The 6th IEEE International Conference on Intelligent Data Acquisition and Advanced Computing Systems: Technology and Applications
15-17 September 2011, Prague, Czech Republic

Modelling of Radio-Frequency Communication Channels Using Volterra Model

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Abstract—**This paper presents an efficient technique of continuous communication channel's experimental research for identification of amplitude-frequency characteristics (AFC). The technique is based on Volterra model and founded on the application of the approximating method of identification of nonlinear dynamic system. The identification is carried out by means of compiling the linear combinations of responses from researched system to the test polyharmonic signals with different amplitudes. Designed firmware, realizing the methodology of identification, is used for construction of informational model of communication channel in the form of first and second orders AFC's on the basis of input-output experiment's data, using harmonic and biharmonic test signals.**

Keywords—communication channels; telecommunication systems; nonlinear dynamic systems; identification; Volterra models; Volterra series; multidimensional transfer functions; multifrequency performances; polyharmonic signals

I. INTRODUCTION

One of the most important demands required from communication systems is an accuracy of information transmitted from source to recipient. In real conditions to fulfill this demands we have to eliminate an errors caused by external interference form communication channel (CC) at receiver's entrance; internal noise of receiver; and signal distortion during transmission. In connection with this problems for the last ten years intensively developing field related to the methods of signals' optimal reception which take into account characteristics of the hardware and CC. The expediency of communication system's application depends on how effectively its potential abilities are used.

Modern continuous telecommunication channels are nonlinear dynamic systems [1]. Nowadays for modeling of nonlinear dynamic systems, Volterra series [2]–[5] are widely used. Nonlinear and dynamic properties of a system are completely characterized by the sequence of multidimensional weighting functions - Volterra kernels. Thus a problem of system's identification - model building in the form of Volterra series - lies in determination of multidimensional Volterra kernels. This process is based on experimental research of input-output type system's data.

Application of the models in the form of Volterra series for communication channel's identification and modeling is caused by their essentially important virtues: invariance concerning to the aspect of input effect (i.e. the possibility of the problem's solution for determined and random input signals); explicit relations between input and output variables; versatility –possibility of the research of nonlinear time-continuous and nonlinear impulse systems, stationary and non-stationary, with lumped and distributed parameters, stochastic systems, and also multidimensional systems (systems with many inputs and many outputs); possibility of conducting research both in analytical and computing aspect; simultaneous and compact registration of nonlinear and inertial properties of the systems; interpretability of the linear systems as subclass of the nonlinear ones that allows to spread temporal and spectral methods to nonlinear systems, and operate with concepts of the multidimensional weighting and transfer functions, the amplitude and phase-frequency characteristics.

Model building of the nonlinear dynamic system in the form of Volterra series lies in the choice of test effects' form and algorithm development. This test effects' measured responses, will allow defining Volterra kernels or their Fourier-images (multidimensional amplitude-frequency characteristic and phase-frequency characteristic) accordingly for modeling in the temporal or frequency domains [4], [5].

The aim of this work is to identify the continuous communication channel in the form of Volterra series in the frequency domain, i.e. the determination of its multifrequency characteristics on the basis of input-output experiment's data, using test polyharmonic signals.

II. VOLTERRA MODELS IN THE TIME AND FREQUENCY DOMAINS

In general case "input-output" type ratio for nonlinear dynamic system can be presented by Volterra series [2], [3]:

$$y[x(t)]=\sum_{n=1}^{\infty}y_n[x(t)]=\sum_{n=1}^{\infty}\int_0^{\infty}\dots\int w_n(\tau_1,\tau_2,\dots\tau_n)\prod_{r=1}^{n}x(t-\tau_r)d\tau_r \quad (1)$$

where $x(t)$ and $y[x(t)]$ are input and output signals of system respectively; $w_n(\tau_1,\tau_2,\dots,\tau_n)$ – weight function or n–order Volterra kernel; $y_n[x(t)]$ – n-th partial component of object response.

In practice, Volterra series are replaced by polynomial and generally limited to several first members of the series. Identification of nonlinear dynamic system in the form of a Volterra series consists of determination of n-dimensional impulse transition functions $w_n(\tau_1, \dots, \tau_n)$ or their Fourier's-images $W_n(j\omega_1, \dots, j\omega_n)$ - n-dimensional transfer functions, accordingly for system modeling in time or frequency domain.

The multidimensional (n-dimensional) Fourier transform for a Volterra kernel of n–th order (1) is written as:

$$W_n(j\omega_1,j\omega_2,...,j\omega_n)=F_n\langle w_n(t_1,t_2,...,t_n)\rangle =$$
$$=\int_{-\infty}^{+\infty}\dots\int w_n(t_1,t_2,...,t_n)\exp\left(-j\sum_{i=1}^{n}\omega_i t_i\right)\prod_{i=1}^{n}dt_i \quad (2)$$

where $F_n\langle\ \rangle$ – n-dimensional Fourier transform; j - imaginary unit. Then the model of nonlinear system on the basis of Volterra series in frequency domain can be represented as:

$$y[x(t)]=\sum_{n=1}^{\infty}F_n^{-1}\left\langle W_n(j\omega_1,j\omega_2,...j\omega_n)\prod_{i=1}^{n}X(j\omega_i)\right\rangle_{t=t_1=t_2=...=t_n} \quad (3)$$

where $F_n^{-1}\langle\ \rangle$ - inverse n-dimensional Fourier transform; $X(j\omega_i)$ - Fourier-image of an input signal.

Identification of nonlinear system in frequency domain coming to determination of absolute value and phase of multidimensional transfer function at given frequencies - multidimensional amplitude-frequency characteristic $|W_n(j\omega_1, j\omega_2 \dots, j\omega_n)|$ and phase-frequency characteristic $\arg W_n(j\omega_1, j\omega_2, \dots, j\omega_n)$ which are defined by formulas:

$$|W_n(j\omega_1,j\omega_2,\dots,j\omega_n)|=\sqrt{\begin{array}{l}[\mathrm{Re}(W_n(j\omega_1,j\omega_2,\dots,j\omega_n))]^2\\+[\mathrm{Im}(W_n(j\omega_1,j\omega_2,\dots,j\omega_n))]^2\end{array}} \quad (4)$$

$$\arg W_n(j\omega_1,j\omega_2,\dots,j\omega_n)=\mathrm{arctg}\frac{\mathrm{Im}(W_n(j\omega_1,j\omega_2,\dots,j\omega_n))}{\mathrm{Re}(W_n(j\omega_1,j\omega_2,\dots,j\omega_n))} \quad (5)$$

where Re and Im - accordingly real and imaginary parts of a complex function of n variables.

III. APPROXIMATING METHOD OF IDENTIFICATION OF THE SYSTEMS IN THE FORM OF VOLTERRA MODELS IN FREQUENCY DOMAIN

During identification of a Volterra kernel of n–th order (n>1) a significant effect on accuracy is rendered by adjacent members of a Volterra series. Therefore it is necessary to apply special methods, allowing to minimize this effect [6], [7]. The idea of one of such methods lies in construction of such an expression of system responses to N ($N \geq n$) test input signals with the given amplitudes, which with certain accuracy (accurate within to the thrown members of order N+1 and above) would be equal to n–th member of a Volterra series:

$$y_n[x(t)]=\sum_{j=1}^{N}c_j y[a_j x(t)]=\sum_{n=1}^{\infty}\left(\sum_{j=1}^{N}c_j a_j^n\right)\int_{-\infty}^{\infty}\dots$$
$$\int w_n(\tau_1,\tau_2,...\tau_n)\prod_{l=1}^{n}x(t-\tau_l)d\tau_l, \quad (6)$$

where a_j - amplitudes of test signals, random nonzero and pairwise different numbers; c_j - real coefficients which are chosen so, that in a right part (6) would become *null* all first N members, except n- th, and the multiplier at n-multiple integral would became equal to unit. This condition leads to a solution of a system of the linear algebraic equations concerning coefficients $c_1, c_2, \dots, c_N$:

$$\left\{\sum_{j=1}^{N}c_j\cdot a_j^n=\delta_k^n=\begin{cases}1, & n=k;\\0, & n\neq k;\end{cases}\right. \text{ where } 1\leq k\leq N, \quad (7)$$

This system always has a solution, and the unique one, as the system determinant differs from Vandermonde determinant with an only multiplier $a_1 a_2 \cdot \ldots \cdot a_N$. Thus, with any real numbers a_j, that different from *null* and pairwise different, it is possible to find such numbers c_j at which the linear combination (6) of system responses is equal *to* n-th member of a Volterra series accurate within to the thrown terms members of the series.

It is possible to build numberless assemblage of modes for expressions (6), by taking various numbers $a_1, a_2, \dots, a_N$ and defining (7) coefficients $c_1, c_2, \dots, c_N$ by them.

The choice of amplitudes a_j should provide the convergence of series (1) and minimum error during extraction of a partial component $y_n[x(t)]$ according to (6) defined by reminder of series (1) - members of degree N+1 and above. If $x(t)$ – is a test effect with maximum admissible amplitude at which a series (1) converges, amplitudes a_j should be, by their absolute values, no more than unit: $|a_j| \leq 1$ for $\forall j$=1, 2... n.

In [8] it is shown that the amplitudes of the test polyharmonic signals offered for usage in an approximating method of identification [6] are not optimal. It also justified the choice of amplitudes for the test effects, providing the minimum inaccuracy for an estimation of multidimensional transfer function (multidimensional amplitude-frequency characteristic and phase-frequency characteristic) of the identified system.

The test polyharmonic effects for identification in the frequency domain are represented by signals in the form of expression:

$$x(t)=\sum_{k=1}^{n} A_k \cos(\omega_k t+\varphi_k) \tag{8}$$

where n - the order of transfer function being estimated; A_k, ω_k and φ_k - accordingly amplitude, frequency and a phase of k-th harmonics. In research it is supposed that every amplitude of A_k to be equal, and phases φ_k equal to zero. Thus the test signal can be written in the complex form:

$$\begin{aligned} x(t) &= A\sum_{k=1}^{n}\cos(\omega_k t)=\frac{A}{2}\sum_{k=1}^{n}(e^{j\omega_k t}+e^{-j\omega_k t})= \\ &=\frac{A}{2}\left(\sum_{k=1}^{n}e^{j\omega_k t}+\sum_{k=1}^{n}e^{-j\omega_k t}\right). \end{aligned} \tag{9}$$

Then the n-th partial component in the response of system can be noted in an aspect:

$$\begin{aligned} y_n(t) &= \frac{A^n}{2^{n-1}}\sum_{m=0}^{[\frac{n}{2}]} C_n^m \underbrace{\sum_{k_1=1}^{n}\dots\sum_{k_n=1}^{n}}_{n} e^{j\left(\sum_{l=1}^{m}\omega_{k_l}-\sum_{r=m+1}^{n}\omega_{k_r}\right)t} \times \\ &\times [W_n(-j\omega_{k_1},\dots,-j\omega_{k_m}, j\omega_{k_{m+1}},\dots,j\omega_{k_n})]\times \\ &\cos\left(\begin{aligned} &(-\sum_{l=0}^{m}\omega_{k_l}+\sum_{l=m+1}^{n}\omega_{k_l})t+ \\ &+\arg W_n(-j\omega_{k_1},\dots,-j\omega_{k_m}, j\omega_{k_{m+1}},\dots,j\omega_{k_n}) \end{aligned}\right) \end{aligned} \tag{10}$$

here [] means function of extraction of an integer part of number.

The component with frequency $\omega_1+ \dots + \omega_n$ is selected from the response to a test signal (6):

$$\begin{aligned} &A^n \cdot |\, W_n(j\omega_1, \dots, j\omega_n)\,| \cos[(\omega_1+\dots+\omega_n)\,t+ \\ &+\arg W_n(j\omega_1, \dots, j\omega_n)], \end{aligned} \tag{11}$$

On the basis of expression's analysis (10) it is defined that during determination of multidimensional transfer functions of nonlinear systems it is necessary to consider the imposed constraints on choice of the test polyharmonic signal frequencies which provide an inequality of combination frequencies in output signal harmonics (10).

In [9] the theorem of test frequencies choice has been proved. According to this theorem: for unambiguity of filtration from *n-th* partial component of identified system's response of a harmonic with combinational frequency $\omega_1 + \omega_2 + \dots + \omega_n$ it is necessary and enough that it did not equal to other combinational frequencies of an aspect $c_1\omega_1+c_2\omega_2+ \dots +c_n\omega_n$, thus factors c_i – are integral numbers (i=1, 2, … n), and satisfy conditions: the set potency c_i<0 accepts values from 0 to $\left[\frac{n}{2}\right]$; $\sum_{i=1}^{n}|c_i|\le n$; $n-\sum_{i=1}^{n}|c_i|=2p, p\in\mathbf{N}$, where **N** – set of natural numbers.

Consequently to such restrictions for a choice of frequencies for test polyharmonic signals, values of transfer functions in these "pricked out" points of multidimensional space of frequencies can be received only by means of interpolating. During practical realization of the method of identification it is necessary to minimize the number of such ambiguous points on an interval of multidimensional transfer function's construction, i.e. to provide a minimum of restrictions for a choice of frequencies of a test signal. The offered condition for a choice of frequencies [9] defining a possibility of univalent filtration of required harmonics, allows to expand a set of admissible test frequencies to the maximum. So, while defining the second order of the transfer function, it is required to provide not five as in [7], but three inequalities between frequencies of an input signal: $\omega_1\neq0$, $\omega_2\neq0$ and $\omega_1\neq\omega_2$. Defining the third order of the transfer function, it is required to provide 15 inequalities between frequencies of the input signals (instead of 45 - in [7]): $\omega_1\neq0$, $\omega_2\neq0$, $\omega_3\neq0$, $\omega_1\neq\omega_2$, $\omega_1\neq\omega_3$, $\omega_2\neq\omega_3$, $2\omega_1\neq\omega_2+\omega_3$, $2\omega_2\neq\omega_1+\omega_3$, $2\omega_3\neq\omega_1+\omega_2$, $2\omega_1\neq\omega_2-\omega_3$, $2\omega_2\neq\omega_1-\omega_3$, $2\omega_3\neq\omega_1-\omega_2$, $2\omega_1\neq-\omega_2+\omega_3$, $2\omega_2\neq-\omega_1+\omega_3$ and $2\omega_3\neq-\omega_1+\omega_2$.

IV. The Technique and Hardware-Software Tools of Radiofrequency Communication Channel's Identification

Experimental research of an Ultra High Frequency range communication channel for the purpose of identification of its multifrequency performances, characterizing nonlinear and dynamic properties of the channel are fulfilled. The Volterra model in the form of the second order polynomial is used. Thus physical communication channel properties are characterized by transfer functions of $W_1(j\omega)$ and $W_2(j\omega_1, j\omega_2)$ − by the Fourier-images of weighting functions $w_1(t)$ and $w_2(t_1, t_2)$.

Implementation of identification method on the IBM PC computer basis has been carried out using the developed software in C++ language with the usage of such classes as CWaveRecorder, CWavePlayer, CWaveReader, CWaveWriter which allow to provide rather convenient interacting with MMAPI Windows. The software allows automating the process of the test signals' forming with the given parameters (amplitudes and frequencies). Also this software allows transmitting and receiving signals through an output and input section of computer's soundcard, to produce segmentation of a file with the responses to the fragments, corresponding to the communication channel's responses being researched on test polyharmonic effects with different amplitudes.

In experimental research two identical SOUTHBELL's VHF-radio stations (a range of operational frequencies 30–300 MHz) and IBM PC with Realtek's sound card, based on ALC850 chip were used. Sequentially amplitude-frequency characteristics of the first and second orders were defined. The method of identification with an order of approximation N=4 was applied. Structure charts of identification procedure - determinations of the first and second orders amplitude-frequency characteristics of communication channel are presented accordingly on fig. 1 and fig. 2. The general scheme of a hardware-software complex of the communication channel's identification, based on the data of input-output type experiment is presented in fig. 3.

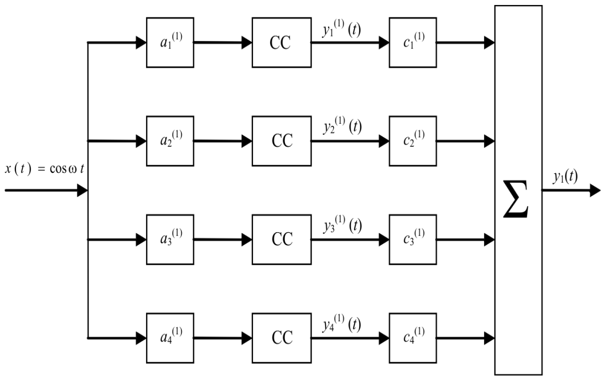

Figure 1. The structure chart of identification procedure of the first order amplitude-frequency characteristics

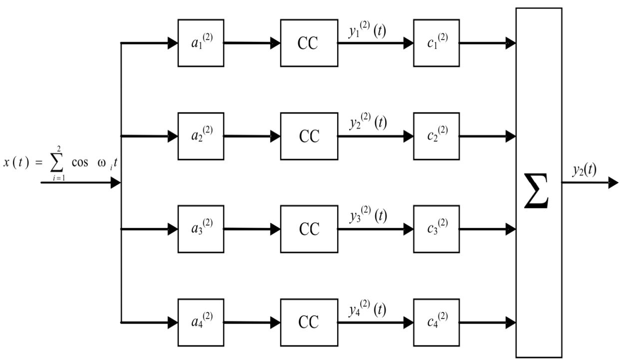

Figure 2. The structure chart of identification procedure of the second order amplitude-frequency characteristics

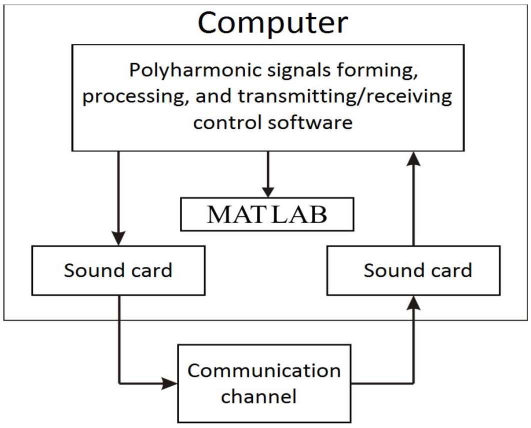

Figure 3. The general scheme of the experiment

The communication channel's received responses to the test signals (fig. 4), compose a group of the signals, which amount is equal to the used order of approximation N (N=4). In each following group the signals' frequency increases by magnitude of chosen step. To form the test signals, amplitudes and corresponding to them coefficients given in [8] are used.

Maximum allowed amplitude in described experiment with use of sound card was 0,677V (defined experimentally). The used range of frequencies was defined by the sound card pass band, and frequencies of the test signals has been chosen from this range, taking into account restrictions specified above. Such parameters were chose for the experiment: start frequency − 200 Hz; final frequency − 1600 Hz; a frequency change step – 27 Hz; to define amplitude-frequency characteristic of the second order determination, an offset on frequency $\omega_2 - \omega_1$ was equal 25, 50 and 100 Hz.

The weighed sum is formed from received signals-responses of each group (fig. 1 and 2). As a result we get partial components of response of the communication channel $y_1(t)$ and $y_2(t)$.

For each partial component of response a Fourier transform (the FFT is used) is calculated, and from received spectra only an informative harmonics (which amplitudes represents values of required characteristics of the first and second orders amplitude-frequency characteristics) are taken.

The first order amplitude-frequency characteristic $|W_1(j\omega)|$ is received by extracting the harmonics with frequency ω from the spectrum of the partial response of the communication channel $y_1(t)$. The second order amplitude-frequency characteristic $|W_2(j\omega_1, j\omega_2)|$ we received extracting the harmonics with summary frequency $\omega_1 + \omega_2$ from the spectrum of the partial response of the communication channel $y_2(t)$ to the test signal $A\cdot\cos\omega_1\cdot t + A\cdot\cos\omega_2\cdot t$.

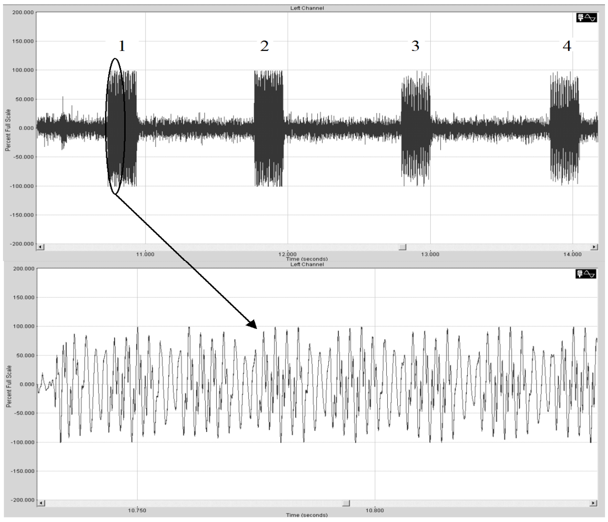

Figure 4. The accepted group of signals – responses corresponding to the amplitudes of the test signals: 1 - a =-1; 2 - a=1; 3 - a =-0,644; 4 - a=0,644

The results received after digital data processing of the experiments for the first and second orders. Amplitude-frequency characteristics definition after the procedure of smoothing are presented in fig. 5-8.

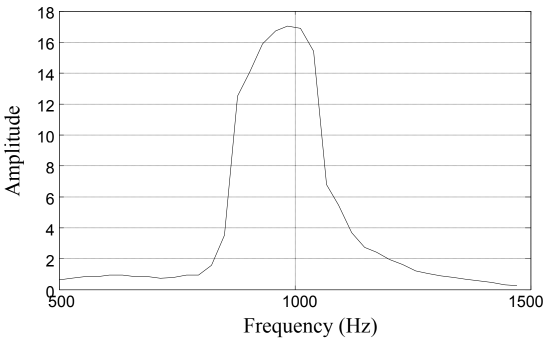

Figure 5. Amplitude-frequency characteristic of the first order after smoothing

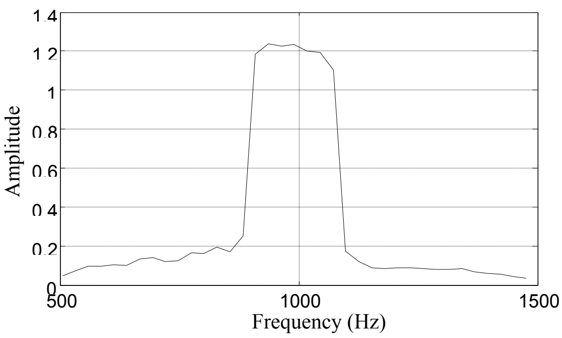

Figure 6. The subdiagonal section of the second order amplitude-frequency characteristic after smoothing at $\Delta\Omega$=25 Hz

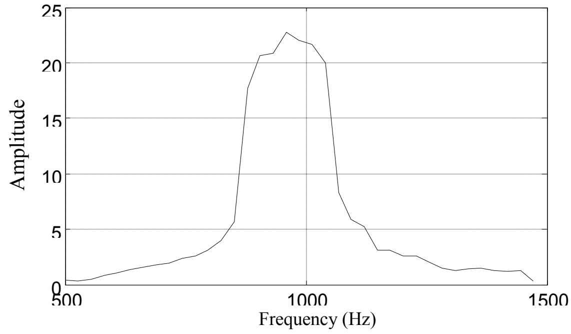

Figure 7. The subdiagonal section of the second order amplitude-frequency characteristic after smoothing at $\Delta\Omega$ = 50 Hz

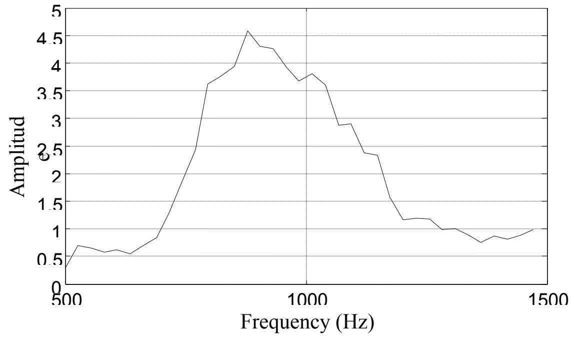

Figure 8. The subdiagonal section of the second order amplitude-frequency characteristic after smoothing at $\Delta\Omega$ = 100 Hz

V. Conclusions

The technique of experimental research of continuous CC of the telecommunication system is developed for the identification of its characteristics regarding to the nonlinear and dynamic properties on the basis of the models in the form of Volterra series in the frequency domain. The technique is based on application of the approximating method of identification of the nonlinear dynamic system using the compilation of the linear combinations of the system responses to the input polyharmonic signals of different amplitudes.

The hardware-software tools are developed implementing the technique of identification. They are applied to create an information model of the CC in the form of the first and second orders AFCs on the basis of the data of input-output type experiment with the usage of the test harmonic and biharmonic signals.

The received results of research show essential nonlinearity of the CC that leads to distortions of the signals in a radio section, reduces an important parameters of the telecommunication system: accuracy of the reproduction signals, channel bandwidth, noise immunity.

In further research the received frequency characteristics of the CC will be used for the synthesis of the compensators of the nonlinearity distortions in the telecommunication system.

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