Polyharmonic test signals application for identification of nonlinear dynamical systems based on volterra model

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Polyharmonic Test Signals Application for Identification of Nonlinear Dynamical Systems Based on Volterra Model

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***Abstract*— The new criterion for selecting the frequencies of the test polyharmonic signals is developed. It allows uniquely filtering the values of multidimensional transfer functions – Fourier-images of Volterra kernel from the partial component of the response of a nonlinear system. It is shown that this criterion significantly weakens the known limitations on the choice of frequencies and, as a result, reduces the number of interpolations during the restoration of the transfer function, and, the more significant, the higher the order of estimated transfer function.**

Keywords*— *frequency limitations; nonlinear system; polyharmonic signals; Volterra kernels; frequency characteristics

I. INTRODUCTION

Nowadays, integro-power Volterra series are widely used to model complex nonlinear dynamical systems [1]. The problem of constructing a model (identification) of a system in the form of Volterra series consists in determining of multidimensional weighting functions – Volterra kernels or multidimensional transfer functions – Fourier transforms of Volterra kernels based on data of experimental input-output system tests in the time [2-4] or frequency [4-8] domain.

In [4], the method of nonparametric identification of nonlinear dynamical system in the frequency domain using polyharmonic signals as test actions is considered. In this case, the multidimensional amplitude- and phase-frequency responses of system are found. At determining of multidimensional amplitude- and phase-frequency responses it is necessary to impose certain limitations on choice of the frequencies of test polyharmonic signals and, consequently, the transfer functions values at these "forbidden" points in the multidimensional frequency space can be obtained only by application of the interpolation procedure. In practical implementation of the method, it is necessary to minimize the number of such uncertainty points in the multidimensional transfer functions definition interval, i.e. strive to provide a minimum of limitations on the choice of test signal frequencies.

The new frequency selections less restrictive than existing [4-7] are proposed. They significantly reduce the number of uncertainty points.

II. MAIN PART

In general, the "input-output" ratio for continuous nonlinear dynamical system with zero initial conditions can be represented as the integral power Volterra series [1–3]:

$$y[x(t)] = \sum_{n=1}^{\infty} y_n[x(t)] = \sum_{n=1}^{\infty} \int_0^{\infty} \cdots \int w_n(\tau_1,\ldots,\tau_n) \prod_{r=1}^{n} x(t-\tau_r)\, d\tau_r, \quad (1)$$

where $x(t)$ and $y[x(t)]$ — input and output signals of the system, respectively; $w_n(\tau_1,\ldots,\tau_n)$ — weighting function, or Volterra kernel of n^{th} order, that is symmetric with respect to real variables $\tau_1, \ldots, \tau_n$; $y_n[x(t)]$ — n^{th} partial component of response of the system; t — current time.

The construction of a model of a nonlinear dynamical system in the form of Volterra serie consists in choosing the type of test actions $x(t)$ and development of an algorithm that would allow for the measured output signals $y(t)$ derive the partial components $y_n[x(t)]$ [9], and to determine using them the Volterra kernel $w_n(\tau_1,\ldots,\tau_n)$ or its Fourier–images $W_n(j\omega_1,\ldots,j\omega_n)$ respectively for modelling of the system in time or frequency domain.

Fourier–image of the Volterra kernel of n^{th} order [2]:

$$W_n(j\omega_1,\ldots,j\omega_n) = F_n\langle w_n(t_1,\ldots,t_n)\rangle = \int_0^{+\infty} \cdots \int w_n(t_1,\ldots,t_n) \exp\left(-j\sum_{i=1}^{n}\omega_i t_i\right) \prod_{i=1}^{n} dt_i, \quad (2)$$

where $F_n\langle\ \rangle$ — n–dimensional Fourier transform.

Nonlinear system model based on the VS in frequency domain can be represented as:

$$y[x(t)] = \sum_{n=1}^{\infty} F_n^{-1}\left\langle W_n(j\omega_1,\ldots,j\omega_n) \prod_{i=1}^{n} X(j\omega_i)\right\rangle_{t_1=\ldots=t_n=t}, \quad (3)$$

where $F_n^{-1}\langle\ \rangle$ – inverse n-dimensional Fourier transform;

$X(j\omega_i)$ – Fourier-image of the input signal.

Nonlinear dynamical system identification in the frequency domain comes to determination at set frequency of the values of the amplitude and phase of multidimensional transfer function – Multidimensional amplitude- and phase-frequency responses, which represent the modulus and phase of the multidimensional Fourier transform of n^{th} order Volterra kernel respectively,

$$|W_n(j\omega_1,\ \dots,\ j\omega_n)| = \sqrt{[\mathrm{Re}(W_n(j\omega_1,\dots,j\omega_n))]^2 + [\mathrm{Im}(W_n(j\omega_1,\dots,j\omega_n))]^2}\ ,\qquad(4)$$

$$\arg W_n(j\omega_1,\dots,j\omega_n) = arctg\,\frac{\mathrm{Im}(W_n(j\omega_1,\dots,j\omega_n))}{\mathrm{Re}(W_n(j\omega_1,\dots,j\omega_n))}\ ,\qquad(5)$$

where Re and Im – the real and imaginary parts of the complex function of n-variables, respectively.

In frequency domain, the test polyharmonic actions are used for identification. They are represented as such signals [4]:

$$x(t) = \sum_{k=1}^{n} A_k \cos(\omega_k t + \varphi_k),\qquad(6)$$

where n – order of estimated transfer function; A_k, ω_k and φ_k — respectively, the amplitude, frequency and phase of the k^{th} harmonic. To simplify the calculations, amplitudes A_k are assumed to be identical, and phases φ_k – to be equal to zero.

At identification of the nonlinear systems in frequency domain, limitations on the choice of the frequencies of test polyharmonic signal are necessary. For example, the partial component of the second order [4]

$$\begin{aligned} y_2[x(t)] &= \int_{-\infty}^{\infty}\!\!\int w_2(\tau_1,\tau_2)x(t-\tau_1)x(t-\tau_2)d\tau_1 d\tau_2 = \\ &= \frac{A^2}{2}\{|W_2(j\omega_1, j\omega_2)|\cos[(\omega_1+\omega_2)t + \arg W_2(j\omega_1, j\omega_2)] + \\ &+ |W_2(j\omega_2, j\omega_1)|\cos[(\omega_1+\omega_2)t + \arg W_2(j\omega_2, j\omega_1)] + \\ &+ |W_2(j\omega_1, j\omega_1)|\cos[2\omega_1 t + \arg W_2(j\omega_1, j\omega_1)] + \\ &+ |W_2(j\omega_2, j\omega_2)|\cos[2\omega_2 t + \arg W_2(j\omega_2, j\omega_2)] + \\ &+ |W_2(j\omega_1, -j\omega_2)|\cos[(\omega_1-\omega_2)t + \arg W_2(j\omega_1, -j\omega_2)] + \\ &+ |W_2(j\omega_2, -j\omega_1)|\cos[(\omega_2-\omega_1)t + \arg W_2(j\omega_2, -j\omega_1)] + \\ &+ |W_2(j\omega_1, -j\omega_1)|\cos(\arg W_2(j\omega_1, -j\omega_1)) + \\ &+ |W_2(j\omega_2, -j\omega_2)|\cos(\arg W_2(j\omega_2, -j\omega_2))\}. \end{aligned}\qquad(7)$$

If to extract the harmonic components with frequency $\omega_1+\omega_2$ from this expression, and taking into account the symmetry of Volterra kernel [1], we practically need to derive only one harmonic with a doubled amplitude, then we obtain values for the module and the phase of the Volterra kernel Fourier transform of the second order. It is necessary to apply filtration to determine the transfer function, i.e. derive the harmonic with frequency $\omega_1+\omega_2$. To uniquely identify the informative harmonics, it is necessary the frequencies of the harmonic components in (7) to be different. It is necessary to impose some limitations on the choice of the frequencies of the test polyharmonic signal, which ensure the inequality of the combination frequencies in the harmonics of the output signal. Similar restrictions are also necessary in the case of determination of the n^{th} order transfer function ($n>2$). The third partial component also contains informative and "extra" harmonics [4].

Limitations on the choice of the frequencies of the test polyharmonic signal, considered in [4-7] are different.

Limitation on the choice of frequencies [5-7], which requires incommensurability of the frequencies of the test signal, i.e. the inequalities between any linear combinations of frequencies with arbitrary integer coefficients are equivalent to requiring the numerical irrationality of all frequencies except one:

$$\forall\{\omega_i | i=\overline{1,n}\}\ \ \exists\{\omega_p | p=\overline{1,n-1}\} \subset \{\omega_i | i=\overline{1,n}\}$$
$$\Big|\ \ \{\omega_p | p=\overline{1,n-1}\} \subset \mathbf{R}\setminus\mathbf{Q},$$

where R — the set of real numbers;

Q — the set of rational numbers;

R\Q — difference of sets R and Q — the set of irrational numbers.

The admissibility of only one rational frequency follows from the fact that already for two frequencies it is possible to choose such integer multipliers that will give the same result when multiplied by these frequencies. As such factors, it can be used the numerator and the denominator of their ratio. Indeed, let $\omega_1=A$ and $\omega_2=B$, where A and B — rational numbers. Then $\frac{A}{B}=\frac{m}{n}$ — rational number; $m\in$Z and $n\in$N, where Z and N — respectively, the set of integers and natural numbers; and $n\omega_1=nA=mB=m\omega_2$. To prove the impossibility of an even larger number of rationally-numerical frequencies, it is sufficient to consider any two sums

It should be noted that this restriction, in addition to the irrationality of frequencies, also requires the irrationality of their ratio, i.e. $\forall p_1 \neq p_2 \Rightarrow \frac{\omega_{p_1}}{\omega_{p_2}} \in \mathbf{R}\setminus\mathbf{Q},$, because otherwise this restriction is also violated.

Another limitation on the frequency range [4], also requires an inequality between any linear combination of frequencies with different integer coefficients, but only for the coefficients, whose absolute value of sum is no more than order of determined transfer function, i.e.

$$\sum_{i=1}^{n} a_i\omega_i \neq \sum_{i=1}^{n} b_i\omega_i \ \text{ at }\ \sum_{i=1}^{n}|a_i| \leq n \ \text{ and }\ \sum_{i=1}^{n}|b_i| \leq n.$$

Since this restriction is related to the order of the determined transfer function, it is weaker than the previous one.

Both restrictions define only sufficient conditions for the frequencies selection, because the possibility of unambiguous frequency filtering, but not vice versa, follows from performance of any of them. Let us consider an example of a weaker restriction [4]. Let the order of the determined Volterra kernel is n=2. Then, in accordance with (7), the second partial component will contain the following combination frequency:

$$\omega_1+\omega_2;\ 2\omega_1;\ 2\omega_2;\ \omega_1-\omega_2;\ -\omega_1+\omega_2;\ 0. \quad (8)$$

Assuming that any combination frequencies other than the first are equal, for example, the third and fourth one: $2\omega_2=\omega_1-\omega_2$, i.e. $\omega_1=3\omega_2$, then the expressions (8) take the form:

$$4\omega_2;\ 6\omega_2;\ 2\omega_2;\ 2\omega_2;\ -2\omega_2;\ 0. \quad (9)$$

Comparing combination frequency in (9), it is easy to see that the first of them does not coincide with any other. Thus, if this restriction is violated, the possibility of unambiguous filtering of the harmonic with the first combination frequency listed in (8) remains unchanged.

In the general case, for arbitrary n, in the partial component for the definition of transfer function only harmonics of one combination frequency are used, therefore, there is no need in inequality of all combination frequencies among themselves, i.e. weaker restrictions on the choice of frequencies are possible. The following theorem gives the minimum of restrictions.

The theorem on the choice of frequencies. For the uniqueness of the filtration from the n^{th} partial component of the harmonic response with the combinational frequency

$$\omega_1+\omega_2+\ldots+\omega_n \quad (10)$$

it is necessary and sufficient that the latest does not equal the combinational frequencies of the form: $k_1\omega_1+k_2\omega_2+\ldots+k_n\omega_n$, where coefficients $\{ k_i \mid i=1,2,\ldots, n \}$ satisfy the conditions:

— power of a finite set of negative coefficients (k_i<0) can take values from 0 to $\left[\frac{n}{2}\right]$, where [] — function of extraction of an integer part of a number;

— sum of absolute values of coefficients k_i is now more than order n of determined kernel: $\sum_{i=1}^{n}|k_i| \le n$;

— sum of absolute values of coefficients k_i is comparable by modulo 2 with the order n - the order of the kernel being determined: $\sum_{i=1}^{n}|k_i| \equiv n \pmod 2$, i.e. $n-\sum_{i=1}^{n}|k_i| = 2\ell,\ \ \ell \in \boldsymbol{N}$.

Proof. All the combination frequencies of the n^{th} partial component can be found by substituting the expression (6) in the n^{th} member of VS (1) for the test polyharmonic signal in complex form

$$x(t) = A\sum_{k=1}^{n}\cos(\omega_k t) = \frac{A}{2}\sum_{k=1}^{n}\left(e^{j\omega_k t} + e^{-j\omega_k t}\right) = = \frac{A}{2}\left(\sum_{k=1}^{n} e^{j\omega_k t} + \sum_{k=1}^{n} e^{-j\omega_k t}\right). \quad (11)$$

After substitution

$$y_n[x(t)] = \int_0^{\infty}\!\!\ldots\!\int w_n(\tau_1,\ldots\tau_n)\prod_{i=1}^{n} x(t-\tau_i)d\tau_i = = \frac{A^n}{2^n}\int_0^{\infty}\!\!\ldots\!\int w_n(\tau_1,\ldots\tau_n)\prod_{i=1}^{n}\left(\sum_{k=1}^{n} e^{j\omega_k(t-\tau_i)} + \sum_{k=1}^{n} e^{-j\omega_k(t-\tau_i)}\right)d\tau_i \quad (12)$$

For further transformations, a generalization of the binomial formula is used [10]:

$$\prod_{i=1}^{n}(\alpha_i+\beta_i) = \sum_{\{s_m | m=\overline{0,n}\}} \alpha_{s_1}\ldots\alpha_{s_m}\beta_{s_{m+1}}\ldots\beta_{s_n} = \sum_{m=0}^{n}\sum_{\{s_m\}}^{C_n^m}\prod_{l=1}^{m}\alpha_{s_l}\prod_{r=m+1}^{n}\beta_{s_r}, \quad (13)$$

where $\{s_m \in \boldsymbol{N} | m=\overline{0,n}\}$ – all possible combinations of n by m;

$\{s_r | r=\overline{m+1,n}\}$ – combination that complements all possible combinations of n by m to a permutation of n natural numbers.

Since here each combination $\{s_m | m=\overline{0,n}\}$ corresponds only one complementing one, then each summand in equation (13) represents the sum of $C_n^m = \frac{n!}{m!(n-m)!}$ terms. We transform into (12) the product of n terms, each of which consists of $2n$ summand, according to the formula (13):

$$y_n[x(t)] = \frac{A^n}{2^n}\int_0^{\infty}\!\!\ldots\!\int w_n(\tau_1,\ldots\tau_n) \left(\sum_{m=0}^{n}\sum_{\{s_m | m=\overline{0,n}\}}^{C_n^m}\prod_{l=1}^{m}\sum_{k=1}^{n} e^{j\omega_k(t-\tau_{s_l})}\prod_{r=m+1}^{n}\sum_{k=1}^{n} e^{-j\omega_k(t-\tau_{s_r})}\right)\prod_{i=1}^{n} d\tau_i. \quad (14)$$

Taking into account the symmetry of the kernels $w_n(\tau_1,\ldots\tau_n)$ instead of combinations – $\{s_m | m=\overline{0,n}\}$ can be considered C_n^m of set of number $\{\overline{1,m}\}$, where $m=\overline{0,n}$. At m=0 these sets of combinations and numbers are empty sets. Then

$$y_n[x(t)] = \frac{A^n}{2^n}\int_0^{\infty}\!\!\ldots\!\int w_n(\tau_1,\ldots\tau_n) \left(\sum_{m=0}^{n} C_n^m\prod_{l=1}^{m}\sum_{k=1}^{n} e^{j\omega_k(t-\tau_l)}\prod_{r=m+1}^{n}\sum_{k=1}^{n} e^{-j\omega_k(t-\tau_r)}\right)\prod_{i=1}^{n} d\tau_i. \quad (15)$$

After multiplying the sums in the products we obtain

$$y_n[x(t)] = \frac{A^n}{2^n}\int_0^{\infty}\!\!\ldots\!\int w_n(\tau_1,\ldots\tau_n) \left(\sum_{m=0}^{n} C_n^m\sum_{k_1=1}^{n}\ldots\sum_{k_n=1}^{n} e^{j\omega_{k_1}(t-\tau_1)}\ldots e^{j\omega_{k_m}(t-\tau_m)} e^{-j\omega_{k_{m+1}}(t-\tau_{m+1})}\ldots e^{-j\omega_{k_n}(t-\tau_n)}\right)\prod_{i=1}^{n} d\tau_i. \quad (16)$$

Because of the reduction of such terms in the exponents, we obtain (16)

$$y_n[x(t)] = \frac{A^n}{2^n}\int_0^{\infty}\!\!\ldots\!\int w_n(\tau_1,\ldots\tau_n) \left(\sum_{m=0}^{n} C_n^m\sum_{k_1=1}^{n}\ldots\sum_{k_n=1}^{n} e^{j\left(\sum_{l=1}^{m}\omega_{k_l} - \sum_{r=m+1}^{n}\omega_{k_r}\right)t} e^{-j\left(\sum_{l=1}^{m}\omega_{k_l}\tau_{k_l} - \sum_{r=m+1}^{n}\omega_{k_r}\tau_{k_r}\right)}\right)\prod_{i=1}^{n} d\tau_i. \quad (17)$$

After the transformations we obtain

$$y_n[x(t)] = \frac{A^n}{2^n}\sum_{m=0}^{n} C_n^m \sum_{k_1=1}^{n}\cdots\sum_{k_n=1}^{n} e^{j\left(\sum_{l=1}^{m}\omega_{k_l} - \sum_{r=m+1}^{n}\omega_{k_r}\right)t} \times$$

$$\times \int_0^{\infty} \cdots \int w_n(\tau_1,\dots\tau_n)\, e^{-j\left(\sum_{l=1}^{m}\omega_{k_l}\tau_{k_l} - \sum_{r=m+1}^{n}\omega_{k_r}\tau_{k_r}\right)} \prod_{i=1}^{n} d\tau_i. \qquad (18)$$

The replacement of the integrals by the corresponding Fourier-images Volterra kernel gives

$$y_n[x(t)] = \frac{A^n}{2^n}\sum_{m=0}^{n} C_n^m \sum_{k_1=1}^{n}\cdots\sum_{k_n=1}^{n} e^{j\left(\sum_{l=1}^{m}\omega_{k_l} - \sum_{r=m+1}^{n}\omega_{k_r}\right)t} W_n\left(-j\omega_{k_1},\dots -j\omega_{k_m}, j\omega_{k_{m+1}},\dots j\omega_{k_n}\right). \qquad (19)$$

After combining complex conjugate functions in expression (19), the n^{th} partial component of the nonlinear dynamical system response

$$y_n[x(t)] = \frac{A^n}{2^{n-1}}\sum_{m=0}^{\left[\frac{n}{2}\right]} C_n^m \sum_{k_1=1}^{n}\cdots\sum_{k_n=1}^{n} \left|W_n\left(-j\omega_{k_1},\dots -j\omega_{k_m}, j\omega_{k_{m+1}},\dots j\omega_{k_n}\right)\right| \times$$

$$\times \cos\left(\left(-\sum_{l=0}^{m}\omega_{k_l} + \sum_{l=m+1}^{n}\omega_{k_l}\right)t + \arg W_n\left(-j\omega_{k_1},\dots -j\omega_{k_m}, j\omega_{k_{m+1}},\dots j\omega_{k_n}\right)\right) \qquad (20)$$

A special case of application of this formula when n=2 is known [4], and a result similar to the expression (20) is represented by expression (7). Thus, the frequencies in the partial component are expressed by the formula:

$$-\sum_{l=0}^{m}\omega_{k_l} + \sum_{l=n+1}^{n}\omega_{k_l}, \quad \text{where} \quad m = \overline{0,\left[\frac{n}{2}\right]}. \qquad (21)$$

The coefficients in the expression (21) for combination frequency have the following properties:

— the sum of the moduli of the coefficients does not exceed n;

— the number of negative summands of m in combination frequency satisfies the inequality $0 \le m \le \left[\frac{n}{2}\right]$;

— among the frequencies $\left\{\omega_{k_i} \middle| k_i = \overline{1,n}\right\}$ possible repetitive frequency, i.e. in (20) it is possible to reciprocally "cancelation" of the same frequencies with coefficients of opposite sign. An example of this is the expression (7), where the last two summands have a degenerate combination frequency ω=0. Therefore, the sum of the moduli of the coefficients is equal to the order of the determined transfer function minus the doubled number of possible "cancellation", because at one "cancellation" two combination frequencies are mutually destroyed. This is equivalent to the simultaneous parity or oddness of the sum of the moduli of coefficients and order of determined transfer function, or the comparability of the latest by modulo 2.

A single-valued filtration of harmonics with combination frequency (21) is possible only if it is not equal to the remaining combination frequencies in the partial component. According to (21), the partial component contains harmonics only with combination frequency, given in the statement of the theorem. Therefore, from single-valued filtration follows the inequality of combination frequency (10) to the remaining combinational frequencies. Thus, the *necessity* of the formulated conditions on the choice of frequencies is proved.

The proof of the *sufficiency* is based on the fact that the combination frequency (10) inequality to another combination frequency from the statement of the theorem, which follows from (20), makes it possible to uniquely filter the harmonic with the desired combination frequency from the partial response component of the identifiable nonlinear dynamical system.

Limitation on the choice of frequencies [5] is not applicable in practice, since it requires irrationality of frequencies, which is impossible with the use of a computer to determine transfer function. This follows from the limitations of the bit grid of the computer, i.e. using of rational numbers only.

The limitation on the choice of frequencies given in [4] can be applied in practice, but as shown in (8) and (9), unnecessarily limits the choice of the used frequencies. Generally speaking, this restriction is applicable not only for filtering harmonics with frequencies (10), but also for any other harmonics in the partial component of the response. The application of this restriction in the determination of transfer function of the second order requires fulfillment of five inequalities satisfaction for frequencies of the input signal:

$\omega_1 \ne 0$, $\omega_2 \ne 0$, $\omega_1 \ne \omega_2$, $3\omega_1 \ne \omega_2$ и $\omega_1 \ne 3\omega_2$.

During the determination of the third order transfer function, it is already required to ensure the fulfillment of 45 inequalities between the frequencies of the input signal:

$\omega_1 \ne 0$, $\omega_2 \ne 0$, $\omega_3 \ne 0$, $\omega_1 \ne \omega_2$, $\omega_1 \ne \omega_3$, $\omega_2 \ne \omega_3$, $\omega_1 \ne \omega_2+\omega_3$, $\omega_2 \ne \omega_1+\omega_3$, $\omega_3 \ne \omega_1+\omega_2$,

$2\omega_1 \ne \omega_2$, $2\omega_2 \ne \omega_1$, $2\omega_3 \ne \omega_1$, $2\omega_1 \ne \omega_3$, $2\omega_2 \ne \omega_3$, $2\omega_3 \ne \omega_2$, $2\omega_1 \ne \omega_2+\omega_3$, $2\omega_2 \ne \omega_1+\omega_3$, $2\omega_3 \ne \omega_1+\omega_2$,

$2\omega_1 \ne \omega_2-\omega_3$, $2\omega_2 \ne \omega_1-\omega_3$, $2\omega_3 \ne \omega_1-\omega_2$, $2\omega_1 \ne -\omega_2+\omega_3$, $2\omega_2 \ne -\omega_1+\omega_3$, $2\omega_3 \ne -\omega_1+\omega_2$,

$3\omega_1 \ne \omega_2$, $3\omega_2 \ne \omega_1$, $3\omega_3 \ne \omega_1$, $3\omega_1 \ne \omega_3$, $3\omega_2 \ne \omega_3$, $3\omega_3 \ne \omega_2$,

$3\omega_1 \ne \omega_2+2\omega_3$, $3\omega_2 \ne \omega_1+2\omega_3$, $3\omega_3 \ne \omega_1+2\omega_2$, $3\omega_1 \ne 2\omega_2+\omega_3$, $3\omega_2 \ne 2\omega_1+\omega_3$, $3\omega_3 \ne 2\omega_1+\omega_2$,

$3\omega_1 \ne \omega_2-2\omega_3$, $3\omega_2 \ne \omega_1-2\omega_3$, $3\omega_3 \ne \omega_1-2\omega_2$, $3\omega_1 \ne -2\omega_2+\omega_3$, $3\omega_2 \ne -2\omega_1+\omega_3$,; $3\omega_3 \ne -2\omega_1+\omega_2$,

$4\omega_1 \ne \omega_2+\omega_3$, $4\omega_2 \ne \omega_1+\omega_3$ и $4\omega_3 \ne \omega_1+\omega_2$.

The proposed new frequency limits are designed to isolate only harmonics with combination frequency (10). Their application will allow expanding maximally the set of admissible frequencies used in identification. In addition, it allows reducing the number of logical conditions (inequalities) in choosing frequencies that determine the possibility of unambiguous filtering of the harmonic with the required combination frequency. Thus, the application of this constraint in the determination of the second order transfer function requires fulfillment of not five but three inequalities between the frequencies of the input signal:

$\omega_1 \ne 0$, $\omega_2 \ne 0$ and $\omega_1 \ne \omega_2$.

At determining of the third order transfer function, it is required to provide the fulfillment of only 15 inequalities between the frequencies of the input signal:

$\omega_1 \ne 0$, $\omega_2 \ne 0$, $\omega_3 \ne 0$, $\omega_1 \ne \omega_2$, $\omega_1 \ne \omega_3$, $\omega_2 \ne \omega_3$, $2\omega_1 \ne \omega_2+\omega_3$, $2\omega_2 \ne \omega_1+\omega_3$, $2\omega_3 \ne \omega_1+\omega_2$,

$2\omega_1 \ne \omega_2-\omega_3$, $2\omega_2 \ne \omega_1-\omega_3$, $2\omega_3 \ne \omega_1-\omega_2$, $2\omega_1 \ne -\omega_2+\omega_3$, $2\omega_2 \ne -\omega_1+\omega_3$ and $2\omega_3 \ne -\omega_1+\omega_2$.

III. CONCLUSIONS

Thus, the proved theorem on the choice of the frequencies of the test polyharmonic signal in the identification of nonlinear dynamical system using multidimensional transfer functions significantly weakens the known conditions [4]. The use of new conditions reduces the number of interpolations during the recovery of transfer function, and this is more significant, the higher the order of the estimated transfer function.

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