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A. Tkachev, PhD., Assoc. Prof.,
O. Tkachev, PhD,
I. Prokopovych, DSc, Prof.
Odessa Polytechnic National University, 1 Shevchenko Ave., Odessa, Ukraine, 65044; e-mail: tavtkach@ukr.net

OPERATION OF PRE-STRESSED SPAN BEAMS OF BRIDGE CRANES TAKING INTO ACCOUNT LOAD COMBINATIONS

А.В. Ткачов, О.А. Ткачов, І.В. Прокопович. **Робота попередньо-напружених прольотних балок мостивих кранів з урахуванням комбінацій навантажень.** В роботі розглядаються питання, пов'язані з підвищенням несучої здатності прольотних балок кранів мостового типу засобом попереднього напруження. Відомо, що робота прольотної будови крана с попередньо-напруженими балками проходить при таких самих умовах, режимах та вантажопідйомностях, як і звичайних кранів, де несуча здатність їх прольотних будов повинна бути забезпечена високою міцністю і жорсткістю у двох площинах – головній вертикальній площині та в горизонтальній. Але дослідження напружено-деформованого стану крану з попередньо-напруженим мостом, працюючим в горизонтальній площині, не проводились. Особливість цього питання складається в розробці нової математичної моделі попередньо напруженої головної балки, а також в отриманні рівняння кривої прогинів цієї балки при роботі її в горизонтальній площині. Був проведений аналіз її напружено-деформованого стану у площині підвісу вантажу, а також при одночасній дії вертикальних та горизонтальних сил. Вказаний аналіз дозволив встановити вплив горизонтальних інерційних навантажень на деформований стан попередньо напруженого моста. Встановлено, що при роботі мостового крану з вантажем у середині прольоту прогини прольотної балки не виходять за межі регламентуючих прогинів звичайного кранового моста, що позитивно впливає на деформований стан головної балки та полегшує роботу крана у цілому. Отримані у цій роботі результати надалі можуть бути використані для проектування кранів мостового типу з попередньо напруженими прольотними балками з метою підвищення їх вантажопідйомності, продовження терміну їхньої служби без демонтажу. А також для вдосконалення діючих конструкцій та інженерних засобів розрахунку, як на стадіях проектування, так і в умовах реальної експлуатації.
Ключові слова: мостовий кран, напружено-деформований стан, попередньо напружені балки, прогини балки

A. Tkachev, O. Tkachev, I. Prokopovych. **Operation of pre-stressed span beams of bridge cranes taking into account load combinations.** The paper considers issues related to increasing the bearing capacity of span beams of bridge cranes by means of pre-stressing. It is known that the work of the span structure of the crane with pre-stressed beams takes place under the same conditions, modes and capacities as conventional cranes. The load-bearing capacity of their span structures must be provided with high strength and rigidity in two planes – the main vertical plane and the horizontal one. However, studies of the stress-strain state of the crane with a pre-stress bridge operating in the horizontal plane were not performed. The peculiarity of this question is the development of a new mathematical model of the pre-stressed main beam, as well as obtaining the equation of the deflection curve of this beam when working in a horizontal plane. An analysis of its stress-strain state in the plane of the load suspension, as well as the simultaneous action of vertical and horizontal forces. This analysis allowed establishing the effect of horizontal inertial loads on the deformed state of the pre-stressed bridge. It is established that when operating a bridge crane with cargo in the middle of the span, the deflections of the span beam do not go beyond the regulatory deflections of a conventional crane bridge, which has a positive effect on the deformed condition of the main beam and facilitates the crane as a whole. The results obtained in this work can be further used for the design of bridge cranes with pre-stressed span beams in order to increase their load capacity, extend their service life without dismantling. Also for improvement of operating designs and engineering means of calculation, both at design stages, and in the conditions of real operation.
Keywords: bridge crane, stress-strain state, pre-stressed beams, deflections of the beam

Introduction

Overhead cranes, the spans of which are pre-stressed, are widely used in industry [1]. Such cranes have a number of advantages, namely, significantly less weight and smaller dimensions compared to conventional cranes, as well as a lower cost of the crane metal structure, which can be 75...80 % of the cost of the crane as a whole [2]. However, such cranes have a disadvantage – they are more deformable, which leads to a decrease in their use [3, 4]. It is known that the operation of a crane span with pre-stressed beams occurs under the same conditions, modes and load capacities as conventional cranes. The bearing capacity of their span structures must be ensured by high strength and rigidity in two planes – in the plane of the load and in the plane of suspension [5].

To do this, when designing a crane bridge, the calculation is carried out in accordance with the established design combinations of loads – "*a*" and "*b*". With a combination of loads "*a*", the opera-

tion of the crane corresponds to the mode of lifting the load or its braking when lowering. The crane bridge in this case is loaded in the *xoy* plane of the load suspension with vertical loads. Prevention of destruction of the crane bridge and the appearance of residual deformations with a given probability is guaranteed if the following conditions are met:

$$\sigma_a = \frac{(-EJ_1)y''}{W_1} \leq Rm, \quad y \leq [y].$$

The combination of loads "*b*" corresponds to the operation of the crane travel mechanism when the load is suspended. In this case, the crane bridge is loaded in two planes and is subjected to oblique bending. The condition for preventing its destruction is as follows [6]:

$$\sigma_b = \frac{(-EJ_1)y_1''}{W_1} + \frac{(-EJ_2)z''}{W_2} \leq Rm, \quad (y+z) \leq [y],$$

where EJ_1, EJ_2 – stiffness of the crane bridge when bending in the suspension plane and out of the cargo suspension plane, respectively;

W_1, W_2 – moments of inertia of the section in the same planes, respectively;

R – design resistance of steel;

m – working conditions coefficient;

$y, z, [y]$ – span beam deflections, in planes *xoy*, *xoz* permissible deflection, respectively.

Thus, when operating a pre=stressed crane bridge under conditions of oblique bending, the bridge must be provided with high strength and rigidity in two planes – in the main vertical plane *xoy* and in the horizontal plane – *xoz*. This condition must be taken into account in matters related to the calculation and design of cranes with pre-stressed bridges. It requires additional research in relation to the actual operating conditions of the crane. In this regard, issues related to the calculation and design of such structures require special attention and are very relevant.

Analysis of publications on the topic of research

The analysis of publications shows that the mathematical models of span beams were subjected to separate studies, numerical modeling of pre-stressed beams was carried out [7, 8]. At the same time, studies of beams with oblique bending were carried out only for conventional crane bridges exposed to transverse and horizontal inertial loads [9, 10]. Such mathematical models cannot be used in our case, since the beams are also subjected to longitudinal compressive forces. This, in turn, requires the development and consideration of a different mathematical model.

In addition, for pre-stressed beams, studies have been carried out on static stiffness issues. However, the operation of the bridge span was considered only in the vertical plane [11, 12].

It should be noted the works where the bridge preformation was determined as the difference between the deflections due to the action of transverse forces and the deflection of the beam due to the action of the longitudinal pre-stressing forces and self-stressing in the tightening. We also cannot consider this, since pre-stressed beams are systems that do not obey the superposition principle. In addition, this approach will not always be correct and gives only approximate results. From all it follows that there have been no publications related to the operation of a pre-stressed beam for combinations of loads "*b*".

This, in turn, requires the development and consideration of a new mathematical model, where the maximum approximation of the design scheme to the real operating conditions of the crane is put forward in the first place.

Purpose and objectives of the study

The purpose of this work is to further study the stress-strain state of a pre-stressed main beam operating simultaneously in two planes. The issues considered in it are those in which the nature of the action of loads on the beam is put forward in the first place with the maximum approximation of the design scheme to the real structural form.

To achieve this goal, it is necessary to solve the following tasks: to analyze the already known mathematical models of an overhead crane with pre-stressed main beams; to develop a mathematical model of a pre-stressed crane bridge, taking into account the operation of the main beams in the horizontal plane; to investigate and analyze the crane bridge during its simultaneous operation in the vertical and horizontal planes; to analyze the obtained results.

Materials and research methods

When developing a mathematical model, in accordance with the requirements put forward for the operation of a crane bridge, we believe that it is necessary to consider two types of calculation schemes for the bridge: the first – for the combination of loads "*a*" and the second – for the combination "*b*". When drawing up design schemes, we assume that all elements of the crane are solid bodies; the beam operates in the elastic stage and is based on ideal hinges.

The first case (combination "*a*") was considered in detail by the author in [13], where the design scheme can be represented by the corresponding scheme in Fig. 1, where the following are indicated: l – beam length; x – current coordinate of the location for determining the horizontal deflection y; a – the distance from the right support of the beam to the place of action of the vertical force F.
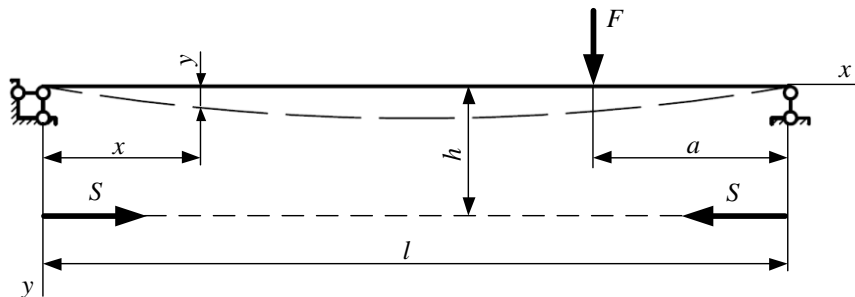

Fig. 1. Calculation scheme of a span beam in a vertical plane

Where the expressions for the deflection curve of the crane bridge and the moments bending it in the *xoy* plane will have the form:

$$y = -hU + \frac{F}{S}\left(\frac{\sin ka \sin kx}{k \sin kl} - \frac{ax}{l} \right);$$

$$M = -hS(U-1) + \frac{F \sin ka \sin kx}{k \sin kl}, \text{ at } 0 \le x \le (l-a);$$

$$y = -hU + \frac{F}{S}\left(\frac{\sin k(l-a) \sin k(l-x)}{k \sin kl} - \frac{(l-a)(l-x)}{l} \right);$$

$$M = -hS(U-1) + \frac{F \sin k(l-a) \sin k(l-x)}{k \sin kl}, \text{at } l \ge x \ge (l-a),$$

where indicated:
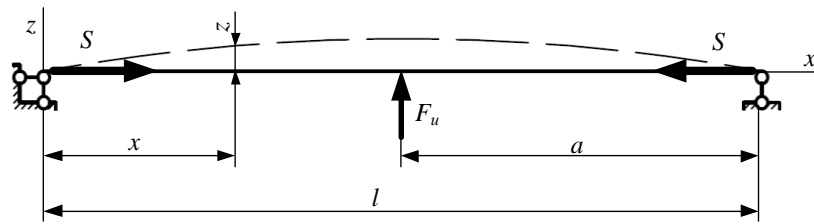
$$U = \cos kx + \sin kx \, tg(0{,}5kl) - 1, \quad k^2 = \frac{S}{EJ_1}.$$

With a combination of loads "*b*", a pre-stressed crane bridge is acted upon (in the *xoz* plane) by a horizontal inertial load F_u determined by the masses of the bridge m_m, bogie m_t, load m and acceleration γ:

$$F_u = (m_m + m_t + m) \cdot \gamma.$$

The design scheme of the span beam, when it works in a horizontal plane, is shown in Fig. 2.

Fig. 2. Calculation scheme of the span beam in the horizontal plane

Denote:

$$z^2 = \frac{S}{EJ_2}.$$

Then, the differential equations of the deflection arrows for the left and right parts of the pre-stressed crane bridge, at $a = 0.5l$, will have the form, respectively:

$$\frac{d^2z}{dx^2} + k^2 z = -\frac{0.5\,F_u x}{EJ_2}, \text{ at } 0 \le x \le (l-a);$$

$$\frac{d^2z}{dx^2} + k^2 z = -\frac{0.5\,F_u (l-x)}{EJ_2}, \text{ at } x \ge (l-a).$$

The full integral of this equation will be:

$$z = C_1 \cos kx + C_2 \sin kx - \frac{0.5 F_u x}{S};$$

$$z = C_3 \cos kx + C_4 \sin kx - \frac{0.5 F_u (l-x)}{S}.$$

We determine the integration constants C_1, C_3 from the conditions at the ends of the beam, where the deflections are equal to zero, and C_2, C_4 – at the place of action of the force F_u on the beam:

$$C_1 = 0;$$

$$C_2 = \frac{F_u \sin 0.5kl}{Nk \sin kl};$$

$$C_3 = \frac{F_u \sin 0.5kl}{Nk};$$

$$C_4 = -\frac{F_u \sin 0.5kl}{Nk\, tgkl}.$$

In our case, the force F_u is applied in the middle of the beam, so the deformation curve is symmetrical. Thus, we can consider one of the two sections of the beam. After simple transformations, we obtain the corresponding expression for the beam deflection arrow:

$$z = \frac{F_u}{S}\left(\frac{\sin 0.5kl\;\sin kx}{k\,\sin kl} - 0.5x\right).$$

From here we obtain the expression for the bending moment:

$$M = -EJ_2\left(\frac{d^2z}{dx}\right) = \frac{F_u}{S}\left(\frac{\sin 0.5kl\;\sin kx}{k\,\sin kl} - 0.5x\right).$$

Research results

According to the expressions obtained, for the current load capacities – $F_1 = 0.5$ t; $F_2 = 0.63$ t; $F_3 = 1.0$ t, beams with a span and combinations of loads "a" and "b", mathematical studies of its deformed state were carried out.

The eccentricity with which the longitudinal forces S act on the beam is $h = 200$ mm. The results of the obtained deformations are shown in Table 1 and in Figs. 3. In the table, it is necessary to understand that: F_a – forces acting on the beam with the combination "a" in the middle part of the span – $x = 0.5l$ and above the support – $x = l$; F_b – the force acting on the beam with the combination "b" in the middle part of the beam – $x = 0.5l$. Bridge deformations are presented in the form of conditional beam deflections for the 4K operating mode group:

$$\frac{y/l}{[y/l]}; \frac{z/l}{[y/l]},$$

and allowable value of conditional deflections – $[y/l] = 2 \cdot 10^{-3}$.

On Fig. 3 for the ratio of forces $F/S = 1.0$; 1.5; 1.75; 2, , respectively, the arrows of the deflections of the beam are presented: for the load capacity $F = 0.63$ t – graphs 1, 2, 3, 4 and for the load capacity $F = 1 t$ – graphs 5, 6, 7, 8.

An analysis of the results obtained with a combination of loads "a" has established that, with the ratio of forces $F/1.5$ acting on the beam, the use of prestressing allows to reduce the deflections of the span beam to a minimum when the vertical load is in the middle of the span $x = 0.5l$ and above the support $x = l$.

Thus, from Table 1 and Fig. 3 it can be seen that when applying an eccentric compressive force S, the deflections of the beam can be reduced by 50...70 %, which positively affects the deformed state of the span bridge.

Table 1

Conditional deflections of the main beam

$\frac{F_i}{S_i}$	Load position F_i	Load capacity, F_i, t								
		$F_1 = 0.5$		$F_2 = 0.63$			$F_3 = 1.0$			
		$\frac{F_b}{S_i}$	$\frac{F_a}{S_i}$	№ graph	$\frac{F_b}{S_i}$	$\frac{F_a}{S_i}$	№ graph	$\frac{F_b}{S_i}$	$\frac{F_a}{S_i}$	
$\frac{F_i}{0.75}$	$0.5l$	0.5	0.49	–	0.66	0.64	–	0.81	0.97	
	l		–0.08			–0.08			–0.08	
$\frac{F_i}{1.0}$	$0.5l$	0.6	0.42	1	0.73	0.6	5	0.86	0.94	
	l		–0.11			–0.11			–0.11	
$\frac{F_i}{1.25}$	$0.5l$	0.65	0.37	–	0.8	0.56	–	0.9	0.9	
	l		–0.16			–0.16			–0.16	
$\frac{F_i}{1.5}$	$0.5l$	0.68	0.35	2	0.84	0.54	6	0.94	0.89	
	l		–0.18			–0.18			–0.18	
$\frac{F_i}{1.75}$	$0.5l$	0.74	0.29	3	0.92	0.46	7	1.0	0.8	
	l		–0.25			–0.25			–0.25	
$\frac{F_i}{2.0}$	$0.5l$	0.82	0.27	4	1.0	0.40	8	1.2	0.75	
	l		–0.3			–0.3			–0.3	

In addition, from Table 1 and Fig. 3 it can be seen that with a combination of loads "b", an increase in the magnitude of compressive forces leads to a significant increase in deformations in the horizontal plane. Acceptable for the crane bridge may be the ratio of forces $F/1.5$ (Fig. 3).
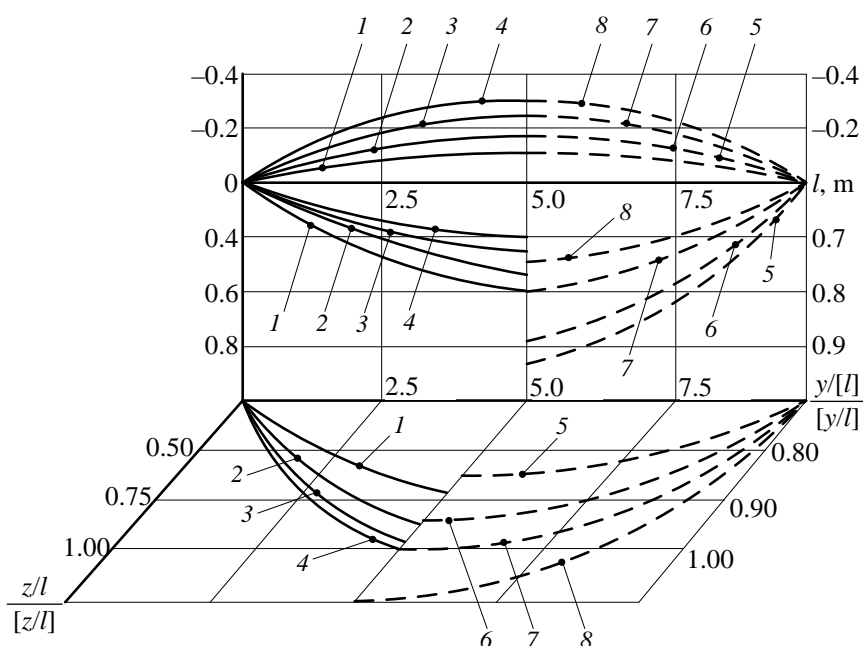
Fig. 3. Deformations of the span beam in the vertical and horizontal planes

At the same time, we note that the beam deformations during oblique bending go beyond the limits of deflections from the temporary load of a conventional bridge without unloading devices. Which, in turn, requires special attention when designing pre-stressed crane bridges.

Conclusions

A new mathematical model of a pre-stressed crane bridge was proposed and investigated when it works with combinations of loads "*c*". Its analysis showed that in order to prevent the destruction of the crane bridge and the appearance of residual deformations, it becomes necessary to abandon the traditional sections of span beams of bridges and use sections with reinforcement in the horizontal plane.

The results obtained in this work can be used in the modernization of structures in order to increase the carrying capacity, increase the service life without dismantling, as well as to improve existing structures and engineering methods of calculation during design and in real operation.

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Ткачьов Анатолій Вікторович; Anatoliy Tkachev, ORCID: https://orcid.org/0000-0002-2456-7533
Ткачьов Олексій Анатолійович; Oleksii Tkachev, ORCID: https://orcid.org/0000-0002-4173-6175
Прокопович Ігор Валентинович; Ihor Prokopovych, ORCID: https://orcid.org/0000-0002-8059-6507