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MODELING THE EFFECT OF STOCHASTIC DEFECTS FORMED IN PRODUCTS DURING MACHINING ON THE LOSS OF THEIR FUNCTIONAL DEPENDENCIES

A.V. Usov, M.V. Kunitsyn, D.V. Klymenko, V.M. Davydiuk. Моделювання впливу стохастичних дефектів, що утворюються у виробках під час механічної обробки, на втрату їх функціональних залежностей. В статті проводиться дослідження впливу спадкових дефектів, сформованих у поверхневому шарі виробів із металів гетерогенної структури на якість оброблених фінішними методами поверхонь, побудовані на базі комплексного підходу, що спирається на результати детермінованої теорії розвитку дефектів та методи теорії ймовірностей. Оброблюваний шар виробу розглядається як середовище, ослаблене випадковими дефектами, що не взаємодіють між собою: структурними змінами, тріщинами, включеннями, що визначають параметри яких є випадковими величинами з відомими законами їх ймовірнісного розподілу. Досліджуються причини появи структурних змін, тріщиноутворення на оброблюваному поверхневому виробі залежно від різних типів ймовірнісного розподілу розмірів: довжини, глибини залягання дефектів, їх орієнтації. З цих позицій розглядаються технологічні можливості їх усунення шляхом визначення галузі поєднань технологічних параметрів, що забезпечують необхідну якість оброблених поверхонь. Моделювання термомеханічних процесів в оброблюваній поверхні, що містить спадкові дефекти, проводиться на підставі рівнянь термопружності з розривними граничними умовами в місцях їхнього скупчення. В дослідженнях використаний апарат крайових задач рівнянь математичної фізики, метод сингулярних інтегральних рівнянь для розв'язку задач механіки руйнування, інтегральні перетворення Фур'є – Лапласа для одержання точних розв'язків, метод побудови розривних функцій. Отримано залежності, що визначають інтенсивність напружень у вершинах спадкових дефектів. Розроблено методику прогнозування характеру тріщиноутворення залежно від ймовірнісного розподілу дефектів, значень теплового потоку, що надходить у поверхневий шар виробу, що обробляється. Встановлено, що збільшення однорідності матеріалу призводить до зростання значення теплового потоку, що відповідає фіксованій ймовірності руйнування.

Ключові слова: обробка поверхонь, ймовірність руйнування, модель руйнування, розподіл, дефект матеріалу, параметри тріщини, тепловий потік, поверхневий шар, інтенсивність напружень

A. Usov, M. Kunitsyn, D. Klymenko, V. Davydiuk. Modeling the effect of stochastic defects formed in products during machining on the loss of their functional dependencies. The article investigates the influence of hereditary defects formed in the surface layer of products from metals of heterogeneous structure on the quality of surfaces treated with finishing methods. The research is based on an integrated approach based on the results of the deterministic theory of defect development and methods of probability theory. The treated layer of the product is considered as a medium weakened by random defects that do not interact with each other, namely: structural changes, cracks, inclusions, the parameters of which are random variables with known laws of their probability distribution. The causes of structural changes, crack formation on the treated surface product depending on different types of probability distribution of dimensions are investigated: length, depth of defects, and their orientation. From these positions, technological possibilities of their elimination by definition of branch of combinations of the technological parameters providing necessary quality of the processed surfaces are considered. Modeling of thermomechanical processes in the treated surface containing hereditary defects is carried out based on thermoelastic equations with discontinuous boundary conditions in the places of their accumulation. The research used the apparatus of boundary value problems of mathematical physics equations, the method of singular integral equations for solving problems of fracture mechanics, Fourier-Laplace integral transformations for obtaining exact solutions, the method of constructing discontinuous functions. The dependences determining the intensity of stresses in the vertices of hereditary defects are obtained. A method for predicting the nature of crack formation depending on the probability distribution of defects, the values of heat flux entering the surface layer of the processed product has been developed. It is established that the increase in the homogeneity of the material leads to an increase in the value of heat flux, which corresponds to a fixed probability of failure.

Keywords: surface treatment, failure probability, fracture model, distribution, material defect, crack parameters, heat flux, surface layer, stress intensity

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Introduction

Machining of products includes the grinding operation is the main finishing operation of heavy-duty parts of high-strength steels and alloys, including gears, shafts, dies, and screw belts. Consequently, roughness significantly affects their performance properties.

A distinctive feature of the grinding operation is the release of a large amount of heat, the highest honor of which is perceived by the treated surface of the part and causes structural changes in some parts – burns, and cracks. These defects help to reduce the initial surface hardness, the formation of tensile residual stresses, and reduce contact endurance and fatigue strength of parts.

For a large group of metals and alloys subject to structural changes in processing them by grinding, the distinct type of marriage are defects such as cracks, which significantly reduce the performance properties of products.

The nature and intensity of cracking are primarily determined by the thermophysical properties of the processed materials, their structure, and heredity of previous technological operations, grinding modes, and the characteristics of the tool used. During the operation of parts, the surface layer of which contains grinding cracks, the destruction of products occurs at the places of their accumulation.

Grinding defects on machined parts dramatically reduce their performance and are unacceptable.

Therefore, determine the technological conditions of defect-free grinding from materials prone to structural changes, cracks, and cracking. Furthermore, it determines the need to study the formation mechanism of grinding defects.

Analysis of recent research and publications

In modern technological machining processes, grinding is widely used as a finishing operation, which ensures high accuracy and quality of the machined surface of the part. A feature of the grinding process is removing material by a large number of tool grains at high pressure and temperature [1, 2]. Unlike turning and milling, the cutting edges of the grinding wheel grains have a stochastic geometry and are randomly distributed on the cutting part of the tool [2, 3, 4]. This feature creates difficulties in the analysis of the grinding process. It can be an obstacle to predicting the results of processing and optimization of modes and parameters of the process. One of the most severe problems of technological development of modern mechanical engineering is the need to ensure constant compliance between the properties of new structural materials to be machined, which are increasingly exacerbated in the operation of products made of modern materials [5, 6, 7, 8]. Often the weakest link in the system's "material – working environment", which determines the allowable operating conditions and resources of the system as a whole, is the surface layer of the processed material, which determines the high importance of developing new methods and technologies of surface treatment [9, 10].

In this regard, the establishment of patterns of structure and properties of the treated surfaces of products from functionally gradient materials depending on the parameters of their production and operation in a wide range of changes in processing conditions is quite relevant [11]. Much attention is paid to the problem of ensuring qualitative characteristics, which primarily include the absence of structural changes (burns) and cracking on the treated surfaces [12]. It is primarily due to the widespread use in technology of modern high-strength materials that are prone to brittle fracture. Hereditary defects that form in the surface layer of products during their manufacture, starting from the work piece and ending with finishing methods of processing their working surfaces, contribute to the loss of functional properties [13]. Defects of different origins in the material structure and microcracks, and foreign inclusions, which cause a high concentration of stresses in the places of their accumulation, play a unique role in this [14]. The size, orientation, location, and distribution of defects in the surface layer are usually random. Defects and randomness in the treated surfaces are interrelated. The joint consideration of these factors in ensuring the quality characteristics of the working surfaces of products is an urgent problem, especially now that there is much information about the loss of load-bearing capacity of products whose working surfaces were treated by finishing methods [15]. The work [16, 17] is devoted to increasing grinding efficiency with a tool made of cubic boron nitride by controlling the contact interaction in the processing zone. Improving the wear resistance of the grinding tool due to the directed thermal effect on the state of the cutting grains of the tool is investigated in [18].

Purpose and objectives of the study

This study aims to develop the theory and recommendations on technological methods of significantly reducing grinding defects such as burns and cracks in processing parts made of materials and alloys, the surface layer of which has hereditary defects of structural or technological origin.

Achieving this goal requires setting and solving the following tasks:

1. To study the mechanism of formation of defects in the surface layer of parts made of materials and alloys prone to defect formation during their processing by grinding, considering previous operations and hereditary inhomogeneities that occur.
2. To develop a mathematical model that describes the thermomechanical processes in the surface layer during the grinding of parts of materials and alloys, considering their inhomogeneities that affect the formation of technological defects and determining the criteria for defect formation.
3. Check the adequacy of the results obtained on products made of materials prone to defects during processing by finishing methods.

Materials and methods of research

Theoretical research is carried out based on scientific bases of technology of mechanical engineering and thermophysics of mechanical and physical and technical processing processes, theories of thermoelasticity, the complex approach of modern deterministic theories of fracture mechanics, and methods of probability theory. In addition, the research uses the apparatus of boundary value problems of equations of mathematical physics and the method of singular integral equations for solving destruction problems.

First, we need to build the first part of the model describing thermomechanical processes and the impact of stochastic defects formed in products on finishing operations on the loss of their functional properties, namely the equations describing thermomechanical processes in the treated layer of products on finishing operations.

When choosing and substantiating the mathematical model, it was taken into account that both thermal and mechanical phenomena accompany the process of grinding parts. However, the predominant effect on the stress-strain state of the surface layer has temperature fields. Given that the bulk of the surface layer of the metal during grinding is in the elastic state, we can use the model of the thermoelastic body, which reflects the relationship of mechanical and thermal phenomena at the final values of heat fluxes. Since information on the propagation of temperatures and stresses along the depth and direction of movement of the tool is essential for the study of the thermomechanical state of polished surfaces, a flat problem is considered [4].

The influence of inhomogeneities in phase transformations of unstable structures, intergranular films, contour boundaries of hereditary austenitic grains, carbide stitching, nonmetallic inclusions, shells, flocs, and other defects arising in the surface layer and defects in the form of conditional cracks, which has the form (Fig. 1).

The system of equations that determine the thermal and stress-strain state when grinding the surface of parts with coatings, the upper layer of which has inhomogeneities such as inclusions and microcracks, contains:

The equation of nonstationary thermal conductivity:

$$\frac{\partial T}{\partial \tau} = a^2 \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right); \quad \begin{matrix} 0 \leq x < \infty; \\ -\infty < y < \infty. \end{matrix} \quad (1)$$

Lame elasticity equation in displacements:

$$\frac{\partial \Theta}{\partial x} \frac{1}{1-2\mu} + \Delta \bar{v} = B^T \frac{\partial T}{\partial x}; \quad \bar{v}(x, y) = \frac{v}{2G}; \quad \bar{v}(x, y) = \frac{v}{2G}; \quad (2)$$

where $T(x, y, \tau)$ is the temperature at the point with coordinates (x, y) and at any time τ ; a is the thermal conductivity of the material; a_t is a temperature coefficient of linear expansion; μ, G are La-

rial; $c\sqrt{\tau}$ is the heat flux from a single grain; v_g, v_{kp}, t_{gr} is the grinding modes, $2a^*$ is the length of the arc of contact of the tool with the part; l^* is the distance between the cutting grains.

The maximum values of the instantaneous temperature T_M , from single grains to the constant component – T_k , were theoretically and experimentally confirmed.

For the layer being processed and having structural and technological inhomogeneities, the discontinuity conditions of the solution, depending on the type of defect, will be:

inclusions: in crack – like defects:

$$\begin{aligned} \bar{v} = 0, \sigma_x \neq 0; & \quad \sigma_x = 0, \bar{v} \neq 0; \\ \bar{v} = 0, \tau_{xy} \neq 0; & \quad \tau_{xy} = 0, \bar{v} \neq 0, \end{aligned} \quad (8)$$

where $\bar{v}, \bar{v}, \sigma_x, \tau_{xy}$ are jumps of shear and stress components.

The problem's solution was carried out by the method of discontinuous solutions [21]. These solutions satisfy the Fourier equations of thermal conductivity and Lamé elasticity everywhere except the defect boundaries. When crossing the boundary of the field of displacements and stresses suffer jumps of the first kind, i.e., their jumps $\bar{v}, \bar{v}, \sigma_x, \tau_{xy}$ appear.

The problems of thermoelasticity (1) – (8) were solved using integral Fourier transforms on the variable γ and Laplace on τ to the functions $T(x, y, \tau)$, $\sigma_x(x, y, \tau)$.

Classical strength criteria evaluated the equilibrium state of the deformable surface layer.

Of the available failure criteria that consider the local physical and mechanical properties of inhomogeneous materials, the most appropriate is the criteria of the force approach associated with the use of the concept of stress intensity factor (SIF). When the load leads to the fact that the stress intensity K_I becomes equal to the limit value K_{Ic} , the crack-like defect turns into the main crack [19, 20, 21, 22].

The development of technological criteria for the control of defect-free grinding is carried out based on the established functional relationships between the physical and mechanical properties of the processed materials and the main technological parameters.

The quality of the treated surfaces will be ensured if with the help of control technological parameters to choose such processing modes, lubricating and cooling media and tool characteristics that the current values of grinding temperature $T(x, y, \tau)$ and heat flux $q(y, \tau)$, stresses σ_M and grinding forces P_x, P_y coefficient K_{Ic} do not exceed their limits.

Implementation of the system of limiting inequalities by the values of the temperature and the depth of its distribution in the form of [9, 10]:

$$T(x, y, \tau) = \frac{C}{2\pi\lambda} \sum_{k=0}^n H\left(\tau - \frac{kl}{v_{kp}}\right) H\left(\frac{L+kl}{v_{kp}}\right) \int_{\gamma_1}^{\gamma_2} f(x, y, \tau, \tau') d\tau' \leq [T]_M; \quad (9)$$

$$T([h], y, \tau) = \frac{C}{2\pi\lambda} \sum_{k=0}^n H\left(\tau - \frac{kl}{v_{kp}}\right) H\left(\frac{L+kl}{v_{kp}}\right) \int_{\gamma_1}^{\gamma_2} \psi(x, \tau, \tau') d\tau' \leq [T]_{s.p.}; \quad (10)$$

$$T_k(0, y, \tau) = \frac{Cv_{kp}}{\pi\lambda l \sqrt{v_g}} \int_{a-e}^{\tau} \int_a^e \frac{\chi(\eta, t) e^{\frac{(y-\eta)^2}{4(\tau-t)}}}{2\sqrt{\pi(\tau-t)}} \times \quad (11)$$

$$\times \left\{ \frac{1}{\sqrt{\pi(\tau-1)}} + \gamma e^{\gamma^2(\tau-t)} [1 + \Phi(\gamma\sqrt{\tau-t})] \right\} d\eta dt \leq [T];$$

$$T_k^{\max}(L, 0) = \frac{Cv_{kp}\alpha}{\lambda l v_q^2} \sqrt{\frac{\alpha}{\pi}} \left[1 - \exp\left(-\frac{v_q \sqrt{Dt_{shl}}}{\alpha}\right) \right] \leq [T], \quad (12)$$

which avoids the formation of grinding burns and can be the basis for the design of grinding cycles by thermal criteria.

Processing of materials and alloys without grinding cracks can be provided to limit the limit values of stresses formed in the zone of intensive cooling of stress:

$$\sigma_{\max}(x, \tau) = 2G \frac{1+\nu}{1-\nu} \alpha_t T_k \operatorname{erf} \left(\frac{x}{2\sqrt{\alpha\tau}} \right) \leq [\sigma_{\text{sch}}]. \quad (13)$$

In the case of the dominant influence of hereditary inhomogeneity on the intensity of the formation of grinding cracks, it is necessary to use criteria, the structure of which includes determining relationships of technological parameters and properties of the inhomogeneities themselves. As such, we can use the limitations of the stress intensity factor [23, 24, 25]:

$$K = \frac{1}{\pi\sqrt{l}} \int_{-e}^e \sqrt{\frac{l+t}{l-t}} \{\sigma_x, \sigma_y\} dt \leq K_{1c}, \quad (14)$$

or providing with the help of technological parameters of the limiting value of heat flux, which maintains the balance of structural defects:

$$q^* = \frac{P_z \nu_{kp} \alpha_s}{\sqrt{Dt_{\text{shl}}}} \leq \frac{\sqrt{3}\lambda K_{1c}}{Hl\sqrt{\pi l \sigma}}. \quad (15)$$

Defective grinding conditions can be implemented using information about the material structure being processed. Thus, in the case of the predominant nature of structural imperfections of length $2l$ of their regular location relative to the contact zone of the tool with the part, you can use as a criterion the equilibrium condition of the defect in the form:

$$l_0 < \frac{Da\lambda^2 \nu_q^u K_c^2}{\pi^2 \left[C\nu_{kp} G(1+\nu)a_t \left(1 - 2xp \left(\frac{\nu_q \sqrt{Dt}}{a\tau} \right) \right) \right]}, \quad (16)$$

where ν_{kp}, ν_q, t are the grinding modes; D, C are tool parameters; λ, a are thermophysical characteristics of the treated coating; K_c is the crack resistance of this coating; G is the modulus of elasticity; ν is the Poisson's ratio; a_t is the temperature coefficient of linear expansion; l is the characteristic linear size of the structural parameter (structure defect).

These inequalities correlate the longitudinal characteristics of the temperature and force fields with the control and technological parameters. Furthermore, they specify the range of combinations of these parameters that meet the obtained thermomechanical criteria. At the same time, the properties of the processed material are taken into account, and product quality assurance is guaranteed.

Based on the received criterion ratios, it is possible to provide the quality of a superficial layer of details at grinding, taking into account the maximum processing productivity.

Next, we need to build the second part of the model, which describes the thermomechanical processes and the impact of stochastic defects formed in products at finishing operations on the loss of their functional properties, namely to calculate the probabilities and criteria of equilibrium of stochastically distributed defects in the surface layer.

Modern materials used in technology have a complex structure formed by interacting particles. Depending on the scale of consideration (structural level), such particles can be atoms of different elements, vacancies, dislocations, and their networks, then – crystals, blocks of crystals, grains, polycrystalline aggregates, radicals, fibers, lamellar or three-dimensional inclusions, micro- and macrocracks.

Particles may differ in chemical composition, physical properties, geometry, and relative position. Larger particles in an adequately ordered structure may contain more minor defects.

As a result, there is a tremendous local heterogeneity of authentic materials. The reduction of the theoretical strength of authentic materials to the technical level is a consequence of defects - gaps in the continuity and homogeneity of the building, which arise from the formation of materials and their products (structural and technological defects).

The complexity of deformation and destruction that occur in the microvolumes of the body does not allow taking into account the impact on these processes of all the imperfections characteristic of this material. These defects refer to different structural equations. Therefore, their effect on strength and fracture is different.

In studying the causes of cracking during the grinding of materials, the most dangerous defects are considered – cracks (actually cracks, crevices, elongated sharp cavities) and sharp rigid inclusions. Such defects include extraneous elastic inclusions with very small or substantial elastic and strength characteristics compared to the characteristics of the base material (matrix). For example, low-strength graphite plate inclusions in the ferrite matrix of cast iron with some approximation can be interpreted as cracks.

The detailed introduction of defects, given by their defining parameters, is the first feature of the material model under consideration. The second feature is related to the static nature of the defect risk distribution.

$a_i^{(r)}$ ($i = 1, 2, 3, \dots, n$) denote the geometric parameters of defects of a specific r -th grade; n_r is the number of defining parameters for a given defect, where r is the type of defect. They determine the defects' size, configuration, and location (orientation). For example, for isolated flat elliptical cracks or rigid inclusions, we have five independent parameters (two ellipse axes and three angular orientation parameters for circular defects – three (radius and two angular parameters)).

As the strength of the continuum, take the value of the resistance of the material of development (K_c) or nucleation (K_o) of cracks.

According to the model, the quantities $a_i^{(r)}$, K_c are random, varying within certain limits. Suppose that for this material we know the function of the typical probability distribution of the values $F_r(a_1^{(r)}, a_2^{(r)}, \dots, a_n^{(r)}, K_c)$ or total probability density $f_r(a_1^{(r)}, a_2^{(r)}, \dots, a_n^{(r)}, K_c)$, which are related by the relation [26, 27]:

$$f(x_1, x_2, \dots, x_n) = \frac{\partial^n F(x_1, x_2, \dots, x_n)}{\partial x_1 \partial x_2 \dots \partial x_n}. \quad (17)$$

The appearance of these functions depends on the structure and technology of the material. Determinants can be stochastically independent or dependent. It, in particular, may be a consequence of the manufacturing technology (for example, in the thermo-machining of the alloy Alnico 8HC (R1-1-13) (Al – 7%; Ni – 14%; Co – 38%; Cu – 3%; Ti – 8%; Fe – other), between size and orientation and inclusions, there is a specific correlation). In the case of stochastic independence of defect parameters, the total distribution of parameters is equal to the product of partial distributions of each parameter separately:

$$f_r(a_1^{(r)}, a_2^{(r)}, \dots, a_n^{(r)}, K_c) = f_{n_1}(a_1^{(n_1)}) f_{n_2}(a_2^{(n_2)}), \dots, f_{n_n}(a_n^{(n_n)}) f(K_c).$$

For isotropic materials, uniform distribution of defects in all possible orientations is provided.

Consider a body of size V , and the symbol V can mean volume, area, length, and so on. Let some units of size V_0 of the material contain an average of n_0 primary defective elements. Then a

body of size V will contain on average $n = n_0 \frac{V}{V_0}$ primary elements. A body of size V can be consid-

ered as a random sample of volume n from the available set of primary elements of the material. Since, as accepted [26, 27], the ultimate load for a body is equal to the ultimate load of its least strong element (weak link), the distribution function of the ultimate load $F_n(P_1, \eta)$ for bodies of volume V can be found according to the formula for the distribution of the minimum member of the samples consisting of n elements of the general set of elements described by the function $F_1(P_1, \eta, \xi)$ [28]:

$$F_n(P_1, \eta, \xi) = 1 - [1 - F_1(P_1, \eta, \xi)]^{n_0 \frac{V}{V_0}}, \quad (18)$$

where $F_n(P_1, \eta, \xi)$ is the probability of failure of the element in the stress field P_1 , $P_2 = \eta$, $P_3 = \xi P_1$.

The same result is obtained by determining the probability of destruction of at least one element in the set of $n_0 \frac{V}{V_0}$ defective elements, and the probability of destruction of each of them separately is equal to $F_1(P_1, \eta, \xi)$ at fixed P_1, η, ξ . Formula (18) is used in many works on the statistical theory of brittle strength, based on the hypothesis of a weak link. The value of the function $F_n(P_1, \eta, \xi)$ is equal to the probability P of local destruction of a body of size V under the action of a given homogeneous complex stress field:

$$P(P_1, \eta, \xi) = F_n(P_1, \eta, \xi). \quad (19)$$

Determining this probability is one of the main tasks of choosing the technological parameters of defect-free processing.

For sufficiently large $n_0 \frac{V}{V_0}$ an asymptotic representation can be used for the function $F_n(P_1)$, which leads to a Weibull-type distribution [26, 27]:

$$F_n(P_1) = 1 - \exp \left[-cn_0 \frac{V}{V_0} (P_1 - P_1, \min)^m \right], \quad (20)$$

where $C > 0$, $m > 0$ are limitations, which depend on the number of defects of the value determined experimentally for a given type of material and load. Determining these values on the basis of the function $F_n(P_1)$ makes it possible to establish the parameters on which they depend, in particular, to establish their explicit relationship with the characteristics of material defects and the type of stress state that causes failure.

The above scheme of determining the probability of destruction of the body under the action of a complex homogeneous stress state is valid in the case when defects of one kind weaken the body material. Moreover, if the body is weakened by defects of different varieties that do not interact with each other, the result is easily generalized.

Let the material be weakened by defects $\gamma^{(r)}$ of different varieties. The average number of n_r^0 defects of each variety per unit size V_0 is assumed to be known, where $i = 1, 2, 3, \dots, \gamma$. Then, similar to the previous one, we can determine the function $F_{1r}(P_1, \eta, \xi)$ of the strength distribution of elements weakened by one defect of each variety separately. We need to know the appropriate deterministic conditions for destroying an element with a defect of each variety and the probability distributions of the defining parameters of defective elements of each variety. Since the value of F_{1r} determines the probability of destruction of an element with one defect of this variety, and $1 - F_{1r}$ is the probability of non-destruction of such an element, the probability of failure of any element with defects of this grade in a body of size V is $(1 - F_{1r})^{n_0 \frac{V}{V_0}}$, and the probability of failure of any element with a defect of any kind

$\prod_{r=1}^{\gamma} (1 - F_{1r})^{n_0 \frac{V}{V_0}}$. Then the possibility of destruction of at least one element of the body with any defect, ie, provided the probability of local destruction of the whole body, determines the formula [28]:

$$P(P_1, r, \xi) = F_n(P_1, r, \xi) = 1 - \prod_{r=1}^{\gamma} \left(1 - F_{1r}^{n_0 \frac{V}{V_0}} \right). \quad (21)$$

Let us consider an example of the calculation of statistical parameters of destruction at thermal influence. Let the half-plane weakened by uniformly scattered random cracks that do not interact with each other be under the action of uniform heat flux of intensity q (Fig. 1). The laws of the joint distribution of the half-length l_k and the angle of orientation φ_k will be known. At a specific value of heat flux (let us call it the limited value q^*) develops at least one crack, i.e., the process of destruction begins. The condition for the development of a single crack with given geometric parameters is established in [26, 27] by formula (22):

$$q^* = \frac{A}{l\sqrt{l}|\sin\varphi|}; A = 2\sqrt{\frac{2}{\pi}} \frac{K_c}{\alpha t E}.$$

We determine the probability distribution of the limiting heat flux and some of its statistical characteristics.

From formula (22) it is seen that due to the randomness of the length l and the orientation of the cracks φ , which vary within some limits $0 \leq l \leq \alpha$, $|\varphi| \leq \pi/2$, the value of q^* the limiting heat flux on the element, the half-plane with one crack is also random. The function $F_1(q^*)$ of the probability distribution is found based on formula (20):

$$F_1(q^*) = \iint_0 f(\varphi, l) d\varphi dl, \quad (22)$$

where R is a two-dimensional domain of possible values of random variables and l in which the relation $f(\varphi, l) \frac{A}{q|\sin\varphi|} \leq q^*$; $f(\varphi, l)$ is the density distribution of probabilities of length l and orientation φ of cracks.

Assuming the values of l and φ are statistically independent $f(\varphi, l) = f(\varphi) \cdot f(l)$ we obtain the formula:

$$F_1(q^*) = \int_{L\varphi} f_2(\varphi) \left[1 - F_3\left(\frac{A}{q|\sin\varphi|}\right)^{\frac{2}{3}} \right] d\varphi. \quad (23)$$

Here the $L\varphi$ is the integration domain of all possible values of φ ($|\varphi| \leq \frac{\pi}{2}$), which executes the relation:

$$0 \leq \left(\frac{A}{q|\sin\varphi|}\right)^{\frac{2}{3}} \leq d. \quad (24)$$

We assume that the distribution of cracks in orientation is uniform, ie, $f_3(\varphi) = \frac{1}{\pi}$, and in length has the form [29, 30]:

$$\frac{f_3(l)(S-1)a^{S-1}}{(x+a)S}, S > 1a > c, \quad (25)$$

where S is the crack shape parameter that determines the structural heterogeneity of the material (the larger S , the more likely small cracks, ie the material is more homogeneous), a is the scale parameter or [28]:

$$f_3(l) = \frac{r+1}{d} \left(1 - \frac{l}{d}\right)^r, r \geq 0, l \leq d, \quad (26)$$

where r is the fracture parameter of the material (the larger r , the more likely small cracks).

The distribution function $F_3(l)$ will be written:

$$F_3(l) = 1 - \left(1 - \frac{l}{d}\right)^{r+1}. \quad (27)$$

Substituting the expressions for $F_3(l)$ and $f_3(\varphi)$ in formula (23) we obtain:

$$F_1(q^*) = \frac{2}{\pi} \left(aq^{\frac{2}{3}}\right)^{S-1} \int_0^{\frac{\pi}{2}} \left(\frac{\sin^{\frac{2}{3}} \varphi}{A^{\frac{2}{3}} + aq^{\frac{2}{3}} \sin^{\frac{2}{3}} \varphi}\right)^{\frac{2}{3}} d\varphi. \quad (28)$$

For distribution (27), when $0 \leq l \leq d$, $q_{\min} = Aa^{\frac{3}{2}}$ and relation (28) holds for $\tilde{\varphi} \leq |\varphi| \leq \pi/2$, where:

$$\varphi = \arcsin\left(Aq^{-1}d^{\frac{3}{2}}\right). \quad (29)$$

In this case, the function $F_1(q^*)$ can be represented as:

$$F_1(q^*) = \frac{2}{\pi} \int_{\varphi}^{\frac{\pi}{2}} \left[1 - \frac{1}{d} \left(\frac{A}{q \sin \varphi}\right)^{\frac{2}{3}}\right]^{r+1} d\varphi. \quad (30)$$

If the half-plane contains n cracks, we assume that the limiting heat flux for the half-plane is equal to the smallest value of the limiting heat fluxes of its elements (weak link hypothesis) [29, 30]. Then the distribution function of the limiting heat flux of the half-plane with n cracks is determined by the following formula:

$$F_n(q^*) = 1 - [1 - F_1(q^*)]^n. \quad (31)$$

In this case q^* the value of the function $F_n(q^*)$ is equal to the probability of destruction of the material at a flow rate not exceeding the specified value q^* :

$$P = F_n(q^*). \quad (32)$$

Results and discussion

Based on formulas (27) – (32) the function $F_n(q^*)$ was calculated for different values of S , r of crack size distributions and two values of the number of defects in the surface layer: $n=10$; $n=100$. The calculations are shown in Fig. 2 and Fig. 3 for $q_{2\min} = 0$, and $q_{2\min} > 0$ in the form of curves of the probability of failure P on the applied heat flux.

The graphs show that the increase in the homogeneity of the material leads to an increase in the value of heat flux, corresponding to a fixed probability of failure. On the other hand, increasing S or r decreases the probability of failure, which corresponds to a given value of heat flux. An increase in the value of S or r leads to an elongation of the area of change of random heat flux, for which the probability of destruction is low.

The magnitude of stress intensity coefficients for defects such as cracks is influenced by the size and orientation of these defects, the depth of their occurrence and mutual location, the magnitude of heat flux. The stochastic model of crack formation during the grinding of metals of heterogeneous structure is based on an integrated approach based on the results of the deterministic theory of the development of individual defects and methods of probability theory. The surface layer is considered as a medium weakened by random non-interacting defects – cracked inclusions, which determine the parameters of which are random variables with known laws of their probability distribution.

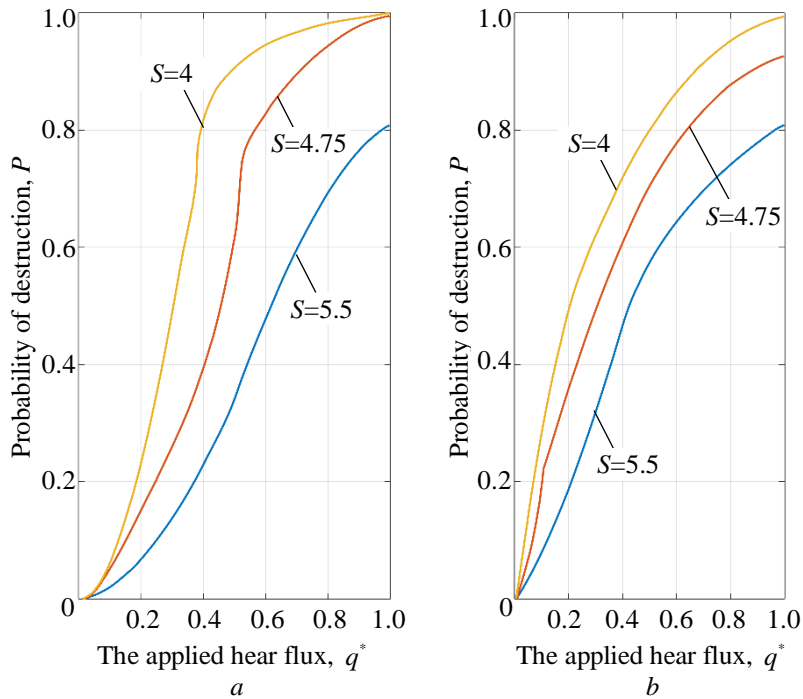


Fig. 2. The dependence of the probability of failure P on the applied heat flux q^* for two values of the number of defects: $a - n = 100$; $b - n = 10$; $P = F_n(q^*)$ has a statistical distribution

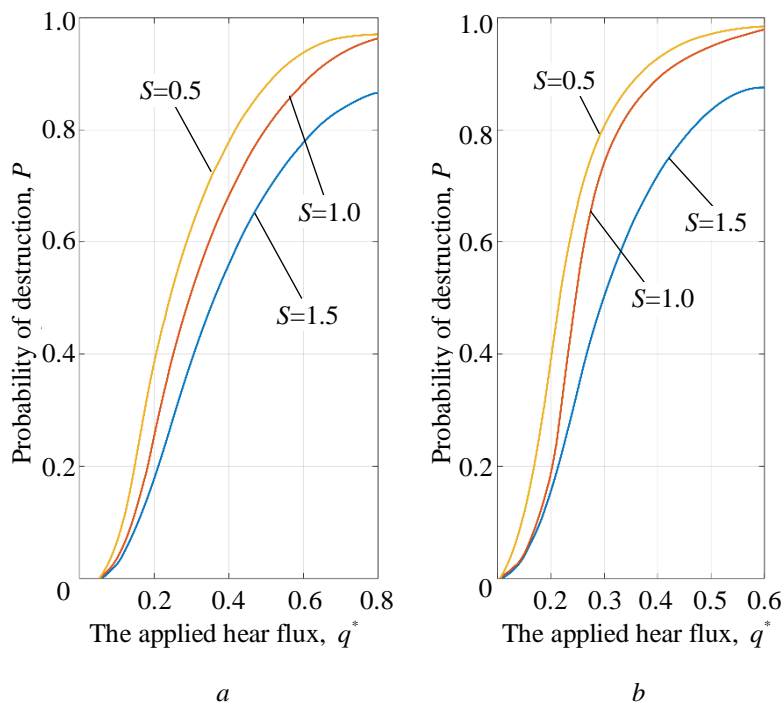


Fig. 3. The dependence of the probability of failure P on the applied heat flux q^* for two values of the number of defects: $a - n = 100$; $b - n = 10$; $f(l)$ has a generalized β – distribution

The probability of destruction of the surface layer depends on different types of the probability distribution of dimensions/length, depth/defects, and orientation. The probabilistic characteristics of

the limiting heat flux are considered from these positions. It is established that the increase in the homogeneity of the material leads to an increase in the value of heat flux, which corresponds to a fixed probability of failure.

Conclusions

The scientific problem of establishing the calculated dependences that determine the impact of hereditary defects from previous operations on the quality of the surface layer during grinding to create optimal technological processing conditions taking into account the accumulated defects and inhomogeneities in the surface layer of materials and alloys.

The following results were obtained.

1. The influence of technological and structural heterogeneity of materials on the mechanism of origin and development of defects under the influence of thermomechanical phenomena accompanying diamond-abrasive processing is established.

2. The analytical model of the definition of a thermomechanical condition at grinding of details in which working surfaces contain inhomogeneities of a hereditary origin is developed. Based on this model, the functional relationships of quality criteria with the technological parameters that control are determined.

3. The stochastic model of the process of crack formation during heterogeneous structure materials grinding of is constructed based on which dependences of the probability of cracks on modes and other parameters of the process of grinding are received. It has been calculated that an increase in the homogeneity of the material leads to a decrease in the probability of defects such as cracks and, consequently, an increase in the grinding operation while maintaining the required quality.

In combination with experimental studies, the obtained dependencies allow to theoretically determine the areas of combinations of technological parameters that provide the required quality of the treated surfaces.

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