identifiable entities. However, the model does not reflect the fullness of the properties of the object described. In this case, it is reasonable to argue about identifying the parameters of a model with the already selected structure – parametric identification. The mathematical apparatus of this procedure in the theory of automatic control (TAC) is extensive and, in many cases, described in detail.

The process of "choosing" the structure of the control object model is less formalized. The term "choosing" is not used by accident. This is how, in many cases, the structure of the model for further research is determined. In such a situation, the decision made is heuristic, and its quality depends on the intuition of the researcher. Intuition can be presented as a product of knowledge (experience) not formalized at the moment. As a result, different researchers can obtain different results when solving the same problems. This does not allow for the optimality, or even a possibility, when the results obtained are not adequate to the examined objects and processes. In many sources, the choice of model type is defined as structural identification. However, because of the incomplete overlap between the properties of the model and the object, the term approximation (from Lat. *Proxima*, close or approximate) is closer in essence to the executed procedure. It is possible to ensure that different researchers obtain equivalent results from model approximation through the formalization of this procedure. Thus, it is a relevant task to construct new formalized methods to obtain the approximating models of the examined control objects and the processes occurring in them.

2. Literature review and problem statement

The issues of object modeling are discussed in detail in the areas of research related to technical devices [1, 2]. This is due, among other things, to the development of systems for managing the processes that occur in them. The models used are based on subjective opinions, the experience of some researchers. Subsequently, the validity of the accepted constructs is checked by comparison with experimental data. In this case, as in many others, it is shown that there are no contradictions to the results of the process modeling under assigned conditions but not the object. Practically, a direct problem is solved. While the model can be built when solving the reverse problem: constructing its possible structure that corresponds to the available experimental data. As the modeling apparatus develops, so does the scope of its application in areas not related to technology [3]. Although the cited work considers the structure of a model but, from the point of view of choosing among several, the task of its construction is not addressed. In addition to expanding the scope of application, the issues of automation of the modeling process itself, the automation of the process of finding approximating models of objects and processes, are discussed [4]. However, even in this case, the authors consider, although automated, but the choice of model structure from the list of those available.

A common approximation technique, similar to the technology involving replacing the model with a simpler one, is linearization. There are many forms of it. These include both the simplest, in the form of expanding into a series, and more sophisticated ones. The latter include, for example, harmonic linearization [5], linearization via feedback [6], and a range of other forms. A common drawback of this approach is the need for a baseline model. And the purpose of structural approximation is to find such a model.

Another approach to solving identification tasks is to reduce the dimensionality of modeling space through the nondimensionalization of mathematical models. In the control and identification theory, nondimensionalization methods have been used relatively recently. To date, there are both standalone papers [7] and monographs [8]. Their authors determine the results of reducing the dimensionality of modeling space based on the π-theorem. Study [9] proposed an approach to reduce, as compared to previous papers, the number of model parameters as a result of the nondimensionalization procedure. A distinctive feature of the nondimensionalization methods is the need for a formalized model, or at least its structure. Nondimensionalization methods are effective in identifying model parameters and do not make it possible to determine its structure.

One way to identify is to set up controllers based on the parameters of the ongoing processes to provide the necessary characteristics. For example, [10] applies a model of an object similar to the one considered, and the parameters of an actual process. Based on these data, a controller is adjusted to ensure the optimized process performance. In this case, using an object model like the one in question suggests defining parameters rather than the structure of an object. A possibility to select a criterion for optimizing characteristics indicates the approximation procedure rather than identifying the parameters of the chosen model. The authors of [11] offer a model-free adaptive control platform for an unknown object, using the concept of an equivalent controller of the dynamic linearization. It is proposed to manage without explicit identification of the process. This structure can be used to manage a specific (single) process but does not provide a basis for analyzing the structure of the object. In [12], when setting up a controller, the authors apply certain frequency characteristics of the managed process. However, as is the case in [11], the structure of the object is not built.

A series of papers claim to find the best structure of the model. The search is limited not to the construction but to the choice of structure from a predefined space. For example, it is proposed in [13] to determine the most important parameters of a computational model by using the deterministic and probabilistic approaches. To this end, different input parameters are modeled. The cited work identifies the parameters and selects the model rather than builds its structure. In [14], the structure of a model is assumed to be known and defined by the examined manufacturing process. Similar to the above cases, the model parameters are identified. The problems that occur are caused by the lag of the signal about the measured thickness of the steel strip and the moment of technological influence. In that case, it is not even the identification of the parameters of the model that takes place but the development of a method for neutralizing the effect of a transport lag on the process.

The above works showed the possibility to organize a management system in the absence of an object or process model. In this case, the controller settings are assigned on the basis of the current parameters of managed processes. In fact, this is *a posteriori* management that refers only to the current process. This approach does not make it possible to predict the behavior of an object during its operation when external conditions or even control over the structure of an object change. It can be considered that the parametric identification for the accepted model of the object or process is objective. From these perspectives, it is interesting to consider the possibility to build a formalized structure of a model based on any raw data, such as experimental responses to external impacts.

3. The aim and objectives of the study

The aim of this study is to construct a method to represent an approximation model of a control object as a set of linear differential models. This makes it possible to approximate control objects with a curve that displays oscillatory processes when setting up automatic control systems, which warrants the predefined quality of control.

To accomplish the aim, the following tasks have been set:

– to build an algorithm to represent the approximation model of control objects;

– to devise a technique to determine the approximation model of control objects.

4. An algorithm to represent an approximation model of control objects

Models based on differential equations are used to describe the dynamic characteristics of processes and control objects. One of the most powerful and generally accepted means to solve them is to apply a method of integrated transformations. This approach allows the differential equations to be reduced to algebraic equations and makes it easier to transform and solve them. The variety and diversity of the properties of integrated transformations are determined by their quantity reaching several dozen forms (more than twenty). However, there are similarities. Thus, almost all integrated transformations are reversible; the original can be restored on the basis of an existing representation. In addition, integrated transformations are linear operators. These features have defined the technique and form of the solution resulting from the devised method:

– basic transformations involve simpler expressions in the representation space;

– the result is represented as a combination of linear equation solutions.

The mathematical apparatus of transfer functions is often used to solve differential equations and to study the properties of the control object and automatic control systems. To move from the original functions to transfer functions and back, the theory of automatic control typically employs a Laplace transform.

Below is a sequence of actions that implements an algorithm for representing an approximation model of control objects. The starting data used in solving a direct problem is a verified mathematical model of the control object (set in the form of a system of linear differential equations, in the form of transfer functions, or matrices in the state space). Hereafter, the apparatus of transfer functions is applied to illustrate the Algorithm.

When a specific input impact is set, the reaction of the examined system is determined. Applying a model recorded using a system of linear (linearized) differential equations, such a solution in representation space is written in the following form:

$$f_{ex}(p)=W(p)\cdot f_{ent}(p), \tag{1}$$

where $f_{ent}(p)$ is the representation of the input signal; $f_{ex}(p)$ is the representation of the output signal, $W(p)$ is a transfer function.

The transfer function represents all the equations that are part of the model (model structure) and all their parameters. Using the solution notation as (1) makes it possible to solve the inverse problem: at the assigned input impact $f_{ent}(p)$ and when registering the reaction to it by the examined system $f_{ex}(p)$, to determine the transfer function $W(p)$ of the examined object. The system's response to external influences can be obtained during the experiment. The formalization of its notation can be performed by, for example, the approximation with a series. In this case, when solving the inverse problem, we should argue about determining the transfer function of the object under study in the form of its approximation. On this basis, the algorithm for determining the approximation of the object's transfer function is constructed as follows.

In the first stage, external influence is sent to the examined object's input. It may take different forms. In this case, for certainty, the impact is accepted in the form of a Heaviside function with the following representation

$$f_{ent}(p)=\frac{1}{p}. \tag{2}$$

The second stage registers the system's reaction at the output. The resulting data are approximated by a power series, for example, using a least-square method. Under the terms of a Laplace transform, the system's reaction at a zero point in time is considered in the state of rest. Therefore, the approximation polynomial should not contain a free term. In a general case, its notation can be represented as follows:

$$f_{ex}(t)=\sum_{i=1}^{n}c_i t^i, \tag{3}$$

where t is time, c_i is the coefficients of an approximating polynomial; i is the number of the approximating polynomial term. The number of terms in the series is chosen based on the necessary accuracy of the approximation.

In the third stage, applying a Laplace transform to a series (3) in the representation space yields a power (truncated) series in the following form:

$$f_{ex}(p)=\sum_{i=1}^{n}c_i\frac{i!}{p^{i+1}}=\sum_{i=1}^{n}c_i\cdot i!\cdot\left(\frac{1}{p}\right)^{i+1}. \tag{4}$$

The fourth stage records the defined transfer function in a general form of the rational fraction:

$$W(p)=\frac{b_m p^m+b_{m-1}p^{m-1}+\ \dots\ +b_1 p}{a_n p^n+a_{n-1}p^{n-1}+\ \dots\ +a_1 p+1}. \tag{5}$$

In the fifth stage, substituting (2), (4), and (5) in expression (1), we obtain an expression in the following form:

$$\sum_{i=1}^{n}c_i\cdot i!\cdot\left(\frac{1}{p}\right)^{i+1}=\frac{b_m p^m+b_{m-1}p^{m-1}+\ \dots\ +b_1 p}{a_n p^{n+1}+a_{n-1}p^n+\ \dots\ +a_1 p^2+p}. \tag{6}$$

In the sixth stage, the numerator and denominator in the right-hand part (6) is divided by the size of the term from the denominator at maximum power for p. In this case, it is $a_n p^{n+1}$. The result is:

$$\sum_{i=1}^{n}c_i\cdot i!\cdot\left(\frac{1}{p}\right)^{i+1}=$$
$$=\frac{\frac{b_m}{a_n}\cdot\frac{1}{p^{n-m+1}}+\frac{b_{m-1}}{a_n}\cdot\frac{1}{p^{n-m+2}}+\ \dots\ +\frac{b_1}{a_n}\cdot\frac{1}{p^n}+\frac{b_0}{a_n}\cdot\frac{1}{p^{n+1}}}{1+\frac{a_{n-1}}{a_n}\cdot\frac{1}{p}+\frac{a_{n-2}}{a_n}\cdot\frac{1}{p^2}\ \dots\ +\frac{a_1}{a_n}\cdot\frac{1}{p^{n-1}}+\frac{1}{a_n}\cdot\frac{1}{p^n}}, \tag{7}$$

or

$$\sum_{i=1}^{n} c_i \cdot i! \cdot \left(\frac{1}{p}\right)^{i+1} = \frac{\sum\limits_{j=m}^{0} \frac{b_j}{a_n} \cdot \left(\frac{1}{p}\right)^{n+1-j}}{1+\sum\limits_{i=n-1}^{1} \frac{a_i}{a_n} \cdot \left(\frac{1}{p}\right)^{n-i} + \frac{1}{a_n} \cdot \left(\frac{1}{p}\right)^{n}}. \qquad (8)$$

In the seventh stage, $1/p=z$ is replaced:

$$\sum_{i=1}^{n} c_i \cdot i! \cdot z^{i+1} = \frac{\sum\limits_{j=m}^{0} \frac{b_j}{a_n} \cdot z^{n+1-j}}{1+\sum\limits_{i=n-1}^{1} \frac{a_i}{a_n} \cdot z^{n-i} + \frac{1}{a_n} \cdot z^{n}}. \qquad (9)$$

Expression (9) represents a notation of the Padé approximant. The left-hand part contains a truncated power series with all the known coefficients. The right-hand part contains the rational fraction whose application approximates the series.

In the eighth stage, according to the approximation method by Padé, all unknown coefficients in the denominator of the right-hand part are determined (9). The coefficients are represented in the form of complexes (a_i/a_n) and a ratio $(1/a_n)$. $(1/a_n)$ defines the (a_n), and, with its help, (a_i/a_n) determines the (a_i) values.

In the ninth stage, the numerator coefficients in the right-hand part of equation (9) are determined. They are represented in the form of (b_i/a_n) complexes. Using the (a_n) value, the values of the b_i coefficients are determined. The knowledge of (a_i) and (b_i) coefficients makes it possible to record the transfer function in form (5) and, on its basis, the approximation model in the original space in the form of a set of linear ordinary differential equations.

5. A technique to determine the approximation model of control objects

As a demonstration example, consider the approximation of the dynamic properties of a gas pipeline that fuels the combustion in an energy boiler. The site diagram is shown in Fig. 1, *a*.

Gas flow rate is measured at a specific pipeline cross-section and compared to its specified value. The resulting controlling impact affects the *CV* regulatory valve. The gas comes from collector C, which maintains constant pressure P_1, and, through the adjusting valve, the gas pipeline, the lock valve, and burner B is fed into the second battery (furnace), where constant pressure P_2 is maintained. Typically, the controlling element is located near the collector, the disturbing valve (*DV*) – right next to the burner. Thus, the section of the pipeline, which should be taken into consideration in the calculations, is between the controlling and disturbing valves. Therefore, we accept the estimation scheme shown in Fig. 1, *b*.

When deriving a model, the following assumptions and limitations are taken:

– the model is linear. The model linearization was carried out by expanding the functions into a Taylor series and discarding all terms in the series above the first;

– to match the model to the actual process, the deviations of technological parameters from the rated values should not exceed 10 %;

– in terms of hydro-gas-dynamics, the medium is considered incompressible; that makes it possible to apply a model with concentrated parameters;

– it is described by a system of linear differential equations;

– the solutions to differential equations are in the form of deviations from the rated values, that is, under zero initial conditions.

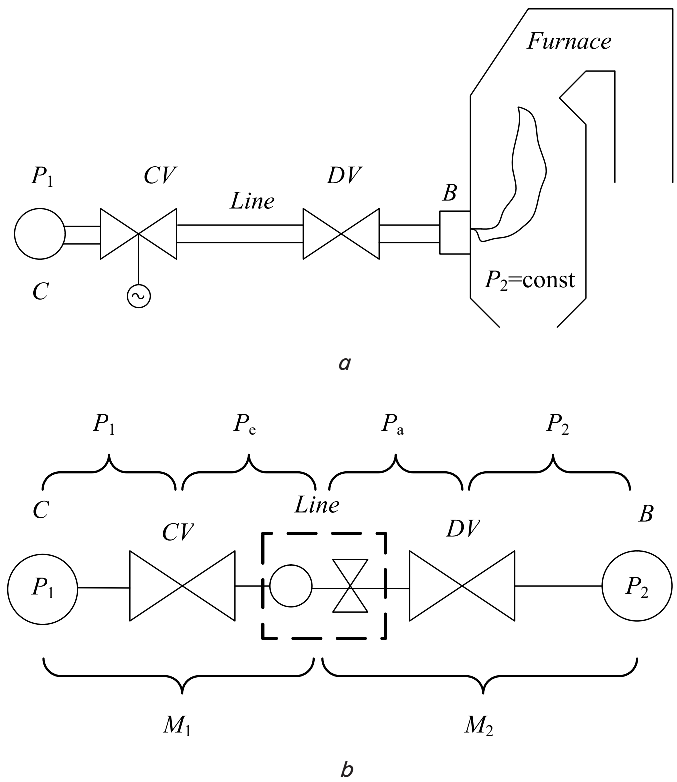

Fig. 1. Schematic of the site that controls gas flow into a boiler

Then one can write down the properties of individual elements of the site:

– for the controlling valve (at subcritical pressure drops)

$$\Delta P_1 - \Delta P_e = a_{CV}\Delta M_1 - b_{CV}\Delta S_{CV}, \qquad (10)$$

at $\Delta P_1=0$;

– for the gas pipeline

$$\frac{M}{2P}TP_e' + \frac{1}{a_l}\Delta P_e = \Delta M_1 - \frac{1}{a_l}\Delta P_a, \qquad (11)$$

$$a_l \frac{\bar{M}}{\bar{P}}TM_2' + \Delta M_2 = -\frac{\bar{M}}{\bar{P}}TP_a' + \Delta M_1; \qquad (12)$$

– for the disturbing valve (at subcritical pressure drops)

$$\Delta P_2 - \Delta P_a = a_{DV}\Delta M_2 - b_{DV}\Delta S_{DV}, \qquad (13)$$

at $\Delta P_2=0$.

For a compressed environment, at subcritical discharge through the valve, the deviation of the pressure drop $\Delta\Delta P_{CV}$ on it is determined from the following expression:

$$\Delta\Delta P_{CV} = a_{CV}\Delta M - \text{в}_{CV}\Delta S_{CV}, \qquad (14)$$

where $a_{CV}=\dfrac{2K_{CV}\bar{M}}{\bar{S}_{CV}^2}$, $v_{CV}=-\dfrac{2K_{CV}\bar{M}^2}{\bar{S}_{CV}^3}$.

A change in pressure loss in a gas pipeline is determined from the following expression:

$$\Delta\Delta P_l = 2K_l\bar{M}\Delta M, \qquad (15)$$

then $a_l = 2K_l\bar{M}$.

In equations (10) to (15), the following designations are adopted:

M – stable rated gas flow rate;

P – stable rated gas pressure;

ΔS_{CV} – area of the pass-through cross-section of the control valve;

ΔS_{DV} – area of the pass-through section of the disturbing valve;

$T=\dfrac{m}{\bar{M}}$ – a time constant time of heat exchange processes associated with changes in the pressure of gas when it moves in the pipe;

α – a coefficient that takes into consideration the imperfection of the heat exchange process.

In the equation system (10) to (15), the ΔS_{CV} value is the input (controlling impact), ΔM_2 is the output value (adjustable value); ΔS_{DV} is the external disturbing influence; ΔM_1, ΔP_e, and ΔP_a are the intermediate parameters of non-interest in this case. When solving the system, they should be excluded.

Excluding these parameters and transforming the system into one equation is a simple, albeit time-consuming, operation. Mathematical considerations associated with the transformation are omitted. After the necessary actions were taken, the following equation was derived:

$$\begin{aligned} &a_2M_2'' + a_1M_2' + a_0\Delta M_2 = \\ &= e_1\Delta S_{CV}' + e_0 + \Delta S_{CV} + z_2\Delta S_{CV}'' + z_1\Delta S_{DV}' + z_0\Delta S_{DV}, \end{aligned} \qquad (16)$$

where

$$a_2 = \left[a_{CV}a_l\left(a_l + 2a_{DV}\right)\right]c^2;\quad c=\left(\frac{\bar{M}}{2\bar{P}}T\right);$$

$$a_1 = 2a_{CV}a_l + 2a_{CV}a_{DV} + 2a_{DV}a_l + a_l^2;$$

$$a_0 = a_{CV} + a_{DV} + a_l;$$

$$e_1 = a_l v_{CV};\quad e_0 = v_{CV};$$

$$z_0 = v_{DV};\quad z_1 = 2\left(a_{CV}+a_l\right)v_{DV}c;\quad z_2 = 2v_{DV}a_{CV}a_lc^2.$$

Thus, one can derive a transfer function describing the properties of a pipeline through the channel "moving the controlling valve – gas flow rate":

$$W_{\Delta S_{PK}\to\Delta M_2}(S) = \frac{e_1S + e_0}{a_2S^2 + a_1S + a_0}; \qquad (17)$$

In practical calculations of controller settings, only the value of the e_0 factor (consistent with the object's transfer factor) and the values of the transfer function denominator coefficients are taken into consideration. Also, when calculating the settings of the controller, it is necessary to take into consideration the inertia of the controlling element. This can be done by adding a first-order inertial link to the control channel.

Thus, a gas pipeline, which delivers gas for burning in the furnace of the boiler, in terms of the control channel can be described as two consistently connected inertial links of the first and second orders with the transfer functions in the following form:

$$W(p) = \left(\frac{1}{1\cdot p+1}\right)\left(\frac{5}{3^2\cdot p^2 + 3\cdot p + 1}\right). \qquad (18)$$

The numerical values of the coefficients were obtained during the start-up tests of a boiler installed in the industrial heating unit.

In addition, the transfer function of form (18) describes in [15] a steam tract when modeling the boiler control system. In the same way, [16] described a model of the electric hydraulic system with a high- and low-pressure turbine valve when designing a controller for the turbine control system of a steam solar power plant.

The form of the transitional characteristic is given in Fig. 2.

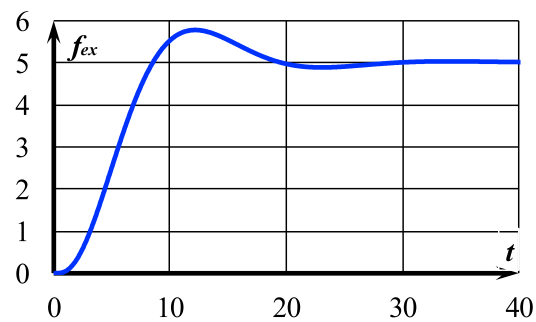

Fig. 2. Transitional response characteristic of an object whose model is represented by transfer function (18) on a stepped disturbance

These data were approximated to produce an expression in form (3) using a least-square method involving a 10-power polynomial. The order of the polynomial is conditioned by the possibility of the adequate approximation of the examined accelerating characteristic (Fig. 1). Depending on the degree of complexity of its form, the order of the polynomial may be different. In a given case, the condition was set for the deviation of values derived from an approximation dependence on the initial data (Fig. 1) not exceeding 2 %.

The relative error of approximation in the examined range does not exceed 0.6 %. The error is determined relative to the magnitude of the stable mode (f_{ex}=5).

Based on these data, we derived the (a_i) and (b_i) coefficients for the approximating transfer functions (5) for different powers of the denominator. For the fourth power, it takes the following form:

$$W(p) = \\ = \frac{5\cdot\left(-1.14766\cdot p^3 + 1.86912\cdot p^2 + 4.03141\cdot p + 0.951226\right)}{57.9304\cdot p^4 + 62.8619\cdot p^3 + 29.8244\cdot p^2 + 7.70912\cdot p + 1}. \qquad (19)$$

Based on the resulting transfer function, a system of approximating linear ordinary differential equations can be built.

To assess the error of approximation, we shall find, by using the inverse Laplace transform, a solution in the space of the originals at a single stepped impact:

$$f_{ex}(t)=(0.246755+2.64605\cdot j)\times$$
$$\times\exp\left[(-0.381805-0.135624\cdot j)t\right]+$$
$$+(0.246755-2.64605\cdot j)\times$$
$$\times\exp\left[(-0.381805+0.135624\cdot j)t\right]-$$
$$-(2.62481+2.61405\cdot j)\times$$
$$\times\exp\left[(-0.160759-0.281611\cdot j)t\right]-$$
$$-(2.62481-2.61405\cdot j)\times$$
$$\times\exp\left[(-0.160759+0.281611\cdot j)t\right]+4.75612\ . \qquad (20)$$

Here, j is an imaginary unit.

The comparison of calculation results based on (20) and the baseline data has shown that the relative error of approximation does not exceed 5.5 % over the entire examined range. The error is determined relative to the magnitude of the stable mode (f_{ex}=5).

6. Discussion of results of studying the applicability of the proposed approximation method for building an object model

Typically, when processing experimentally derived acceleration curves of the control object, the structure of the model is chosen by the researcher (the adjuster) from a set of standard ones based on subjective judgments. Given this, there is ambiguity in determining the structure of the control object model. The proposed approach makes it possible to formalize determining the type of model.

In the construction of the approximating transfer function (19), no requirements were put forward for the nature of the process under consideration and the original "experimental" data in its description. The study results demonstrated that the proposed method could be used to obtain an approximation model of the control object. Under these conditions, the resulting maximum error of approximation of the solution does not go beyond the acceptable limits in engineering calculations. To determine the possible causes of errors, consider (Fig. 3) a change in the initial data and the results of the calculation over a longer period than the one in Fig. 2.

Here:

– dependences (2) to (4) are derived from the analytical solution of form (20) of the approximation models with the transfer function of form (5) when choosing for approximation, respectively, the third, fourth, and sixth powers of the polynomial in the denominator;

– dependence 5 is derived from the tenth-power polynomial of form (3) used to approximate the acceleration characteristic for (18) when simulating the original data.

A model with the transfer function that has the fourth power of the denominator – Fig. 3, dependence (3), describes the examined process with less error than the model with the third power – Fig. 3, dependence (2). However, the sixth power in the denominator of the transfer function leads to a significant increase in the error of approximation. That means having the optimal order of the approximating transfer function (and, accordingly, the model in the space of the originals). One should consider the value of the original data calculated using the approximating polynomial – Fig. 3, dependence (5). They adequately reflect the experimental initial data only up to second 40, Fig. 3, dependence (5). Their further deviation from the initial experimental data may cause an increase in the error of Padé approximant (7) with an increase in the order of the transfer function – Fig. 3, dependence (5). In this case, the error of the data obtained as a result of approximation by the polynomial increases, rather than the error of the mathematical model. Despite the nature of the original data to a Padé approximant – Fig. 3, dependence (5), it should be noted the properties of transitional characteristics of the approximation models – Fig. 3, dependences (2) to (4). They do not contradict the original experimental data over the entire period under consideration.

To confirm that an approximation object model is obtained rather than a process, consider the reaction of the object with the same original model when submitting a disturbing impact of another form to the input. As an approximation model, we shall adopt transfer function (19), and as an input impact – a signal of the $f_{ent}=e^{-t}$ form. Such a signal can be considered the approximation of an actual pulse impact with the representation in the mapping space in the following form:

$$f_{ent}=\frac{1}{p+1}\ . \qquad (21)$$

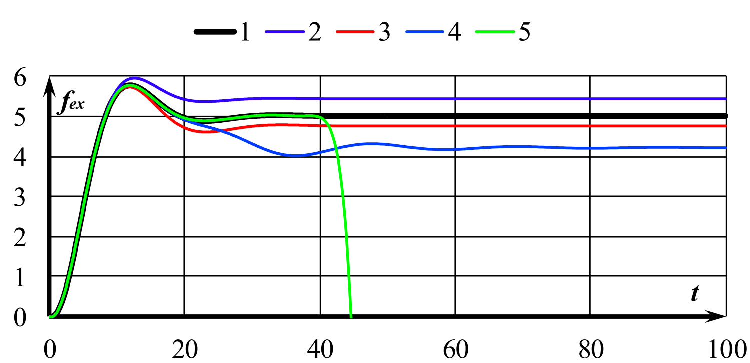

Fig. 3. Displaying the response of a simulated control object to a stepped impact: 1 – an experimental acceleration curve; 2 – a polynomial of the third power of the denominator' transfer function; 3 – a polynomial of the fourth power of the denominator's transfer function; 4 – a sixth-power polynomial of the transfer function of the denominator; 5 – original 10-power approximating polynomial (3)

In this case, the differential equation derived from transfer function (18) and considered as the original model of the object at a given impact will take the following form:

$$9\cdot f_{ex}'''+12\cdot f_{ex}''+4\cdot f_{ex}'+f_{ex}=5\cdot e^{-t}. \qquad (22)$$

Its analytical solution:

$$f_{ex}(t)=-\frac{5}{49}\cdot e^{-t}\cdot\begin{pmatrix}-7t-11\sqrt{3}\cdot e^{\frac{5t}{6}}\cdot\sin\left(\frac{t}{2\sqrt{3}}\right)+\\ +15\cdot e^{\frac{5t}{6}}\cdot\cos\left(\frac{t}{2\sqrt{3}}\right)-15\end{pmatrix}. \qquad (23)$$

The approximation model in the mapping space using transfer function (19) and impact (21) takes the following form:

$$\bar{f}_{ex}(p)= =(5)\cdot\left(\frac{-1.14766\cdot p^3+1.86912\cdot p^2+4.03141\cdot p+0.951226}{57.9304\cdot p^4+62.8619\cdot p^3+29.8244\cdot p^2+7.70912\cdot p+1}\right)\cdot\left(\frac{1}{p+1}\right). \qquad (24)$$

Using the inverse Laplace transform, we find a solution in the space of the originals at a given impact:

$$\begin{aligned}\bar{f}_{ex}(t)&=(0.761855-1.52122\cdot j)\times\\ &\times\exp[(-0.381805-0.135624\cdot j)t]+\\ &+(0.761855+1.52122\cdot j)\times\\ &\times\exp[(-0.381805+0.135624\cdot j)t]-\\ &-(0.753137-1.12878\cdot j)\times\\ &\times\exp[(-0.160759-0.281611\cdot j)t]-\\ &-(0.753137+1.12878\cdot j)\times\\ &\times\exp[(-0.160759+0.281611\cdot j)t]-\\ &-0.0174342\cdot\exp(-t).\end{aligned} \qquad (25)$$

A graphic image of solutions (23) and (25) is shown in Fig. 4.

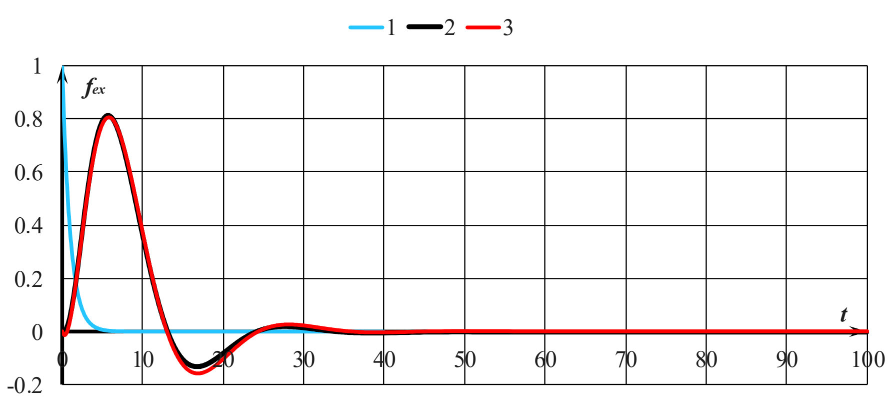

Fig. 4. Displaying the response of a simulated control object to the impact of form (11): 1 – external impact; 2 – analytical model (23); 3 – approximating model (24)

The comparison of calculation results shows an adequate representation by an approximating model of the system's reaction when simulating a pulse influence. The margin of error is not greater than 3.6 % relative to the maximum value of the transitional characteristics. The deviation, as is the case of a stepped impact, can be explained by the error in representing the original data when constructing approximation model (19). Good agreement between calculation results allows us to consider the approximation model of form (19) as a model of an object, not a process.

The method developed to represent an object approximation model makes it possible to formalize the process of determining the type of structure of this model to ensure that it can be performed by the parametric identification.

Comparing the proposed approximation method with those applied in setting the automatic control systems, the following can be noted. The engineering methods currently in use make it possible to find the parameters of transfer functions for objects, in fact, of only two classes. Namely: inertial not oscillatory with a lag, and astatic with a lag. The order of inertia is usually the first or second. The class of oscillatory objects is not considered when setting up industrial controllers. Therefore, when faced with similar transitional processes in the system at an industrial site, the set-up can be guided only by intuition and personal experience of adjusting controllers.

The approximation method discussed in this article makes it possible to approximate a control object and, based on the established parameters of the model, to calculate the settings of the controller, without using specialized mathematical packages, such as MATLAB or Mathcad.

The diagrams in Fig. 3 suggest that the accuracy of the approximation model may be enhanced by changing the way the original data are represented. As one of them, the use of lattice functions may be proposed. This determines the direction of further research in the development of methods for building approximation models.

7. Conclusions

1. Based on approximation in the form of a power series of the experimental acceleration characteristic, using a Laplace transform, we have derived its expression in the representation space in the form of a truncated series. The result of representing this series in the form of a Padé approximant is a rational fraction, a transfer function of a certain linear model. In the original space, it can be represented as several linear differential equations, which are the approximation model of the examined control object.

2. An important part of the method for representing a control object model as a set of linear differential models is the technique to determine an approximation model. The technique, using the model of two sequential inertial links of the first and second order, was used to simulate experimental data and did not influence the results of the

transformations. The solution demonstrated the possibility of approximation of arbitrary original models.

Our study involved the resulting approximation model and an external impact, which differs from the original one in the form of a stepped influence. Regardless of the type of external impact, a small margin of error was achieved from the results of modeling a control object's response compared to the accurate data available. The margin of error is not greater than 3.6 % relative to the maximum value of the transitional characteristic. This indicates that we have obtained the approximating model of an object, not a process.

Applying the method considered when setting up automatic controllers makes it possible to approximate control objects with a non-monotonous acceleration curve, which until now has not been considered in practical engineering calculations. For the specified control objects, the controllers are set up by empirical methods, which does not warrant the predefined quality of the transition process.

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