UDC 514.18:621.165
DOI: 10.15587/1729-4061.2021.224924

DEVELOPMENT OF A METHOD FOR GEOMETRIC MODELING OF CENTRIFUGAL COMPRESSOR IMPELLERS

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The practice of designing centrifugal compressors in which impellers are the main components shows that there are reserves for their further improvement. One of the main reserves consists in improving flow conditions for the compressed medium in the compressor flow path and, above all, in the impeller. A method of geometric modeling flow path of the impellers of centrifugal compressors was proposed which involves the construction of meridional boundaries of impellers and the blade profile on an involute of the cylindrical surface of the outer radius of the impeller. The blade is represented by ruled surfaces. The shroud of the impeller is described by a curve in natural parameterization using cubic dependence of curvature on the arc length. Dependences and length of the arc are determined in the process of modeling the boundary based on the set source data. The problem is solved by minimizing deviations of intermediate curves from the boundary endpoint. The hub is obtained as an envelope of circles inscribed in the meridional channel of the impeller. Radii of the circles are determined taking into account the flow areas of the channel. The camber line of the blade profile on an involute of the cylindrical surface of the outer radius of the impeller is modeled using a curve that is presented in natural parameterization with quadratic law of curvature distribution. A computer code was developed in the Fortran Power Station programming environment that visualizes the obtained numerical results graphically on a computer display in addition to digital information on the modeled boundaries and the blade profile. Graphical results were presented. They confirmed the efficiency of the proposed method of modeling the flow path of centrifugal compressor impellers. The method can be useful to offices involved in the design of centrifugal compressors

Keywords: centrifugal compressor, impeller, shroud and hub, blade, modeling

Received date 08.12.2020
Accepted date 05.02.2021
Published date 19.02.2021

1. Introduction

Centrifugal compressors began to spread in various fields of technology in the early 1920s. Naturally, their scope of use was limited because of their low efficiency. This was caused by the complexity of manufacturing, primarily impellers, in which main energy transformations take place. Centrifugal compressors were constantly improved during their existence. The achievements in understanding the gas-dynamic processes occurring in their settings have contributed to this process. Advances in the technology of manufacturing impellers with blades of complex spatial geometric shapes were of high importance.

Centrifugal compressors are used in several industries: ferrous and nonferrous metallurgy, chemical, petrochemical, coal, gas, mechanical engineering, energy, cryogenic, refrigeration, nuclear technology, and air conditioning [1]. They play an important role in pressure boost units of internal combustion engines. The space industry is a new field of use of centrifugal compressors [2]. These compressors are used in the aviation industry [3] and in low-power utility equipment [4]. The following examples of centrifugal compressors should be pointed out: two-stage compressors for helicopter engines with a degree of pressurizing of 14 [5] and single-stage compressors with a degree of pressurizing of 10 [6].

The known designs of centrifugal compressors having impellers with the medium flow on both impeller sides [7] can double-flow rates of the compressed medium.

Expansion of the use of multi-shaft integrated centrifugal compressors [8] which have certain advantages over single-shaft ones is the current trend in the compressor industry.

The development of new designs of centrifugal compressors is inextricably linked with the solution of a large number of diverse problems including thermogas-dynamics, strength-related, design-and-layout, technological, and other problems in the first place. A comprehensive solution to these problems contributes to the creation of highly efficient centrifugal compressors for various purposes in transport and industrial power engineering.

Despite the progress achieved in the research, design, and manufacture of centrifugal compressors, there are still some reserves for further improvements aimed at the higher efficiency of these compressors. This primarily applies to impellers. The main task consists of improving the velocity field of the compressed medium and ensuring its flow continuity. It is known that speed distribution in centrifugal compressor impellers is significantly influenced by meridional boundaries which form a channel for medium movement in the meridional section of the impeller. The most important attention should be paid to the of the impeller shroud because the curvature of this boundary is much larger than that of the hub. It is the curvature that has a significant effect on the compressed medium speed.

No less important factors influencing the efficiency of the centrifugal compressors include the distribution of areas of meridional sections along the impeller path as well as the geometry of the blade profile on an involute of the outer radius of the impeller.

All this determines the relevance of studies in the line of developing new approaches to geometric modeling settings of the centrifugal compressor impellers.

2. Literature review and problem statement

The impeller in which main energy transformations take place is the most important component of a centrifugal compressor. Aerodynamic perfection of impellers in centrifugal compressors is determined by the geometry of meridional boundaries and blade profile on an involute of the cylindrical surface of the impeller's outer radius. Distribution of speed in the setting of the compressor and, as a consequence, the level of energy losses in the impeller depend on meridional boundaries and the blade profile.

When designing centrifugal compressors, uniform distribution of speed and pressure of the compressed medium along the flow path is sought for. This is realized, first of all, by appropriately modeling the shroud and hub of the impeller.

The influence of meridional boundaries of the centrifugal compressor impeller on its effective performance was studied in [9]. Meridional profile was formed by circular curves, i. e. arcs of circles for one of the impellers, and by arcs of elliptical curves for other impellers. It has been experimentally proven that a compressor with meridional boundaries in the form of elliptical arcs has the best effective performance. This is a consequence of a smooth change of the boundary curvature which is not the case when using circular curves with constant curvature. However, the formation of meridional contours by arcs of circles or ellipses at their joint point leads to an abrupt change in curvature which negatively affects the distribution of flow velocity and can lead to flow tear-off.

In [10], the meridional profile of the impeller of a centrifugal compressor was proposed to be described by Bezier curves on which four intermediate points were used in addition to two extreme points that did not change their positions. Thus, six points of the characteristic Bézier broken line were used at each boundary of the meridional circumference of the impeller.

Application of Bezier curves to modeling the meridional profile of the centrifugal compressor impeller should be considered promising. However, with the seemingly simple Bézier method analytically based on Bernstein's polynomials, its effective use is associated with certain problematic aspects. This relates to the appropriate placement of nodal points of the characteristic polyline which outlines the modeled curve in the first approximation. In addition, it should be borne in mind that the moving of even one node leads to a restructuring of the entire curve.

The position of intermediate points was determined in [10] by solving an optimization problem by an evolutionary, rather cumbersome, method. Efficiency indicators were taken as the target function which was determined when calculating the three-dimensional flow of the compressed medium in the compressor.

The authors of [11] described the meridional profile of a centrifugal compressor impeller by means of Bezier curves with eight nodal points but only two points were moved later. Their expedient position was determined by solving the optimization problem using a genetic algorithm in conjunction with an artificial neural network. Note that the authors modeled the meridional boundaries of the impeller without taking into account flow areas of the meridional channel which significantly slowed down obtaining a satisfactory result.

B-splines which are a more general case of Bézier curves were used in the simulation of the impeller blades in a centrifugal compressor [12]. B-splines, like Bézier curves, also depend on nodal points with all problematic issues inherent in Bézier curves. However, they are characterized by the fact that a change of a node position does not affect the curve as a whole. At the same time, when using B-splines, it is necessary to additionally determine the curve exponent.

Modeling of the setting of the centrifugal compressor impeller using non-uniform rational B-splines (NURBS) was proposed in [13]. To solve a similar problem, it was also proposed in [14] to use NURBS. Note that NURBS curves are based on preset nodal points and weights. Appropriate location of nodal points and, in particular, determination of weights is a problematic issue. Its solution requires the use of optimization methods with a very large number of parameters to be optimized. For a one-extreme function, this problem is solved relatively quickly and accurately. However, goal functions have several extrema in practice. Searching for a global extremum is quite a time-consuming task that does not always end effectively. It happens very often that an optimization algorithm ends the search on the local extremum.

It was proposed in [15] to represent an initial section of the hub of the impeller channel by segments of straight lines with ensuring zero order of smoothness. The issue of designing an impeller blade of a centrifugal compressor with a certain thickness was considered in [16]. The use of linear sections at the meridional boundaries is impractical from a gas-dynamic point of view, especially when the flow is diffuse in nature. Segments of straight lines are poorly streamlined.

Based on the above analysis, it can be concluded that the outer meridional boundary should be modeled with a smooth curvature distribution. It is desirable that the equation of the applied curve allows double differentiation. These derivatives are necessary when calculating the flow of the medium being compressed. The hub of the meridional channel should be determined taking into account the areas of passage. There is no information on the construction of a blade profile in the considered publications.

3. The aim and objectives of the study

The study objective consisted of developing a method of geometric modeling of an impeller flow path for a centrifugal combined-flow compressor having blades of spatial shape limited by ruled surfaces. Generatrixes of these surfaces are perpendicular to the impeller rotation axis. This will enable the impeller fabrication both by casting and milling.

To achieve this goal, the following tasks were set:

– construct shroud of the meridional section of a centrifugal compressor impeller using a curve represented in natural parameterization and the cubic law of curvature distribution depending on arc length;

– construct hub of the meridional section of the impeller as an envelope of circles inscribed in a channel formed between shroud and hub of the meridional section of the impeller;

– construct the blade profile on an involute of a cylindrical surface of the outer radius of the impeller with ensuring the possibility of describing the blade with ruled surfaces;

– construct a spatial image of the centrifugal compressor impeller.

4. Modeling of the shroud of the centrifugal compressor impeller

Construction of meridional boundaries of the centrifugal compressor impeller should start from the shroud because its curvature is larger than that of the hub which significantly affects the distribution of velocities in the flow part. This boundary should smoothly change its curvature and have zero values at the start and endpoints.

As a result of the gas-dynamic calculation of the centrifugal compressor, the overall dimensions of the impeller sets become known. In particular, this determines the position of extreme points P_1 and P_2 of the modeled shroud. To construct this envelope angles φ_1 and φ_2 are set for design reasons. These are the angles of inclination of tangents at the impeller entrance and exit, respectively.

Similar to [17] where a method of geometric modeling of the *S*-shaped camber line of the axial compressor blade profile was proposed, the outer meridional boundary of the centrifugal compressor impeller will be described by a curve represented in natural parameterization. Boundary conditions for modeling this impeller boundary will be as follows:

a) at $x=0$; $y=0$; $\mathrm{d}y/\mathrm{d}x=\mathrm{tg}\varphi_1$; $k=0$;

b) at $x=x_m-y=0$; $\mathrm{d}y/\mathrm{d}x=\mathrm{tg}\varphi_2$; $k=0$.

Under these boundary conditions, the specified meridional boundary will be represented by a curve with its curvature k obeying the law of cubic distribution depending on the curve arc length:

$$k=as^3+bs^2+cs+d, \tag{1}$$

where s is the length of the modeled curve arc; a, b, c, d are unknown coefficients that are used in the process of constructing the desired curve line.

To construct the curve line, it is necessary to know four coefficients and the arc length S. To find these unknowns, there are three expressions [18]:

$$\varphi(s)=\varphi_1+\int_0^s k(s)\mathrm{d}s; \tag{2}$$

$$x(s)=x_1+\int_0^s \cos\varphi(s)\mathrm{d}s;$$

$$y(s)=y_1+\int_0^s \sin\varphi(s)\mathrm{d}s, \tag{3}$$

where φ_1, x_1, y_1 are the angle of inclination of the tangent and coordinates of the starting point of the modeled curve, respectively.

Since the curvature of the curve at the start and endpoints is zero, the coefficient d will also have zero value.

Find the coefficient a from expression (1) for $s=S$:

$$a=-\left(\frac{b}{S}+\frac{c}{S^2}\right),$$

where S is the length of the curve of the impeller's outer meridional section.

Integrate expression (2) taking into account dependence of curvature distribution (1) and obtain the expression for determining the angle of tangent inclination to the outer meridional boundary of the centrifugal compressor impeller. For the endpoint of the modeled boundary, this expression will take the following form:

$$\varphi_2=\varphi_1+\frac{aS^4}{4}+\frac{bS^3}{3}+\frac{cS^2}{2}. \tag{4}$$

Find coefficient a from the following expression:

$$a=\frac{4(\varphi_2-\varphi_1)}{S^4}-\frac{4b}{3S}-\frac{2c}{S^2}.$$

Having two expressions for the coefficient a, determine the coefficient b and finally the coefficient a:

$$a=-\frac{12(\varphi_2-\varphi_1)}{S^4}+\frac{2c}{S^2};$$

$$b=\frac{12(\varphi_2-\varphi_1)}{S^3}-\frac{3c}{S}.$$

Thus, the number of unknowns for modeling the curve is reduced to two: the coefficient c and the curve arc length S. These unknowns are found by solving the minimization problem associated with bringing the curve to a given endpoint P_2.

To demonstrate the possibility of modeling curves with zero values of curvature at the start and endpoints, Fig. 1 shows three curves that have zero curvature values at the endpoints. The curves were modeled with a gradual increase in the angle φ_1 from 30° to 50° in steps of 10°. The angle φ_2 decreased with the same step from –40° to –60°. At the endpoints of the curves, there are segments of lines that are tangent to them.

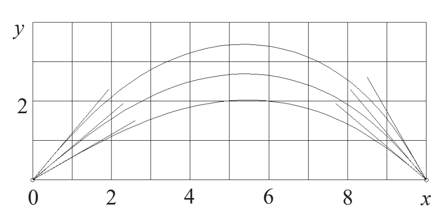

Fig. 1. Examples of curves constructed with zero curvature values at the endpoints

In order to visually confirm zero curvature values at the endpoints of the curve, Fig. 2 shows graphs of the distribution of curvature depending on the relative length of the curves.

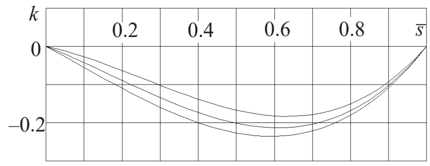

Fig. 2. Graphs of curvature distribution of test curves with zero values at endpoints

Fig. 3 shows a sketch of the centrifugal compressor impeller with a shroud.

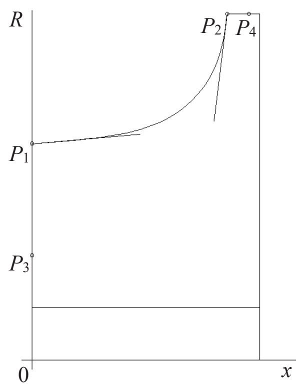

Fig. 3. Sketch of the meridional section of the impeller with the shroud of the channel

Tangents to the boundary are drawn at points P_1 and P_2. Values of the angles φ_1 and φ_2 can be visually estimated with their help. Points P_1 and P_2 correspond to the start and endpoints of the hub of the impeller.

5. Modeling the hub of the centrifugal compressor impeller

The hub of the channel of the compressor setting is constructed in such a way as to provide the desired law of change of flow areas f along the channel midline. Since areas f_1 at the inlet and f_2 at the outlet of the impeller are known from the results of gas-dynamic calculation of the compressor, the simplest solution consists of adopting a linear law of distribution of flow areas. However, the calculations showed that there is a so-called redundancy of areas under this law of distribution of flow areas at the impeller exit. This leads to the intersection of the hub of the channel with a plane drawn through the point P_4 parallel to the impeller disk. A similar situation is undesirable because it reduces the strength characteristics of the impeller and can cause its destruction.

Under these circumstances, adopt the quadratic law of area distribution. However, these actions require specifying the third additional point. The flow area at this point will be defined as some value of the half-sum of areas f_1 and f_2. Therefore, the area f_ξ will be found from the following expression:

$$f_\xi = \xi \frac{f_1 + f_2}{2},$$

where ξ is the coefficient that affects the flow area distant from the starting point of the shroud of the meridional section of the impeller by a distance σ and is defined as a fraction of the outer boundary arc length.

Therefore, take the law of distribution of areas in the form:

$$f = a_0 + a_1 s + a_2 s^2,$$

where a_0, a_1, a_2 are unknown coefficients to be determined; s is the relative length of the arc of the outer meridional envelope.

We have $a_0 = f_1$ for $s=0$, $a_0 + a_1 + a_2 = f_2$, for $s=1$ and finally $a_0 + a_1\sigma + a_2\sigma^2 = f_\xi$ for $s=\sigma$. When solving a system of linear equations with two unknowns, the following is obtained:

$$a_1 = (f_\xi - f_1)(1+\sigma)/\sigma;$$

$$a_2 = \left[\sigma(f_2 - f_1) - (f_\xi - f_1)(1+\sigma)\right]/\sigma.$$

Next, determine tangent inclination angle φ for some value of length s of the arc of the outer meridional envelope. To this end, substitute length s of the arc to expression (4). The center of the inscribed circle is located on the line perpendicular to the tangent at a distance r. Find radius r of the inscribed circle for the following reasons.

Express the flow area as $f = 4\pi r_m r$ where r is the radius of the circle inscribed in the meridional channel of the impeller; r_m is the distance from the impeller axis to the center of the inscribed circle. Next, express r_m through radius r_A which determines the position of point A, that is, the point of contact of the inscribed circle with the shroud of the channel $r_m = r_A - r\sin\alpha$. The angle α is equal to $\alpha = \pi/2 - \varphi$. As a result of these actions, we will have an expression for the calculation of the radius of the circle inscribed in the channel:

$$r = \frac{r_A - \sqrt{r_A^2 - f\sin\alpha/\pi}}{\sin\alpha}. \quad (5)$$

Knowing the radius of the inscribed circle, find coordinates of its center.

Fig. 4 shows a test example of an impeller channel with inscribed circles.

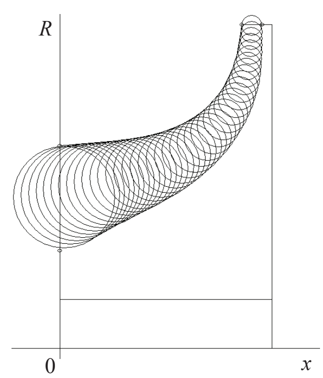

Fig. 4. Impeller channel with inscribed circles

Coordinates of the hub of the impeller channel are determined using equations of families of circles of radius r with centers located on the midline of the channel:

$$F(x,y,s) = (x - x_m)^2 + (y - y_m)^2 - r^2 = 0; \quad (6)$$

$$F'(x,y,s) = -2(x-x_m)x'_m - 2(y-y_m)y'_m - 2rr' = 0, \qquad (7)$$

where x_m, y_m are coordinates of the points of the channel midline.

Solve the system of equations (6), (7) with respect to coordinates x and y of the envelope points. As a result, the envelope equation is obtained in a parametric form: $x=x(s)$, $y=y(s)$.

Express $(x-x_m)$ from equation (7):

$$(x - x_m) = -\frac{rr' + (y-y_m)y'_m}{x'_m} = -\left[rw + q(y-y_m)\right], \qquad (8)$$

where

$$w = \frac{r'}{x'_m}; \quad q = \frac{y'_m}{x'_m}.$$

Substitution of formula (8) to equation (6) and further transformations give a quadratic equation with respect to $(y-y'_m)/r$:

$$\left(\frac{y-y'_m}{r}\right)^2 + 2\frac{wq}{r}\left(\frac{y-y'_m}{r}\right) - \frac{1-w^2}{1+q^2} = 0.$$

The solution of this equation will finally take the following form:

$$y_{1,2} = y_m - \frac{r}{1+q^2}\left(wq \mp \sqrt{1+q^2-w^2}\right). \qquad (9)$$

Substitution of the solution of (9) to equation (8) and further transformations give the following:

$$x_{1,2} = x_m - \frac{r}{1+q^2}\left(w \pm q\sqrt{1+q^2-w^2}\right). \qquad (10)$$

Since the quantities included in the right-hand side of solutions (9) and (10) are functions of the parameter s, they represent the parametric equation of the enveloping family of circles and, therefore, are the problem solution.

Determining the coordinates of points from formulas (9) and (10) is possible provided that the derivatives x'_m, y'_m and r' from the arc length s are known. To calculate them, describe coordinates x and y of the midline of the impeller channel and radii r of the inscribed circles with an interpolation-approximation spline of odd degree depending on the arc length s. The spline is constructed using the Anselon-Laurent algorithm. Its main provisions are set out in [19]. Fig. 5 shows the meridional profile of the centrifugal compressor impeller.

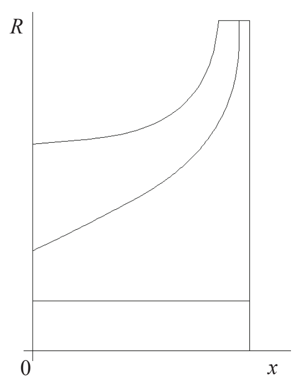

Fig. 5. The meridional profile of the impeller

It can be seen that the inner boundary at the impeller exit does not cross the impeller disk.

6. Construction of the blade profile on an involute of the cylindrical surface of the impeller's outer radius

The camber line T_0T_1 (Fig. 6) of the blade profile on an involute of the outer radius of the impeller is constructed provided that its curvature k is subject to the quadratic distribution law depending on the arc length:

$$k = a_1s^2 + b_1s + c_1, \qquad (11)$$

where s is the length of the modeled curve arc; a_1, b_1 and c_1 are unknown coefficients that are used in the process of curve modeling.

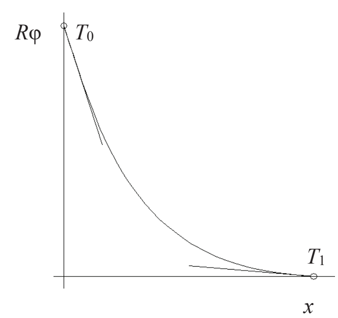

Fig. 6. Camber line of the blade profile on an involute of the cylindrical surface of the outer radius of the impeller

To determine curvature at the camber line endpoint, it is necessary to find three unknown coefficients a_1, b_1 and c_1 of the law of curvature distribution (11) and length S of the modeled camber line arc. To find them, three integral equations (2) and (3) are used.

The fourth condition necessary for the construction of the camber line is ensuring the minimum value of the difference between curvature at the end and starting points of the camber line, namely, k_{T_1} and k_{T_0}.

This allows us to find expression for the coefficient b_1:

$$b_1 = -2a_1S_1.$$

At the starting point of the curve where $s=0$, the coefficient c_1 of the dependence (11) is equal to the curvature of the curve k_{T_0}, that is, $c_1 = k_{T_0}$. Integration of expression (2) taking into account the law of distribution of curvature (11) and that $c_1 = k_{T_0}$, gives the following:

$$\varphi_{T_1} = \varphi_{T_0} + \frac{a_1S_1^3}{3} + \frac{b_1S_1^2}{2} + k_{T_0}S_1.$$

Taking into account dependence for the coefficient b_1, the expression for calculating the coefficient a_1 is obtained:

$$a_1 = \frac{3}{2S_1^2}\left(k_{T_0} - \frac{\varphi_{T_1} - \varphi_{T_0}}{S_1}\right).$$

With this in mind, the expression for the coefficient b_1 will finally take the following form:

$$b_1 = \frac{3}{S_1}\left(\frac{\varphi_{T_1} - \varphi_{T_0}}{S_1} - k_{T_0}\right).$$

Thus, to construct the camber line, it is necessary to find the value of the arc length S_1 and curvature k_{T_0} at point T_0. Using equation (3) to determine the position of the endpoint T_1, determine unknown lengths of the arc S_1 and curvature k_{T_0} at point T_0 by minimizing deviation from the set point.

The angle of inclination of the tangent at point T_1 can be zero, then the blade will be perpendicular to the impeller disk. To ensure the inclination of the blade to the impeller disk, some non-zero value can be set to the angle of tangent inclination at point T_1.

Regarding the angle of inclination of the tangent at point T_0, it must be noted that this angle is calculated taking into account the twist of the blade relative to the radius at the impeller inlet. The angle at the inlet to the impeller obtained as a result of gas-dynamic calculation of the centrifugal compressor should be taken into account.

To obtain a real profile, i. e. a profile of a certain thickness δ, construct a series of circles of radius $\delta/2$ with centers located on the midline. Next, build envelopes of these circles by the method used to form the meridional channel of the impeller.

A schematic representation of the middle surface of the spatial blade of a centrifugal compressor with a ruled surface is shown in Fig. 7. Generatrix of this surface moves along the blade profile on an involute of the cylindrical surface of the impeller outer radius and its centerline around which the impeller rotates. Suction and pressure surfaces are constructed in the same way.

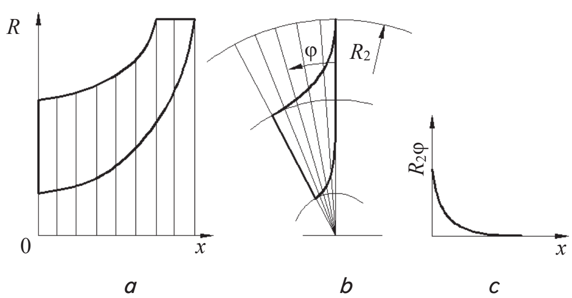

Fig. 7. Diagram of forming the impeller blade surface: *a* — meridional profile; *b* — view of the impeller disk; *c* — the profile camber line on an involute of the outer impeller radius

7. Construction of a spatial model of the impeller

A computer code was developed based on the proposed methods of modeling meridional envelopes and blade profile on an involute of the cylindrical surface of the outer impeller radius of the centrifugal compressor. As a result of the execution of the code, the user obtains results in numerical and graphical forms that are visualized on a computer display.

At the same time, a so-called script file containing AutoCAD commands is formed. Depending on the options of these commands, coordinates of points for constructing straight line segments, circle centers, and radii, etc. are set. To realize more complex actions, commands to move, rotate, group the objects, describe the surfaces, etc. are used. Construction begins with an impeller disk. One of its boundaries is the hub of the channel. To do this, the disk cross-section is formed which creates the disk model when rotated by 360°. Next, a blade model is formed by using shroud and hub and sections of lateral ruled surfaces. The blade is moved to the impeller disk and then the *array* command is applied to the disk a number of times corresponding to the blade number.

The AutoCAD design environment has a built-in AutoLISP functional programming language that converts the information contained in a script file into graphics images. It is clear that all actions are performed in a 3D space of the AutoCAD environment.

Fig. 8 shows an example of a set of cross-sections of the interblade channel of the centrifugal compressor impeller formed by planes perpendicular to the axis of impeller rotation. These data confirm the ruled nature of the lateral surfaces of the blades.

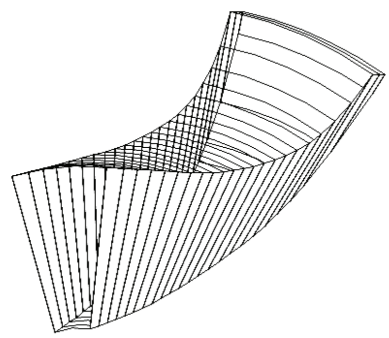

Fig. 8. Flat sections of the interblade channel of the impeller

To further confirm the efficiency of the proposed method, a test example of a spatial model of centrifugal compressor impellers was modeled. Wireframe and conceptual models of the centrifugal compressor impeller are shown in Fig. 9, *a*, and Fig. 9, *b*, respectively. The impeller blades were modeled with a constant thickness.

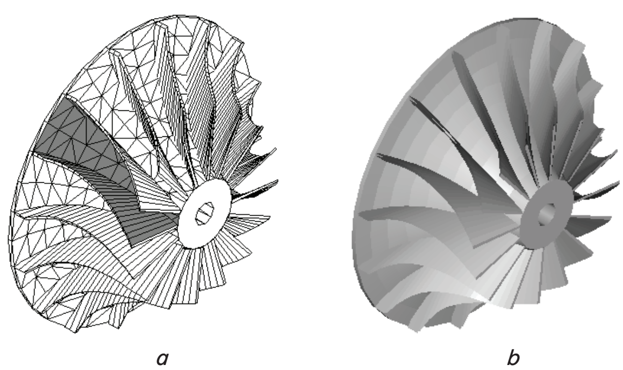

Fig. 9. Spatial models of the impeller: wireframe model (*a*); conceptual model (*b*)

Note that most of the figures in this study are screenshots obtained during the development of the program for modeling centrifugal compressor impellers.

These results are purely illustrative. Their purpose is to confirm the ability of the proposed method to model settings of the centrifugal compressor impellers.

8. Discussion of the proposed method of geometric modeling the centrifugal compressor impellers

A new approach to geometric modeling of meridional boundaries and blade profiles on an involute of the cylindrical surface of the outer radius of the impeller was proposed. The shroud was represented in natural parameterization (3) which provides for the construction of curves according to the law of curvature distribution depending on the arc length. A polynomial dependence of the third degree (1) in which four coefficients and the arc length are unknown was used in this study for modeling the outer meridional boundary. The number of unknowns was reduced to two by integrating the law of curvature distribution (4) according to known angles of tilt of tangents at initial and endpoints of the curve (Fig. 3). Values of these unknowns were found by solving the minimization problem meant for drawing a curve through a set endpoint. The obtained result is shown in Fig. 3. It is a screenshot image.

Based on the obtained shroud of the impeller channel, a set of tangent circles was constructed (Fig. 4). Radii of these circles functionally depend on the flow area of the impeller's meridional channel and the radius of the location of the inscribed circle centers on the channel midline. Radii of the circles are determined by expression (5). The set of circles inscribed in the meridional channel is shown in Fig. 4.

Expressions (9) and (10) to determine coordinates of the hub of the impeller's meridional channel were found by solving the system of equations (6) and (7). Both boundaries of the channel are shown in Fig. 5.

To construct the camber line of the blade profile on an involute of the cylindrical surface of the outer radius of the impeller, a curve represented in natural parameterization and according to the law of curvature distribution in the form of second-degree dependence (11) was used. The unknown coefficients and arc length according to this law were also determined by solving the problem of minimizing the deviation of intermediate endpoints of the curve from the set endpoint. The modeled midline is shown in Fig. 6.

Thus, a mathematical model of the setting of the channel of the centrifugal compressor impeller was obtained. By extending it to the number of channels less than the number of blades, a mathematical model of the setting of the centrifugal compressor impeller was obtained. Flat sections of one of the impeller channels (Fig. 8) with generatrixes of rectilinear shape and three-dimensional impeller images (Fig. 9) were visualized using AutoCAD.

The numerical methods used in the program code have made it possible to obtain results with an error of an order of 10^{-6} which is much less than the capabilities of modern equipment. This makes it possible to process blades of centrifugal compressors with a minimum tolerance of 0.03–0.05 mm.

For strength reasons, it is advisable to prevent the intersection of the hub of the impeller's meridional channel with a plane passing through the upper point of this boundary, perpendicular to the axis of the impeller rotation. This is achieved by an appropriate choice of coefficients ξ and σ of the law of distribution of flow areas of the impeller channel.

The development of a geometric model of the impeller blades bent in the direction opposite to the direction of the impeller rotation may be considered a continuation of the performed study.

9. Conclusions

1. A method of geometric modeling of shroud of channels of the centrifugal compressor impellers based on the natural representation of curves and the cubic law of curvature distribution was proposed. This provides zero values of curvature at the start and endpoints of the modeled curve and helps improve diagrams of the distribution of relative flow velocity of the compressed medium.

2. A method of constructing the hub of the impeller by entering in the channel a set of circles with positions of their centers and radii determined taking into account flow areas of the impeller's meridional channel was developed. The inner boundary of the meridional section of the impeller is obtained as an envelope inscribed in the channel of the circles. The projected increase in efficiency of the centrifugal compressor due to the proposed measures is up to one percent.

3. Spatial impeller blade is represented by ruled surfaces with their guideways being the blade profile built on an involute of the cylindrical surface of the outer impeller radius and two straight parallel axes of the impeller. The camber line of the blade profile on an involute of the cylindrical surface of the outer impeller radius is modeled using the natural parameterization of curves and the quadratic law of curvature distribution. The use of ruled surfaces for the construction of blades simplifies their manufacture by casting or milling in modern machining centers.

4. The constructed spatial images of the centrifugal compressor impeller can be used to visually assess the quality of the developed impeller by viewing it in an AutoCAD environment at various angles. Besides, the availability of spatial images makes it possible to load gas dynamics data into computational software systems to perform appropriate calculations.

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