DOI: https://doi.org/10.15276/hait.05.2022.5
UDC 62-52

The study of the properties of structural models of elements of educational environment

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ABSTRACT

The widespread use of mathematical modeling is often constrained by the insufficient level of technological maturity and the lack of theoretical foundations to justify the effectiveness of different classes of models. One of the ways to improve educational project management systems is to solve problems of modeling the structure and trajectory of changes in the parameters of the system in education management, as a complex poorly structured organizational and technical system. To solve the problem of analysis of structural schemes of complex technical systems, which include education systems, it is proposed to use the analytical method of determining closed cycles in complex control systems. The method is based on the use of specific properties of adjacency matrices. It is shown that the degrees of the adjacency matrix follow the general structure of the oriented graph with certain regularities of mapping the arcs of the graph. This allows you to create a matrix of reach of the studied topological structure with the selection of contours in an oriented graph. A method for identifying cycles in graphs based on the formation of the Boolean sum of degrees of the reach matrix with its subsequent transposition and superposition has been developed, which allows obtaining a mapping of a contour in a graph in the form of a square submatrix filled with units.

Keywords: Complex systems; directed graph; superposition matrix; adjacency matrix; analytical calculation; method of identifying cycles in graphs

***For citation*: Gogunskii V.D., Kolesnikova K. V., Kolesnikov O. Ye., Danso Janet O. The study of the properties of structural models of elements of educational environment. *Herald of Advanced Information Technology*. 2022; Vol. 5 No.1: 52–61. DOI: https://doi.org/10.15276/hait.05.2022.5**

INTRODUCTION

Informatization of society, the growth of the social role of the individual and the intelligence of his work in the rapidly transforming techniques and technologies require constant development and modernization of the educational environment to form and maintain relevant vocational education throughout life for everyone. Knowledge and information in the information society are becoming the main intellectual resource, but as the volume and rate of accumulation of knowledge is constantly and sharply increasing. Modern computerization of education is based mainly on the information approach, remaining essentially on the “manual” management of learning, which does not allow to fully individualizing this process.

Assessment of the level of student achievement is based on the subjective requirements of the teacher with a significant delay in time during the examination session. Lack of operational management of the learning process leads to a decrease in the overall level of training. Only the use of information technology and mathematical modeling tools will allow moving to a personal differentiated approach to student learning. This approach should be based on competency-oriented models and methods of forming the information environment of the university.

Known computer elements of training and control systems, as a rule, perform certain scientific and practical tasks and are not focused on the formation of a comprehensive system of information support and management of the current learning process, considering the modular structure of disciplines. Therefore, research, creation, and implementation of mathematical models for the study of the educational process, in the development of information environment, for the organization of knowledge control and decision-making based on the results of control is extremely important.

The learning process is implemented in a complex poorly structured system, which includes many heterogeneous subsystems that form a complex "web" of communications [1]. The development of an adequate deterministic formal description for such systems, in the general case, has no solution, as it is almost impossible to establish cause-and-effect relationships between results, resources and methods of organizational and technical interaction [2].

The learning process can be analyzed from the standpoint of the theory of self-organization of complex ordered systems, based on the properties of the synergetic approach [3]. Learning systems can be classified as nonlinear systems; because, for example, increasing the management influence in the form of the amount of educational material needed for study does not lead to a clear result in the form of improved quality. It is known that it is impossible to apply "rigid" management methods to complexly organized systems [4]. It is necessary to understand, contributing to their own development trends, how to bring systems to the rails of self-organization when external goals are related to the needs of these systems. The defining postulate of the synergetic approach is that guided development takes the form of self-government. In addition, the widespread dissemination of various types of educational information in electronic forms, on the one hand, leads to a pluralistic nature of ways to achieve learning goals, on the other hand, objectively leads to chaos of educational information [5]. In this regard, the formation of an individual learning strategy often leads to a unique curriculum for the student, with a chaotic accumulation of educational influences [6].

To solve the problem of analysis of structural schemes of complex systems, which are elements of the educational environment, it is proposed to use the method of analytical determination of cycles in complex control schemes. Unlike the well-known method of Leonard Euler, the cycle is determined by analytical calculation, not heuristic search [7]. The basis for the analytical solution of the problem is the use of the characteristic properties of the adjacency matrix [8].

The study of graph theory is associated with the names of prominent scientists Leonard Euler [9], Gustav Kirchhoff [10] and William Hamilton [11]. Leonard Euler determined the conditions for the existence of a cycle in a connected graph. Such a cycle, which contains all the edges of the graph without repetitions, in honor of Leonard Euler is now called Euler. The method known by Euler's theorem allows determining cycles on graphs as a result of heuristic search.

Euler's theorem: "A graph has a cycle if and only if it is connected and the degrees of all its vertices are even", is proved using the technique of coloring those edges of the graph, which have already been passed by traversing the vertices of the graph. This algorithm is difficult to formalize.

Kirchhoff's research used graphs to show the topological structures of electrical networks and created an algorithm for determining the maximum subgraph without loops (called a tree), which allowed to form an independent system of equations of the electric circuit.

Hamilton formulated his research: "Find a simple cycle containing all the vertices of the graph", which allowed the graphs to solve the famous classic problem of the salesman. The Hamilton cycle, in contrast to the Euler cycle, which correlates with the edges of the graph, is formed based on the vertices of the graph.

The solution of one of the most famous problems of the four-color problem technique is also based on the study of the properties of planar (flat) graphs, which allows you to draw maps or design printed circuit boards.

As is known, a system that unites sets of some entities, for example, $S\{s_1, s_2,\dots, s_m\}$, which are vertices of an oriented graph connected by oriented arcs and $G\ \{g_1, g_2,\dots, g_r\}$, can be displayed using the adjacency matrix $[c_{ij}]_S = [i,j]$, each row of which shows the connections of one vertex with other vertices of the graph. The element $c_{ij} = 1$ reflects the arc between the vertices S_i and S_j. If $c_{ij} = 0$, then the arc directly between the vertices of the graph i and j is absent.

The relations between the elements of the sets $S\{s_1, s_2,\dots, s_m\}$ and $G\ \{g_1, g_2,\dots, g_r\}$ can also be described as an incident matrix $[h_{ij}]_S$, $g = [i, j]$, the rows of which correspond vertices, and columns to the arcs of the oriented graph. In this case, the h_{ij} element is equal +1 if S_i is the initial vertex of the arc and (- 1) if S_i – is the final vertex of the arc [12].

Adjacency matrix, which has specific properties, is used to analyze structures [11]. In the case of successive reduction of the adjacency matrix in the degree n = 2, 3 ... the elements of the nth degree (c_{ij}) n show the path containing n arcs between the i-th and j-th vertices of the graph.

Multiplication of matrices is performed according to the usual rule [13]:

$$\left\|c_{ij}^{n+1}\right\| = \left\{\begin{matrix} c_{1.1}^{n} & c_{1.2}^{n} & \cdots & c_{1.m}^{n} \\ c_{2.1}^{n} & c_{2.2}^{n} & \cdots & c_{2.m}^{n} \\ \cdots & \cdots & \cdots & \cdots \\ c_{m.1}^{n} & c_{m.2}^{n} & \cdots & c_{m.m}^{n} \end{matrix}\right\} \times \left\{\begin{matrix} c_{1.1}^{1} & c_{1.2}^{1} & \cdots & c_{1.m}^{1} \\ c_{2.1}^{1} & c_{2.2}^{1} & \cdots & c_{2.m}^{1} \\ \cdots & \cdots & \cdots & \cdots \\ c_{m.1}^{1} & c_{m.2}^{1} & \cdots & c_{m.m}^{1} \end{matrix}\right\} =$$

$$= \left\{\begin{matrix} \sum_{k=1}^{m} c_{1k}^{n} c_{k1}^{1} & \sum_{k=1}^{m} c_{1k}^{n} c_{k2}^{1} & \cdots & \sum_{k=1}^{m} c_{1k}^{n} c_{km}^{1} \\ \sum_{k=1}^{m} c_{2k}^{n} c_{k1}^{1} & \sum_{k=1}^{m} c_{2k}^{n} c_{k2}^{1} & \cdots & \sum_{k=1}^{m} c_{2k}^{n} c_{km}^{1} \\ \cdots & \cdots & \cdots & \cdots \\ \sum_{k=1}^{m} c_{mk}^{n} c_{k1}^{1} & \sum_{k=1}^{m} c_{mk}^{n} c_{k2}^{1} & \cdots & \sum_{k=1}^{m} c_{mk}^{n} c_{km}^{1} \end{matrix}\right\}$$

where: n is the degree of the adjacency matrix; n=1, 2,…; m is the total number of vertices in the scheme.

To display the relationships between the elements of complex system, we use the following simplification: the presence of the relationship, which is determined from the above, we mean the value of the matrix element $[c^n_{ij}] = 1$. In the absence of a connection $[c^n_{ij}] = 0$.

That is, the operations of multiplication of matrices will be performed according to all the rules adopted in mathematics, and at the stage of displaying the results we will perform the transformation:

$$\|c_{ij}^{n+1}\| = \begin{cases} 1, \text{ if } \sum_{k=1}^{m} c_{ik}^{n} c_{kj}^{1} > 0 \; for \; \{\forall i,j \; \in 1,2,\dots,m\}; \\ 0, \text{ if } \sum_{k=1}^{m} c_{ik}^{n} c_{kj}^{1} = 0 \; for \; \{\forall i,j \; \in 1,2,\dots,m\} \end{cases}. \quad (1)$$

Published works on structural analysis of complex schemes provide, often without proof, recommendations in the form of algorithms for finding cycles [14, 15]. Structural analysis of complex systems is used in various fields of knowledge. In [16, 17] a structural analysis of the international standard of competencies for project managers NCB (National Competences Baseline) was carried out. The authors proved that the cycles in the NCB competency matrix are the basis for the formation of knowledge cores. In [18, 19] the modernization of the project-managed organization was performed on the basis of structural analysis. It is theoretically substantiated that the matrix diagram of project value indicators is strongly related [20, 21]. Structural analysis has become the basis for optimizing the structure of the enterprise [22, 23].

These examples show that the theoretical substantiation of methods for analyzing the structures of complex technical systems is an urgent task, since the structure of the system and information connections significantly affect to the results of research.

THE PURPOSE OF THE ARTICLE

The aim of the research is to improve the method of analytical selection of closed loops in oriented graphs of complex topological structures of educational systems.

To achieve this goal the following tasks are defined:

– investigate the properties of degrees of adjacency matrices of oriented graphs;

– to develop a method of identifying cycles in graphs based on the formation of the reachability matrix with its subsequent transposition.

MAIN PART

We will study the methods of representation of different structures using the adjacency matrix; consider the properties of adjacency matrices and its degrees in terms of applying these properties for structural analysis of design systems.

<u>*Lemma 1.*</u> Two arcs, one of which enters and the other exits one vertex, form two elements in the adjacency matrix, offset from the main diagonal by 1 column in the direction of the arcs, if and only if the three vertices of the oriented graph are represented by adjacent columns.

Evidence. According to the rule of displaying oriented graphs in the adjacency matrix, the line numbers correspond to the number of the vertex of the graph from which the arc emanates. And column numbers are the number of the vertex that includes the arc. Note that in any oriented graph having a contour, we can distinguish the linear part of the contour in the direction of the arcs of the oriented graph and the feedback (arc) that forms the cycle.

Since the vertex numbers of the oriented graph play rather the role of vertex identifiers and do not determine the mandatory order in the adjacency matrix, and do not affect the structure of connections between vertices, we assume that the vertices of the oriented graph can be numbered arbitrarily. Therefore, we impose a condition on assigning numbers to the vertices of the graph: in the linear subgraph s S the vertices are numbered in the direction of the arcs of the digraph.

Consider an oriented graph from the following vertices: *a, b, c, d, e, f, g*. Let the vertices of the graph be connected by the relations: $a \to b \to c \to d \to e \to f \to g$. Since the vertices of an oriented graph can be numbered arbitrarily, we assume the following numbering:

$$\begin{aligned} a &\to \{i\}; \\ b &\to \{i+1\}; \\ c &\to \{i+2\}; \\ d &\to \{i+3\}; \\ e &\to \{i+4\}; \\ f &\to \{i+5\}; \\ g &\to \{i+6\}. \end{aligned} \quad (2)$$

In this case, in the adjacency matrix, as a result of (1), the rows and columns corresponding to the vertices *a, b,…, g.*

Will be in series, and the values of the corresponding elements of the adjacency matrix will be as follows:

$$\begin{aligned} c_{i,i+1} &= c_{a,b} = 1; \\ c_{i+1,i+2} &= c_{b,c} = 1; \\ &\dots \\ c_{i+5,i+6} &= c_{f,g} = 1. \end{aligned} \quad (3)$$

The elements of the adjacency matrix defined in (2) are shifted by one column from the main diagonal.

ISSN 2663-0176 (Print)
ISSN 2663-7731 (Online)

That is, the arcs of the linear part of the oriented graph are reflected in the adjacency matrix by a diagonal parallel to the main one and shifted from it by 1 column, provided that the vertices are in the adjacency matrix sequentially in the direction of the oriented graph arcs (Fig. 1).

	a	*b*	*c*	*d*	*e*	*f*	*g*
a	0	1	0	0	0	0	0
b	0	0	1	0	0	0	0
c	0	0	0	1	0	0	0
d	0	0	0	0	1	0	0
e	0	0	0	0	0	1	0
f	0	0	0	0	0	0	1
g	0	0	0	0	0	0	0

Fig. 1. **The adjacency matrix of the fragment of the linear part of the graph**
***Source:* compiled by the authors**

Thus, an arc that does not form a diagonal parallel to the main one does not belong to the linear part of the arcs of the oriented graph. For example, in the case of a contour formed by an arc between the vertices $e \rightarrow b$, in the adjacency matrix element $[c_{eb}] = 1$, or taking into account the numbering (3) we get the value $[s_{i+4,\, i+2}] = 1$ (Fig. 2 and Fig. 3).

As can be seen from Fig. 2 and Fig. 3, the mapping of the arc between the vertices e → b in the adjacency matrix is carried out through the value of the element $[s_{i+4,\, i+2}] = 1$. This element forms a "triangle" with the linear part of the contour of the oriented graph.

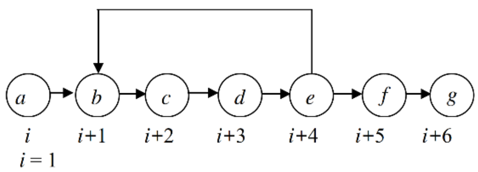

Fig. 2. **Oriented graph by the Adjacency Matrix with the arc e → b, which forms a cycle**
Source: compiled by the authors

The specified property of mapping cycles using the adjacency matrix is the basis for structural analysis.

<u>*Lemma 2.*</u> The elements of all columns of the contour, except the last one, of the adjacency matrix of degree n are shifted in degree n + 1 by one column in the direction of the edges of the oriented graph.

Evidence. Let's use the property of arbitrary numbering of vertices.

The nonzero elements of the adjacency matrix of degree n = 1:

$$c^1_{i,i+1}=1,\ i = k, k+1, \ldots, m-1;\ k \in 1,\ldots,\ m-1;\ c^1_{m,k}= 1, \tag{4}$$

where *k and m* are the initial and final vertices included in the contour, $k < m$.

	a	*b*	*c*	*d*	*e*	*f*	*g*
a	0	1	0	0	0	0	0
b	0	0	1	0	0	0	0
c	0	0	0	1	0	0	0
d	0	0	0	0	1	0	0
e	0	1	0	0	0	1	0
f	0	0	0	0	0	0	1
g	0	0	0	0	0	0	0

Fig. 3. **Oriented graph by the Adjacency Matrix with the arc *e* → *b*, which forms a cycle**
***Source:* compiled by the authors**

Find the elements of the adjacency matrix of degree n + 1:

$$c^{n+1}_{ij} = \sum_{h=k}^{m} c^n_{ih} c^1_{hj}, \quad j = 1, 2, \cdots, m; \quad i = 1, 2, \cdots, m. \tag{5}$$

Let's calculate the values of the elements of one of the rows *s{1, 2,…m}* of the adjacency matrix of degree *n + 1*:

$$\begin{aligned}
c^{n+1}_{s,1} &= c^n_{s,1} \cdot c^1_{1.1} + c^n_{s,2} \cdot c^1_{2.1} + c^n_{s,3} \cdot c^1_{3.1} + \cdots + c^n_{s,m} \cdot c^1_{m.1};\\
c^{n+1}_{s,2} &= c^n_{s,1} \cdot \left(c^1_{1,2}\right) + c^n_{s,2} \cdot c^1_{2,2} + c^n_{s,3} \cdot c^1_{3,2} + \cdots + c^n_{s,m} \cdot c^1_{m,2};\\
c^{n+1}_{s,3} &= c^n_{s,1} \cdot c^1_{1,3} + c^n_{s,2} \cdot \left(c^1_{2,3}\right) + c^n_{s,3} \cdot c^1_{3,3} + \cdots + c^n_{s,m} \cdot c^1_{m,3};\\
&\vdots\\
c^{n+1}_{s,m} &= c^n_{s,1} \cdot c^1_{1,m} + c^n_{s,2} \cdot c^1_{2,m} + \cdots + c^n_{s,m-1} \cdot \left(c^1_{m-1,m}\right) + c^n_{s,m} \cdot c^1_{m,m},
\end{aligned} \tag{6}$$

where m is the element number in the string.

The nonzero elements $s_{i,\, i+1} = 1$ (i = 1, 2,…, m – 1) of the adjacency matrix are highlighted in parentheses.

Rejecting the remaining elements, we obtain in the general case that the value of the element of the line *s{1, 2,… m}* for the linear part of the graph will be determined by the first factor:

$$c^{n+1}_{s,h} = c^n_{s,h-1}; \quad h = 2, 3, \cdots, m. \tag{7}$$

Graphical interpretation of the proof on the example of calculating the element of the matrix $[c^2_{2,4}]$ is shown in Fig. 4.

$$
\begin{array}{c|ccccccc}
 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ \hline
1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
3 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
4 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
5 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
6 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
7 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}
\times
\begin{array}{c|ccccccc}
 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ \hline
1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
3 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
4 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
5 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
6 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
7 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}
=
\begin{array}{c|ccccccc}
 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ \hline
1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
2 & 0 & 0 & 0 & \boxed{1} & 0 & 0 & 0 \\
3 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
4 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
5 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
7 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}
$$

Fig. 4. **The scheme of displacement of the element $[c^1_{2,3}] = 1$ by one column in the element of the matrix $[c^2_{2,4}] = 1$, which is the result of multiplication of matrices**

***Source:* compiled by the authors**

To determine the value of the element $[c^2_{2,4}]$, multiply the elements of row 2 and column 4 and determine the sum. As you can see, only two elements of the 2nd row $[c^1_{2,3}] = 1$ and the 4th column $[c^1_{3,4}] = 1$, have non-zero values. They will give values $[c^2_{2,4}] = [c^1_{2,3}] \times [s^1_{3,4}] = 1$.

In the general case, for an example in Fig. 3, we obtain:

$$
\begin{aligned}
&[c^2_{1,3}] = [c^1_{1,2}];\\
&[c^2_{2,4}] = [c^1_{2,3}];\\
&[c^2_{3,5}] = [c^1_{3,4}];\\
&[c^2_{4,6}] = [c^1_{4,5}];\\
&[c^2_{5,7}] = [c^1_{5,6}].
\end{aligned}
\tag{8}
$$

It is proved that the elements of all columns of the linear part of the oriented graph, except for the last one, of the adjacency matrix of degree n are shifted in degree n + 1 by one column in the direction of the edges of the oriented graph.

<u>*Lemma 3.*</u> In the degree n + 1 of the adjacency matrix, the elements of the last column of the contour of the degree *n* pass into the 1st column of the contour.

Evidence. Let be an oriented graph with a contour represented by a matrix of contiguity with the conditions accepted in Lemma 2.

Consider the formation of any column k of the adjacency matrix of degree n + 1. The elements of column k are calculated:

$$
\begin{aligned}
c^{n+1}_{1,k} &= c^n_{1,1}\cdot c^1_{1,k} + c^n_{1,2}\cdot c^1_{2,k} + \cdots + c^n_{1,m-2}\cdot c^1_{m-2,k} + c^n_{1,m-1}\cdot c^1_{m-1,k} + c^n_{1,m}\cdot c^1_{m,k};\\
c^{n+1}_{2,k} &= c^n_{2,1}\cdot c^1_{1,k} + c^n_{2,2}\cdot c^1_{2,k} + \cdots + c^n_{2,m-2}\cdot c^1_{m-2,k} + c^n_{2,m-1}\cdot c^1_{m-1,k} + c^n_{2,m}\cdot c^1_{m,k};\\
c^{n+1}_{3,k} &= c^n_{3,1}\cdot c^1_{1,k} + c^n_{3,2}\cdot c^1_{2,k} + \cdots + c^n_{3,m-2}\cdot c^1_{m-2,k} + c^n_{3,m-1}\cdot c^1_{m-1,k} + c^n_{3,m}\cdot c^1_{m,k};\\
c^{n+1}_{4,k} &= c^n_{4,1}\cdot c^1_{1,k} + c^n_{4,2}\cdot c^1_{2,k} + \cdots + c^n_{4,m-2}\cdot c^1_{m-2,k} + c^n_{4,m-1}\cdot c^1_{m-1,k} + c^n_{4,m}\cdot c^1_{m,k};\\
&\vdots\\
c^{n+1}_{m,k} &= c^n_{m,1}\cdot c^1_{1,k} + c^n_{m,2}\cdot c^1_{2,k} + \cdots + c^n_{m,m-2}\cdot c^1_{m-2,k} + c^n_{m,m-1}\cdot c^1_{m-1,k} + c^n_{m,m}\cdot c^1_{m,k};
\end{aligned}
\tag{9}
$$

For the system of equations (9) it is necessary to enter the initial conditions: the numbers of the beginning k and the end of the cycle r. For example, for the scheme in Fig. 2, such data will be $k = b = 2$ and $r = e = 5$. Under such conditions, the element of the 2nd factor $[c^1_{5,2}] = 1$. And the penultimate element of the contour in the first factor of the matrix $[c^{(n = 1)}_{4,5}] = 1$, as a consequence of Lemma 1 on the parallelism of the main diagonal of the matrix of elements of the linear part cycle.

Rejecting the zero elements from (9), and taking the values of the elements known by the initial conditions, we obtain for n = 1:

$$
c^2_{4,2} = c^1_{4,5}\cdot c^1_{5,2}. \tag{10}
$$

Graphical interpretation of the proof of Lemma 3 on the example of calculating the element of the resulting matrix $[p^2_{4,2}]$ is shown in Fig. 5.

To determine the value of element $[c^2_{4,2}]$ from (2), multiply the elements of row 4 and column 2 and determine the sum. The elements of the 4th row

$$
\begin{array}{c|ccccccc}
 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ \hline
1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
3 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
4 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
5 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\
6 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
7 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}
\times
\begin{array}{c|ccccccc}
 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ \hline
1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
3 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
4 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
5 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\
6 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
7 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}
=
\begin{array}{c|ccccccc}
 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ \hline
1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
3 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
4 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\
5 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\
6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
7 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}
$$

Fig. 5. **The scheme of "jumping" of the element $[c^1_{4,5}]$ from the contour of the first multiplier to the first column $[c^2_{4,2}] = 1$ as a result of multiplication of matrices**

***Source:* compiled by the authors**

and the 2nd column and the result of multiplication are highlighted in Fig. 5. They are in accordance with () will give the values: $[c^2_{4,2}] = [c^1_{4,5}] \times [c^1_{5,2}] = 1$.

Since the second factor does not change and always $[c^1_{r,k}] = 1$, in the general case for all columns, the elements representing the linear part of the oriented graph, in the case of ascent to the next steps will jump from the penultimate column (r -1) to the first column to the contour.

Lemma 4. The connections between the vertices of the graph through 1… n arcs reflect the degrees of the adjacency matrix from 1 to n, respectively.

Evidence. As defined in Lemma 2, the elements of all columns of the contour, except the last one, the adjacency matrices of degree n are shifted in degree n + 1 by one column in the direction of the edges of the digraph. That is, each n + 1 degree represents the connections from the i-th to the n + 1 vertex of the graph. Thus, the connections obtained on the basis of the 2nd degree of the adjacency matrix reflect the connections in the graph through one transit vertex (dotted line, Fig. 6).

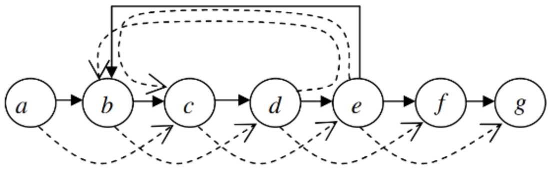

Fig. 6. **Mapping of relations in the adjacency matrix of the 2nd degree through one transit vertex of the graph**
***Source:* compiled by the authors**

As can be seen, the new connections connect those vertices that were connected by two arcs in the initial matrix (Fig. 5, Fig. 6 and Fig. 7). These conclusions are also true for the 3rd degree of the adjacency matrix, with the difference that the detected connections already pass through three arcs and two transit vertices of the graph (Fig. 6 and Fig. 7).

2nd degree		to the top						
		a	*b*	*c*	*d*	*e*	*f*	*g*
from the top	*a*	0	0	**1**	0	0	0	0
	b	0	0	0	**1**	0	0	0
	c	0	0	0	0	**1**	0	0
	d	0	**1**	0	0	0	**1**	0
	e	0	0	**1**	0	0	0	**1**
	f	0	0	0	0	0	0	0
	g	0	0	0	0	0	0	0

Fig. 7. **Mapping of connections in the adjacency matrix of the 2nd degree through one transit vertex of the graph: adjacency matrix of the 2nd degree**
***Source:* compiled by the authors**

As can be seen from Fig. 8, there is a certain pattern in the change of connections that are characteristic of different degrees of the adjacency matrix.

The elements of the adjacency matrix move from right to left (in the direction of the arc of the digraph). At the same time, the penultimate elements of the cycle pass a specific path (with skipping). These properties of the degrees of adjacency matrices allow us to hypothesize the possibility of calculating the contours in an oriented graph.

Let's develop a method of defining contours in a digraph. Assume that the Boolean sum of adjacency matrices of degrees 1 to m is a reachability matrix that forms a graph of all circuit paths, including a closed loop.

Evidence. We use the conclusions of Lemma 4. To obtain the matrix R^n of all paths of the oriented graph or the matrix of reach, we form a Boolean sum of all degrees of the adjacency matrix presented in Fig.8. The elements $[r_{ij}]$ of the reach matrix are determined using disjunction or conjunction operations.

The reachability matrix of the first rank $R^{(1)}$ is identical to the adjacency matrix C^1 of the first degree:

$$[r_{ij}^{(1)}] = [c_{ij}^{1}], \forall\ i, j \in \{1, 2, \ldots, m\}. \tag{11}$$

The reach matrices of the following ranks $R^{(n)}$ for values n> 1 are determined using the reach matrices of ranks (n - 1) and adjacency matrices of the corresponding degrees.

The reachability matrix R^n contains all connections from top *i* to *j* through n arcs of the graph.

As the degrees n of the adjacency matrices increase, the reachability matrix R^n becomes filled with units due to the validity of Lemma 2. The submatrix filled with units shows that all its vertices have a connection in the direction of the arcs of the graph. And this is a description of all possible paths in the digraph in the direction of the arcs of the graph. In some lines, the elements of the main diagonal (MD) of the reachability matrix have the value $[r_{ii}] = 1$, which is a sign that this line contains a description of part of the path in the digraph in the direction of oriented arcs from the element $i \rightarrow i$. The presence of such a path from the element and to and is possible in the cycle of the oriented graph. It should also be noted that some elements of the line i, in which there is a connection $i \rightarrow i$, are not included in the closed loop, because the direction of the arcs of the graph from the vertex and there is a path to the final vertices of the graph, for example for *f* and *g* (Fig. 2).

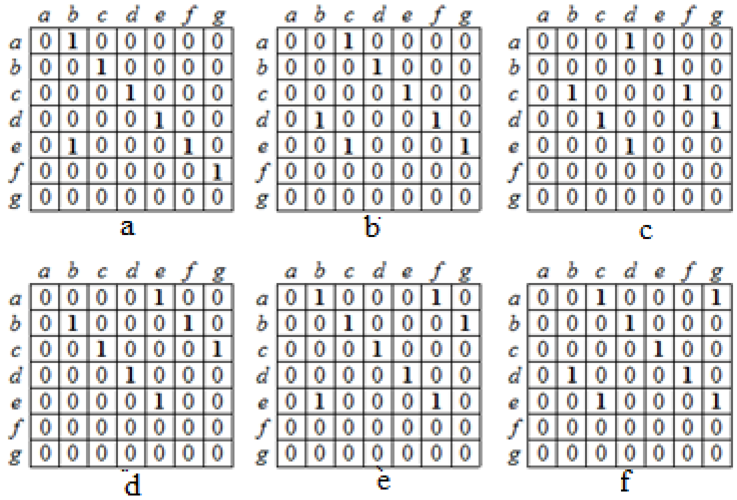

Fig. 8. **Offset of elements of the adjacency matrix in degrees from n = 1 to n = 6: a – n = 1; b – n = 2; c – n = 3; d – n = 4; e – n = 5; f – n = 6**

***Source:* compiled by the authors**

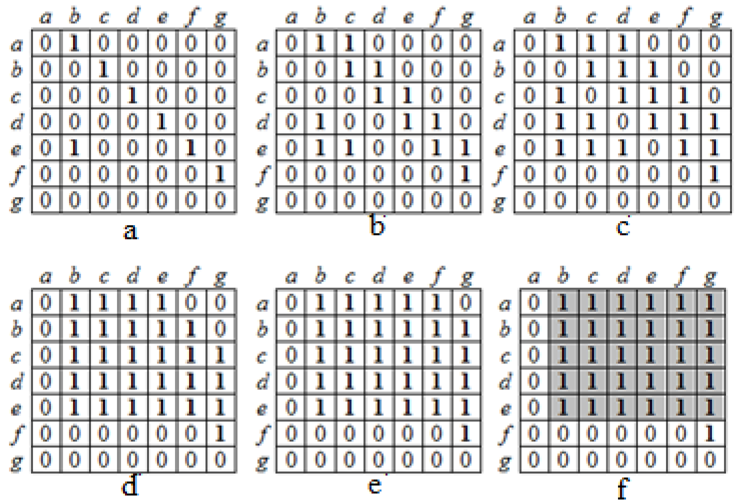

Fig. 9. **Reachability matrix Rn for different n: a – n = 1; b – n = 2; c – n = 3; d – n = 4; e – n = 5; f – n = 6**

***Source:* compiled by the authors**

To determine all subsystems that exist in the graph and are included in the circuit, we perform the change of directions to the inverse of all arcs of the graph by transposing the reachability matrix $R^n \rightarrow (R^n)^T$ followed by superposition $W = R \cap R^T$. The elements of the superposition matrix $W = R \cap R^T$ are formed using the operations of disjunction (logical "OR") or conjunction (logical "AND").

The nonzero elements of the MD of the matrix W point to the line containing all the paths of the circuit. Selected contours, in which all elements are related to all other elements, form the basis of the ergodic subset of the oriented graph. Not only the final result of the superposition matrix W^n is informative, but also the results that show the formation of closed loops.

Perform transposition of the reachability matrix $R^6 \rightarrow (R^6)^T$, which is shown in Fig. 9, followed by superposition $W^6 = R^6 (R^6)^T$:

As can be seen from the results of superposition (Fig. 10), the developed method allows determining the presence of a closed loop in an oriented graph, which includes the following vertices connected by connections: $b \rightarrow c \rightarrow d \rightarrow e \rightarrow b$.

CONCLUSIONS

In conditions of incomplete certainty of the processes taking place in educational systems, to assess their quantitative parameters, it is convenient to use graphical probabilistic models presented as a graph, the vertices of which reflect random variables, and the edges show the relationship between

$$
\begin{array}{c|ccccccc|}
 & a & b & c & d & e & f & g \\
\hline
a & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\
b & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\
c & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\
d & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\
e & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\
f & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
g & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
\end{array}
\quad
\begin{array}{c|ccccccc|}
 & a & b & c & d & e & f & g \\
\hline
a & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
b & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\
c & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\
d & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\
e & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\
f & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\
g & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\
\hline
\end{array}
\quad
\begin{array}{c|ccccccc|}
 & a & b & c & d & e & f & g \\
\hline
a & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
b & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\
c & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\
d & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\
e & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\
f & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
g & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
\end{array}
$$

$$\mathbf{R}^6 \qquad\qquad (\mathbf{R}^6)^T \qquad\qquad \mathbf{W}^6 = \mathbf{R}^6 \cap (\mathbf{R}^6)^T$$

Fig. 10. **The superposition matrix W6 = $R^6 \cap (R^6)^T$, which is obtained on the basis of the reachability matrix R^6 of example on Fig. 9**

Source: **compiled by the authors**

them. To solve the problem of analysis of structural schemes of complex technical systems, which include education systems, it is proposed to use the analytical method of determining closed cycles in complex control systems. The technique is based on the use of specific properties of adjacency matrices. It is shown that the degrees of the adjacency matrix follow the general structure of the oriented graph with certain regularities of mapping the arcs of the graph. This allows creating a matrix of reach of the studied topological structure with the selection of contours in an oriented graph. A method for identifying cycles in graphs based on the formation of the Boolean sum of degrees of the reachability matrix with its subsequent transposition and superposition has been developed, which allows obtaining a mapping of a contour in a graph in the form of a square submatrix filled with "1". Using the proposed analytical method, the resulting matrix of reachability R^n is obtained, which contains all connections from the vertex *i* to the vertex *j* through n arcs of the oriented graph. Based on it, it becomes possible to determine the presence of closed contours in an oriented graph and speak about the ergodicity of the system under consideration. This means that a transition from any state S_i to any state S_j is possible in a finite number of steps.

The proposed method for the analytical determination of closed loops will allow further parametric exploration of the system, adjusting the model to display the properties of a particular educational system, and analyzing the behavior of the system for various initial data.

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Conflicts of Interest: the authors declare no conflict of interest

Received 10.01.2021
Received after revision 04.03.2021
Accepted 14.03. 2021

DOI: https://doi.org/10.15276/hait.05.2022.5

УДК 62-52

Дослідження властивостей структурних моделей елементів освітнього середовища

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АНОТАЦІЯ

Широке застосування математичного моделювання часто стримується недостатнім рівнем технологічної зрілості і відсутністю теоретичних засад щодо обґрунтування ефективності різних класів моделей. Одним із напрямків поліпшення систем управління освітніми проектами є розв'язання проблем моделювання структури та траєкторії зміни параметрів системи при управлінні освітою, як складною слабо структурованою організаційно-технічною системою. Для вирішення задачі аналізу структурних схем складних технічних систем, до яких належать і системи освіти, запропоновано використовувати аналітичний метод визначення замкнених циклів в складних системах управління. Методика заснована на використанні специфічних властивостей матриць суміжності. Показано, що степені матриці суміжності наслідують загальну структуру орієнтованого графу з певними закономірностями відображення дуг графу. Це дозволяє створювати матрицю досяжності досліджуваної топологічної структури з виділенням контурів в орієнтованому графі. Розроблено методику ідентифікації циклів у графах на основі формування булевої суми ступенів матриці досяжності з подальшим її транспонуванням і суперпозицією, що дозволяє отримати відображення контуру в графі у формі квадратної підматриці, заповненої одиницями.

Ключові слова складні системи; орієнтований граф; матриця суперпозиції; матриця суміжності; аналітичний розрахунок

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