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FUNDAMENTAL SOLUTIONS FOR A PIECEWISE-HOMOGENEOUS TRANSVERSELY ISOTROPIC ELASTIC SPACE

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The problem of construction of the fundamental solutions for a piecewise-homogeneous transversely isotropic space is reduced to a matrix Riemann problem in the space of slowly increasing distributions. We propose a method for the solution of this problem. As a result, in the explicit form, we obtain expressions for the components of the vector of fundamental solution and simple representations for the components of the stress tensor and the vector of displacements in the plane of joint of transversely isotropic elastic half spaces subjected to the action of concentrated normal and tangential forces. We study the fields of stresses and displacements in the plane of joint of the half spaces. In particular, for some combinations of materials, we present the numerical values of the coefficients of influence of concentrated forces on the stresses and displacements. We also establish conditions under which the normal displacements are absent in the plane of joint of transversely isotropic elastic half spaces.

Keywords: fundamental solutions, matrix Riemann problem, transversely isotropic inhomogeneous space, generalized functions.

The investigation of stress concentration in the vicinity of interface and internal defects, such as cracks or inclusions, in thermoelastic fields is of significant practical interest. Numerous works are devoted to the investigation of this problem in different media. Thus, in particular, the problems of stationary thermoelasticity for bodies containing heat-permeable disk-shaped inclusions whose surfaces are under the conditions of imperfect thermal contact and the problems with thin thermally active disk-shaped inclusions were considered in [3–7]. The analyzed problems were reduced to hypersingular integral equations of the first and second kinds for which it is possible to obtain the exact solutions.

In [2, 9–12, 15, 18], the nonsymmetric problems of elasticity and thermoelasticity for interface stress concentrators, such as cracks or rigid inclusions in piecewise-homogeneous transversely isotropic spaces, were reduced to systems of two-dimensional singular integral equations (SIE) by the method of singular integral relations (SIR) [29], and a method for their solution was proposed. A similar approach was applied in [8, 13, 14, 20–22] to solve the problems of interface and internal defects in piecewise-homogeneous anisotropic media.

For the mathematical statement and solution of problems of this kind for defects, it is necessary to impose boundary conditions on the defect itself, namely, either the stresses acting on the crack faces or the displacements on the inclusion. In the physical statement of the problems of evaluation of the fields of stresses and displacements in the vicinity of stress concentrators, the values of stresses or displacements are known on the boundary of the domain, at certain internal points, or at infinity (for the unbounded bodies). Hence, the determination of the boundary conditions imposed on the defect is a separate problem.

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Within the framework of the linear theory of elasticity, for the solution of this problem, it is necessary to know the distribution of the fields of stresses and displacements in the corresponding piecewise-homogeneous bodies without defects in the presence of volume forces.

In particular, for piecewise-homogeneous isotropic and transversely isotropic spaces, these solutions were presented in [33] and [32], respectively. However, in [32], the solutions have a quite complicated structure. The Green functions for piecewise homogeneous transversely isotropic spaces in the presence of concentrated heat sources and in the absence of thermodiffusion were constructed in [24]. In the presence of thermodiffusion, they were constructed in [30]. In [23, 31], the Green functions were constructed for a layered thermoelastic medium.

The method of fundamental solutions in the space $\mathfrak{I}'(\mathbb{R}^3)$ of generalized tempered functions proves to be an efficient method for the solution of the indicated problem. In particular, in [16, 17], the problem of construction of the fundamental solutions for piecewise homogeneous two-dimensional anisotropic media was reduced to the matrix Riemann problem for a part of variables in the space $\mathfrak{I}'(\mathbb{R}^3)$, and an approach to its solution was proposed. In the present work, the indicated approach is generalized for the construction, in the explicit analytic form, of fundamental solutions in piecewise-homogeneous transversely isotropic space, which enables us to study the influence of volume loads on the stresses and displacements in the plane of joint of the materials.

1. Statement of the Problem

Assume that the volume forces $\mathbf{P}(x,y,z) = (P_1, P_2, P_3)$ concentrated in some domains of dimension n, $n = 0, 1, 2, 3$, act in an inhomogeneous space formed by two different transversely isotropic half spaces completely joined in the plane $z = 0$. The elastic strained state of the space is described by the following vector:

$$\mathbf{v} = \{v_k(x,y,z)\}_{k=1,\ldots,9} = \{\sigma_x, \sigma_y, \sigma_z, \tau_{yz}, \tau_{xz}, \tau_{xy}, u, v, w\}. \tag{1}$$

By using the equilibrium equations and generalized Hooke's law for the components of the vector $\mathbf{v}$, in the space of tempered generalized functions $\mathfrak{I}'(\mathbb{R}^3)$, we get the following boundary-value problem:

$$\mathbf{D}[z, \partial_1, \partial_2, \partial_3]\mathbf{v} = \mathbf{F}, \quad \mathbf{v}, \mathbf{F} \in \mathfrak{I}'(\mathbb{R}^3). \tag{2}$$

$$v_k(x,y,+0) = v_k(x,y,-0), \quad k = 1,\ldots,9, \quad k \neq 1,2,6, \tag{3}$$

$$v_k(x,y,x)\big|_{(x,y,z)\to\infty} = 0, \quad k = 1,\ldots,9. \tag{4}$$

Here, we introduce the following notation:

$$\mathbf{D} = \left\| \begin{matrix} \mathbf{D}_0 & \mathbf{O}_{3\times 3} \\ -\mathbf{S} & \mathbf{D}_0^{\top} \end{matrix} \right\|, \qquad \mathbf{F}^{\top} = \left\| -P_1, -P_2, -P_3, 0, 0, 0, 0, 0, 0 \right\| \cdot \delta(\Omega),$$

$$\mathbf{S} = \left\| \begin{matrix} \mathbf{S}_1 & \mathbf{O}_{3\times 3} \\ \mathbf{O}_{3\times 3} & \mathbf{S}_2 \end{matrix} \right\|, \qquad \mathbf{D}_0 = \left\| \begin{matrix} \partial_1 & 0 & 0 & 0 & \partial_3 & \partial_2 \\ 0 & \partial_2 & 0 & \partial_3 & 0 & \partial_1 \\ 0 & 0 & \partial_3 & \partial_2 & \partial_1 & 0 \end{matrix} \right\|,$$

$$\mathbf{S}_1 = \left\| \begin{matrix} s_{11} & s_{12} & s_{13} \\ s_{21} & s_{11} & s_{13} \\ s_{13} & s_{13} & s_{33} \end{matrix} \right\|, \qquad \mathbf{S}_2 = \left\| \begin{matrix} s_{44} & 0 & 0 \\ 0 & s_{44} & 0 \\ 0 & 0 & s_{66} \end{matrix} \right\|,$$

$$\partial_1 = \frac{\partial}{\partial x}, \ \partial_2 = \frac{\partial}{\partial y}, \ \partial_3 = \frac{\partial}{\partial z};$$

$s_{kj} = \theta(z)s_{kj}^{+} + \theta(-z)s_{kj}^{-}$; Ω is the domain of concentration of volume forces; $\delta(\Omega)$ is the characteristic function of the domain Ω from $\Im'(\mathbb{R}^3)$; $s_{kj}^{\pm}$ are the coefficients of generalized Hooke's law for the upper $z > 0$ and lower $z < 0$ half-spaces, respectively, and $\mathbf{O}_{3\times 3}$ is the 3×3 null matrix.

2. Construction of the Fundamental Solution

The components of the vector $\mathbf{v}$ (1) are represented in the form

$$v_k(x, y, z) = \sum_{j=1}^{3} w_{kj} * F_j, \tag{5}$$

where the functions $w_{kj}(x, y, z) \in \Im'(\mathbb{R}^3)$ are the components of the system of the fundamental solutions $\mathbf{w}_j = \{w_{kj}\}_{k=1,\ldots,9}$, $j = 1, 2, 3$, of problem (2)–(4), i.e., $\mathbf{w}_j$ are the solutions of the system of boundary-value problems

$$\mathbf{D}[z, \partial_1, \partial_2, \partial_3]\, \mathbf{w}_j = \mathbf{f}^0, \quad \mathbf{w}_j, \mathbf{f}^0 \in \Im'(\mathbb{R}^3), \tag{6}$$

$$w_{kj}(x, y, +0) = w_{kj}(x, y, -0), \quad k = 1, \ldots, 9, \quad k \neq 1, 2, 6, \tag{7}$$

$$w_{kj}\,(x, y, x)\big|_{(x,y,z)\to\infty} = 0, \quad k = 1, \ldots, 9, \tag{8}$$

where $\mathbf{f}^0 = \left\{\delta_{kj}\delta(x - x_0, y - y_0, z - z_0)\right\}_{k=1}^{9}$ and δ_{kj} is the Kronecker delta.

We represent the components of the vectors $\mathbf{w}_j$ in the form

$$w_{kj} = \theta(z)w_{kj} + \theta(-z)w_{kj} = w_{kj}^{+} + w_{kj}^{-},$$

where $w_{kj}^{\pm} \in \mathfrak{I}'(\mathbb{R}_{\pm}^3)$ and $\mathbb{R}_{\pm}^3 = \mathbb{R}^2 \times \mathbb{R}_{\pm}$, and apply the operator of three-dimensional Fourier transform F_3 from $\mathfrak{I}'(\mathbb{R}^3)$ to the matrix equation (6). Thus, by using equalities (7), conditions (8), and the results obtained in [7, 13, 14, 16, 17, 19–22], for

$$W_{kj}^{\pm}(\alpha_1, \alpha_2, \alpha_3) = F_3[w_{kj}^{\pm}] \in \mathfrak{I}'(\mathbb{R}^3),$$

we obtain the following matrix equation:

$$\mathbf{M}^{+}\mathbf{W}_j^{+} = \mathbf{M}^{-}\mathbf{W}_j^{-} + \mathbf{F}_j^0, \quad \mathbf{W}_j^{\pm}, \mathbf{F}_j^0 \in \mathfrak{I}'(\mathbb{R}^3), \quad j = 1, 2, 3. \tag{9}$$

Here, we have denoted

$$\mathbf{W}_j^{\pm} = \{W_{kj}^{\pm}\}_{k=1}^{9}, \quad \mathbf{M}^{\pm} = \mathbf{D}[\pm 0, -i\alpha_1, -i\alpha_2, -i\alpha_3], \quad \mathbf{F}_j^0 = \{\delta_{kj} e_0\}_{k=1}^{9},$$

where $e_0 = \exp(i\alpha_1 x_0 + i\alpha_2 y_0 + i\alpha_3 z_0)$.

The functions $w_{kj}^{\pm} \in \mathfrak{I}'(\mathbb{R}_{\pm}^3)$ admit an analytic representation [7, 13, 14, 16] with respect to the variable α_3 and, therefore, Eq. (9) is the boundary condition of the matrix Riemann problem in the variable α_3.

By using the properties of generalized functions applying using the methods proposed in [7, 13, 14, 16, 17, 19–22], we represent the boundary conditions (9) in the form

$$\mathbf{M}_{\pm}\mathbf{W}_j^{\pm} = \mathbf{F}_j^{\pm}, \quad \mathbf{W}_j^{\pm}, \mathbf{F}_j^{\pm} \in \mathfrak{I}'(\mathbb{R}^3), \quad j = 1, 2, 3, \tag{10}$$

where

$$\mathbf{F}_j^{\pm} = \{f_{kj}^{\pm}\}_{k=1,\ldots,9}, \quad f_{kj}^{\pm} = \theta(\pm z_0)\, e_0 \delta_{kj} \mp \frac{1}{2}\chi_k,$$

$$\boldsymbol{\chi} = \{\chi_k\}_{k=1,\ldots,9} \in \mathfrak{I}'(\mathbb{R}^2), \quad \chi_k = 0, \quad k = 4, 5, 9,$$

$\chi_k(\alpha_1, \alpha_2)$ are unknown functions from $\mathfrak{I}'(\mathbb{R}^2)$. In order to determine these functions, we use the Fourier transforms of conditions (7).

The unknown functions are represented in the form

$$W_{7j}^{\pm} = -(-i\alpha_2)\Psi_{1j}^{\pm} - (-i\alpha_1)\Psi_{2j}^{\pm}, \quad W_{8j}^{\pm} = (-i\alpha_1)\Psi_{1j}^{\pm} - (-i\alpha_2)\Psi_{2j}^{\pm}, \tag{11}$$

$$W_{5j}^{\pm} = -(-i\alpha_2)T_{1j}^{\pm} - (-i\alpha_1)T_{2j}^{\pm}, \quad W_{4j}^{\pm} = (-i\alpha_1)T_{1j}^{\pm} - (-i\alpha_2)T_{2j}^{\pm}, \tag{12}$$

where $\Psi_{kj}^{\pm}$ and $T_{kj}^{\pm}$, $k = 1, 2$, are new unknown functions. Then the matrix equation (10) can be split into two independent equations

$$\mathbf{L}_\pm \mathbf{U}_j^\pm = \mathbf{F}_{j1}^\pm, \quad \text{and} \quad \mathbf{G}_\pm \mathbf{V}_j^\pm = \mathbf{F}_{j2}^\pm. \tag{13}$$

Here, we have introduced the following notation:

$$\mathbf{U}_j^\pm = \{U_{kj}^\pm\}_{k=1,2} = \{T_{1j}^\pm, \Psi_{1j}^\pm\}, \quad \mathbf{V}_j^\pm = \{V_{kj}^\pm\}_{k=1,4} = \{W_{3j}^\pm, T_{2j}^\pm, \Psi_{2j}^\pm, W_{9j}^\pm\},$$

$$\mathbf{F}_{j1}^\pm = \{(-i\alpha_2)f_{1j}^\pm - (-i\alpha_1)f_{2j}^\pm, (-i\alpha_2)f_{7j}^\pm - (-i\alpha_1)f_{8j}^\pm\},$$

$$\mathbf{F}_{j2}^\pm = \{f_{3j}^\pm, (-i\alpha_1)f_{1j}^\pm + (-i\alpha_2)f_{2j}^\pm, (-i\alpha_2)f_{8j}^\pm + (-i\alpha_1)f_{7j}^\pm, f_{6j}^\pm\},$$

$$\mathbf{G}_\pm = \{g_{kj}^\pm\}_{k,j=1,\ldots,4}, \quad g_{11}^\pm = g_{44}^\pm = (-i\alpha_3), \quad g_{12}^\pm = r^2,$$

$$g_{22}^\pm = g_{33}^\pm = (-i\alpha_3)r^2, \quad g_{kj}^\pm = g_{jk}^\pm = 0, \quad k=1,2, \quad j=3,4,$$

$$g_{21}^\pm = -\frac{c_{13}^\pm}{c_{33}^\pm} g_{12}^\pm, \quad g_{23}^\pm = -\frac{\overline{c_{13}^\pm + c_{13}^{\pm\,2}}}{c_{33}^\pm} r^4, \quad g_{32}^\pm = -\frac{1}{c_{44}^\pm} g_{12}^\pm,$$

$$g_{34}^\pm = -g_{12}^\pm, \quad g_{41}^\pm = -\frac{1}{c_{33}^\pm}, \quad g_{43}^\pm = \frac{c_{13}^\pm}{c_{33}^\pm} g_{12}^\pm,$$

$$\mathbf{L}_\pm = \{\ell_{kj}^\pm\}_{k,j=1,2}, \quad \ell_{11}^\pm = (-i\alpha_3)r^{-2}, \quad \ell_{22}^\pm = (-i\alpha_3)r^2,$$

$$\ell_{21}^\pm = -\frac{r^2}{c_{44}^\pm}, \quad \ell_{21}^\pm = -c_{66}r^4, \quad r^2 = \alpha_1^2 + \alpha_2^2.$$

Directly from Eqs. (13), we obtain $\mathbf{U}_j^\pm = \mathbf{L}_\pm^{-1}\mathbf{F}_{j1}^\pm$ and $\mathbf{V}_j^\pm = \mathbf{G}_\pm^{-1}\mathbf{F}_{j2}^\pm$, where $\mathbf{L}_\pm^{-1} = \{\ell_{kj}^{*,\pm}\}_{k,j=1,2}$ and $\mathbf{G}_\pm^{-1} = \{g_{kj}^{*,\pm}\}_{i,j=1,\ldots,4}$. Further, by using representations (11) and (12), as a result of the inverse Fourier transformation, we represent the components of the vectors $\mathbf{u}_j^\pm = \{u_{kj}^\pm\}_{k=1,2} = F_3^{-1}[\mathbf{U}_j^\pm]$ and $\mathbf{v}_j^\pm = \{v_{kj}^\pm\}_{k=1,\ldots,4} = F_3^{-1}[\mathbf{V}_j^\pm]$, $j=1,2$, in the following form:

$$u_{kj} = \vartheta_{1j}\left\{\frac{S_{k1}(r_0^2 + (\xi_0|z-z_0|)^2)^{(2-k)/2}}{\xi_0|z-z_0| + \sqrt{r_0^2 + (\xi_0|z-z_0|)^2}}\right.$$

$$\left. + \frac{\tilde{\beta}_k(r_0^2 + (\hat{\xi}_0 z + \breve{\xi}_0 z_0)^2)^{(2-k)/2}}{\hat{\xi}_0|z| + \breve{\xi}_0|z_0| + \sqrt{r_0^2 + (\hat{\xi}_0 z + \breve{\xi}_0 z_0)^2}}\right\}, \quad k=1,2,$$

$$v_{1j} = -\vartheta_{2j} \sum_{n=1}^{2} \frac{R_{1,2,n}}{\left(r_0^2 + (\xi_n |z - z_0|)^2\right)^{3/2}} + \sum_{n,m=1}^{2} \frac{\beta^2_{1,n,m}}{\left(r_0^2 + \left(\hat{\xi}_n z + \breve{\xi}_m z_0\right)^2\right)^{3/2}},$$

$$v_{kj} = \vartheta_{2j} \sum_{n,m=1}^{2} \left\{ \frac{-R_{k,2,n}\left(r_0^2 + \left(\xi_n |z - z_0|^2\right)\right)^{-1/2}}{|z - z_0|\, \xi_n^+ + \sqrt{r_0^2 + (\xi_n |z - z_0|)^2}} \right.$$

$$\left. + \frac{\beta^2_{k,n,m}\left(r_0^2 + \left(\hat{\xi}_n z + \breve{\xi}_m z_0\right)^2\right)^{-1/2}}{\hat{\xi}_n |z| + \breve{\xi}_m |z_0| + \sqrt{r_0^2 + \left(\hat{\xi}_n z + \breve{\xi}_m z_0\right)^2}} \right\}, \quad k = 2, 4,$$

$$v_{3j} = \vartheta_{2j} \sum_{n,m=1}^{2} \left\{ \frac{-R_{3,2,n}}{|z - z_0|\, \xi_n^+ + \sqrt{r_0^2 + (\xi_n |z - z_0|)^2}} \right.$$

$$\left. + \frac{\beta^2_{3,n,m}}{\hat{\xi}_n |z| + \breve{\xi}_m |z_0| + \sqrt{r_0^2 + \left(\hat{\xi}_n z + \breve{\xi}_m z_0\right)^2}} \right\},$$

$$v_{13} = -\sum_{n=1}^{2} \frac{|z - z_0|\, \hat{R}_n}{\left(r_0^2 + (\xi_n |z - z_0|)^2\right)^{3/2}} + \sum_{n,m=1}^{2} \frac{z\hat{\beta}_{n,m} + z_0 \breve{\beta}_{n,m}}{\left(r_0^2 + \left(\hat{\xi}_n z + \breve{\xi}_m z_0\right)^2\right)^{3/2}},$$

$$v_{k3} = -\sum_{n=1}^{2} \frac{R_{k,1,n}}{\left(r_0^2 + (\xi_n |z - z_0|)^2\right)^{1/2}} + \sum_{n,m=1}^{2} \frac{\beta^1_{k,n,m}}{\left(r_0^2 + \left(\hat{\xi}_n z + \breve{\xi}_m z_0\right)^2\right)^{1/2}}, \quad k = 2, 4,$$

$$v_{33} = \sum_{n=1}^{2} R_{3,1,n} \left(\ln \frac{c}{2} + \ln\left(|z - z_0|\, \xi_n + \sqrt{r_0^2 + (\xi_m |z - z_0|)^2} \right) \right)$$

$$- \sum_{n,m=1}^{2} \beta^1_{3,n,m} \left(\ln \frac{c}{2} + \ln\left(\hat{\xi}_m |z| + \breve{\xi}_m |z_0| + \sqrt{r_0^2 + \left(\hat{\xi}_n z + \breve{\xi}_m z_0\right)^2} \right) \right).$$

Here, we have introduced the following notation:

$$\vartheta_{1j} = \frac{(y - y_0)^{2-j}}{(x - x_0)^{1-j}}, \quad \vartheta_{2j} = \frac{(x - x_0)^{2-j}}{(y - y_0)^{1-j}}, \quad S^{\pm}_{pk} = \frac{\tilde{\ell}^{\pm}_{pk}(\xi_0)}{2\xi_0},$$

$$S_{pk} = \theta(z, z_0) S^-_{pk} + \theta(-z, -z_0) S^+_{pk}, \quad p, k = 1, 2, \quad \tilde{\ell}^{\pm}_{11} = \tilde{\ell}^{\pm}_{22} = \pm \xi_0, \quad \tilde{\ell}^{\pm}_{21} = -\frac{1}{c^{\pm}_{44}},$$

$$\tilde{\ell}^{\pm}_{12} = -c^{\pm}_{66}, \quad \hat{R}_n = \theta(z, z_0) \hat{R}^+_n + \theta(-z, -z_0) \hat{R}^-_n, \quad \hat{R}^{\pm}_n = \xi^{\pm}_n R^{*,\mp}_{1,1,n},$$

$$\tilde{\beta}_p \;=\; -\theta(z,z_0)\,\tilde{\beta}_p^{++} + \theta(z,-z_0)\,\tilde{\beta}_p^{+-} + \theta(-z,z_0)\,\tilde{\beta}_p^{-+} - \theta(-z,-z_0)\,\tilde{\beta}_p^{+-}, \quad p=1,2\,,$$

$$\overset{\frown}{\beta}_{n,m} \;=\; -\theta(z,z_0)\,\overset{\frown}{\beta}{}_{n,m}^{++} + \theta(z,-z_0)\,\overset{\frown}{\beta}{}_{n,m}^{+-} + \theta(-z,z_0)\,\overset{\frown}{\beta}{}_{n,m}^{-+} - \theta(-z,-z_0)\,\overset{\frown}{\beta}{}_{n,m}^{--},$$

$$\breve{\beta}_{n,m} \;=\; -\theta(z,z_0)\,\breve{\beta}_{n,m}^{++} + \theta(z,-z_0)\,\breve{\beta}_{n,m}^{+-} + \theta(-z,z_0)\,\breve{\beta}_{n,m}^{-+} - \theta(-z,-z_0)\,\breve{\beta}_{n,m}^{--},$$

$$\beta_{k,n,m}^{p} \;=\; -\theta(z,z_0)\,\beta_{k,n,m}^{p,++} + \theta(z,-z_0)\,\beta_{k,n,m}^{p,+-} + \theta(-z,z_0)\,\beta_{k,n,m}^{p,-+} - \theta(-z,-z_0)\,\beta_{k,n,m}^{p,--},$$

$$p = 1,2\,,$$

$$\overset{\frown}{\beta}{}_{n,m}^{+\pm} \;=\; \xi_n^{+}\beta_{1,n,m}^{1,+\pm}, \quad \breve{\beta}_{n,m}^{+\pm} \;=\; \xi_m^{\pm}\beta_{1,n,m}^{1,+\pm}, \quad \overset{\frown}{\beta}{}_{n,m}^{-\mp} \;=\; \xi_n^{\mp}\beta_{1,n,m}^{1,-\mp}, \quad \breve{\beta}_{n,m}^{-\mp} \;=\; \xi_m^{-}\beta_{1,n,m}^{1,-\mp},$$

$$\overset{\frown}{\xi}_n \;=\; \theta(z,z_0)\xi_n^{+} + \theta(z,-z_0)\xi_n^{+} + \theta(-z,z_0)\xi_n^{-} - \theta(-z,-z_0)\xi_n^{-}, \quad n = 0,1,2\,,$$

$$\breve{\xi}_m \;=\; \theta(z,z_0)\xi_m^{+} - \theta(z,-z_0)\xi_m^{-} - \theta(-z,z_0)\xi_m^{+} + \theta(-z,-z_0)\xi_m^{-}, \quad m = 0,1,2\,,$$

$$R_{k,p,n} \;=\; \theta(z,z_0)R_{k,p,n}^{*,-} + \theta(-z,-z_0)R_{k,p,n}^{*,+}, \quad k = 2,3,4, \quad p = 1,2\,,$$

$$\tilde{\alpha}_p^{+} \;=\; \sum_{k=1}^{2}\tilde{a}_{pk}^{*}S_{k1}^{+}, \quad \tilde{\alpha}_p^{-} \;=\; \sum_{k=1}^{2}\tilde{a}_{pk}^{*}S_{k1}^{-}, \quad \tilde{\beta}_p^{+\pm} \;=\; \sum_{k=1}^{2}S_{pk}^{+}\alpha_k^{\pm}, \quad \tilde{\beta}_p^{-\pm} \;=\; \sum_{k=1}^{2}S_{pk}^{-}\alpha_k^{-}\,,$$

$$\tilde{\mathbf{A}}_0^{-1} \;=\; \{\tilde{a}_{kj}^{*}\}, \quad \tilde{\mathbf{A}}_0 \;=\; \mathbf{H}^{+} - \mathbf{H}^{-}, \quad \mathbf{H}^{\pm} \;=\; \pm\{S_{kp}^{\pm}\}_{k,p=1,2}, \quad \mathbf{A}_0^{-1} \;=\; \{a_{kj}^{*}\}_{k,j=1,\ldots,4}\,,$$

$$\mathbf{A}_0 \;=\; \mathbf{N}^{+} - \mathbf{N}^{-}, \quad \mathbf{N}^{\pm} \;=\; \pm\{R_{k,p}^{\pm}\}_{k,p=1,\ldots,4}, \quad R_{p,k}^{\pm} \;=\; \sum_{n=1}^{2}R_{p,k,n}^{*,\pm}, \quad \xi_0^{\pm} \;=\; \sqrt{\frac{c_{66}^{\pm}}{c_{44}^{\pm}}}\,,$$

$$R_{p,k,n}^{*,\pm} \;=\; \frac{\tilde{g}_{pk}^{\pm}(\xi_n)}{2\xi_n(\xi_{3-n}+\xi_n)(\xi_n-\xi_{3-n})}, \quad \alpha_{\ell,n}^{p,+} \;=\; \sum_{j=1}^{4}a_{\ell j}^{*}R_{j,p,n}^{*,-}, \quad \alpha_{4,n}^{p,-} \;=\; \sum_{j=1}^{4}a_{4\,j}^{*}R_{j,p,n}^{*,+}\,,$$

$$\beta_{\ell,n,m}^{p,+\pm} \;=\; \sum_{k=1}^{4}R_{\ell,k,n}^{*,+}\alpha_{k,m}^{p,\pm}, \quad \beta_{\ell,n,m}^{p,-\pm} \;=\; \sum_{k=1}^{4}R_{\ell,k,n}^{*,-}\alpha_{k,m}^{p,\pm}, \quad \xi_0 \;=\; \theta(z_0)\xi_0^{+} + \theta(-z_0)\xi_0^{-}\,,$$

$$\tilde{g}_{11}^{\pm}(\xi_n) \;=\; \tilde{g}_{44}^{\pm}(\xi_n) \;=\; \mp\,\xi_n\left(-\overline{c}_{34}\xi_n^{2}+\overline{c}_{13}-c_{13}^{2}-c_{13}c_{44}\right), \quad \overline{c}_{34} \;=\; c_{33}c_{44}\,,$$

$$\overline{c}_{13} \;=\; c_{11}c_{33}, \quad \xi_n \;=\; \theta(z)\xi_n^{+} + \theta(-z)\xi_n^{-}, \quad \tilde{g}_{12}^{\pm}(\xi_n) \;=\; -\left(c_{33}\xi_n + c_{13}^{2}\right)c_{44}\,,$$

$$\tilde{g}_{34}^{\pm}(\xi_n) = \left(c_{33}\xi_n^2 + c_{13}^2\right)c_{44}, \quad \tilde{g}_{41}^{\pm}(\xi_n) = c_{44}\xi_n^2 - c_{11}, \quad \tilde{g}_{42}^{\pm}(\xi_n) = \mp\left(c_{13}^2 + c_{44}\right)\xi_n,$$

$$\tilde{g}_{31}^{\pm}(\xi_n) = \pm\left(c_{13}^2 + c_{44}\right)\xi_n, \quad \tilde{g}_{32}^{\pm}(\xi_n) = c_{33}\xi_n^2 - c_{44}, \quad \tilde{g}_{23}^{\pm}(\xi_n) = \left(\bar{c}_{13} - c_{13}^2\right)c_{44}\xi_n^2,$$

$$\tilde{g}_{14}^{\pm}(\xi_n) = -\left(\bar{c}_{13} - c_{13}^2\right)c_{44}, \quad \tilde{g}_{22}^{\pm}(\xi_n) = \tilde{g}_{33}^{\pm}(\xi_n) = \pm\left(c_{33}\xi_n^2 + c_{13}^2\right)c_{44}\xi_n,$$

$$\tilde{g}_{k,k+2}^{\pm}(\xi_n) = \mp(-1)^k\left(\bar{c}_{13} - c_{13}^2\right)c_{44}\xi_n,$$

$$\tilde{g}_{2k,2k-1}^{\pm}(\xi_n) = (-1)^{k-1}\left(c_{13}\xi_n^2 + c_{11}\right)c_{44}, \quad k = 1,2, \quad c_{kj} = \theta(z)c_{kj}^{+} + \theta(-z)c_{kj}^{-},$$

$$c_{33}^{\pm}c_{44}^{\pm}(\xi_k^{\pm})^4 + \left[c_{13}^{\pm}\left(c_{13}^{\pm} + 2c_{44}^{\pm}\right) - c_{11}^{\pm}c_{33}^{\pm}\right](\xi_k^{\pm})^2 + c_{11}^{\pm}c_{44}^{\pm} = 0, \quad k = 1,2.$$

3. Fields of Stresses and Displacements in the Plane of Joint of the Half Spaces

Setting $z = 0$ in the fundamental solutions, we get the distributions of normal and tangential stresses and displacements in the plane of joint of the half spaces in the case where a concentrated force $\mathbf{P} = (P_1, P_2, P_3)$ $(P_k \geq 0)$, acts at an arbitrary point $M_0 = (x_0, y_0, z_0)$. In particular, if the force $\mathbf{P}$ acts only in the direction of the X-axis, $\mathbf{P} = (P_1, 0, 0)$, or of the Y-axis, $\mathbf{P} = (0, P_2, 0)$, then we get $(j = 1,2)$:

$$\sigma_z = P_j \sum_{n=1}^{2} B_{1,n} \frac{\vartheta_{2j} z_0}{\left(r_0^2 + (\xi_n z_0)^2\right)^{3/2}},$$

$$\tau_{yz} = -P_j \left\{ -\partial_1 \frac{\vartheta_{1j}(-1)^{3-j}\left(r_0^2 + (\xi z_0)^2\right)^{-1/2} S_1}{\xi|z_0| + \sqrt{r_0^2 + (\xi z_0)^2}} \right.$$

$$\left. + \partial_2 \sum_{n=1}^{2} \frac{\vartheta_{2j} B_{2,n}\left(r_0^2 + (\xi_n z_0)^2\right)^{-1/2}}{\xi_n|z_0| + \sqrt{r_0^2 + (\xi_n z_0)^2}} \right\},$$

$$\tau_{xz} = -P_j \left\{ \partial_2 \frac{\vartheta_{1j}(-1)^{j} S_1\left(r_0^2 + (\xi z_0)^2\right)^{-1/2}}{\xi|z_0| + \sqrt{r_0^2 + (\xi z_0)^2}} \right.$$

$$\left. + \partial_1 \sum_{n=1}^{2} \frac{\vartheta_{2j} B_{2,n}\left(r_0^2 + (\xi_n z_0)^2\right)^{-1/2}}{\xi_n|z_0| + \sqrt{r_0^2 + (\xi_n z_0)^2}} \right\},$$

$$u = -P_j\left\{-\partial_2 \frac{S_2\vartheta_{1j}}{\xi|z_0|+\sqrt{r_0^2+(\xi z_0)^2}} + \partial_1\sum_{n=1}^{2}\frac{B_{3,n}\vartheta_{2j}}{\xi_n|z_0|+\sqrt{r_0^2+(\xi_n z_0)^2}}\right\},$$

$$v = -P_j\left\{-\partial_1 \frac{S_2\vartheta_{1j}}{\xi|z_0|+\sqrt{r_0^2+(\xi z_0)^2}} + \partial_2\sum_{n=1}^{2}\frac{B_{3,n}\vartheta_{2j}}{\xi_n|z_0|+\sqrt{r_0^2+(\xi_n z_0)^2}}\right\},$$

$$w = P_j\sum_{n=1}^{2}\frac{B_{4,n}\vartheta_{2j}}{\sqrt{r_0^2+(\xi_n z_0)^2}\left(\xi_n|z_0|+\sqrt{r_0^2+(\xi_n z_0)^2}\right)}.$$

If the force $\mathbf{P}$ acts only in the direction of the Z-axis, $\mathbf{P}=(0,0,P_3)$, then we get

$$\sigma_z = -P_3\sum_{n=1}^{2}\frac{A_{1,n}z_0}{\left(r_0^2+(\xi_n z_0)^2\right)^{3/2}},$$

$$\tau_{xz} = P_3\sum_{n=1}^{2}\frac{A_{2,n}(x-x_0)}{\left(r_0^2+(\xi_n z_0)^2\right)^{3/2}}, \qquad \tau_{yz} = P_3\sum_{n=1}^{2}\frac{A_{2,n}(y-y_0)}{\left(r_0^2+(\xi_n z_0)^2\right)^{3/2}},$$

$$u = P_3\sum_{n=1}^{2}\frac{A_{3,n}(x-x_0)}{\sqrt{r_0^2+(\xi_n z_0)^2}\left(\xi_n|z_0|+\sqrt{r_0^2+(\xi_n z_0)^2}\right)}, \tag{14}$$

$$v = P_3\sum_{n=1}^{2}\frac{A_{3,n}(y-y_0)}{\sqrt{r_0^2+(\xi_n z_0)^2}\left(\xi_n|z_0|+\sqrt{r_0^2+(\xi_n z_0)^2}\right)},$$

$$w = -P_3\sum_{n=1}^{2}\frac{A_{4,n}}{\sqrt{r_0^2+(\xi_n z_0)^2}},$$

where

$$S_p = \theta(z_0)S_{p1}^{+} + \theta(-z_0)S_{p1}^{-}, \qquad A_{p,n} = \theta(z_0)A_{p,n}^{+} + \theta(-z_0)A_{p,n}^{-}, \qquad p=1,2,$$

$$B_{p,n} = \theta(z_0)B_{p,n}^{+} + \theta(-z_0)B_{p,n}^{-}, \qquad A_{1,n}^{+} = -\widehat{R}_n^{+} + \widehat{\beta}_n^{++}, \qquad A_{1,n}^{-} = -\breve{\beta}_n^{+-},$$

$$\widehat{\beta}_n^{\pm\pm} = \sum_{m=1}^{2}\widehat{\beta}_{m,n}^{\pm\pm}, \qquad \breve{\beta}_n^{\pm\mp} = \sum_{m=1}^{2}\breve{\beta}_{m,n}^{\pm\mp}, \qquad \beta_{p,k,n}^{\pm\pm} = \sum_{m=1}^{2}\beta_{k,m,n}^{p,\pm\pm},$$

$$\beta^{\pm\mp}_{p,k,n} = \sum_{m=1}^{2} \beta^{p,\pm\mp}_{k,m,n}, \quad A^{+}_{k,n} = -R^{-}_{k,1,n} + \beta^{++}_{1,k,n}, \quad A^{-}_{k,n} = -\beta^{+-}_{1,k,n}, \quad k = 2,3,4,$$

$$B^{+}_{k,n} = -R^{+}_{k,2,n} + \beta^{++}_{2,k,n}, \quad B^{-}_{k,n} = -\beta^{+-}_{2,k,n}, \quad k = 1,\ldots,4.$$

In Tables 1 and 2, we present the values of the coefficients of influence $A^{\pm}_{p,n}$ from representations (14) for some combinations of transversely isotropic materials [1]; in particular, for the ceramics A ($BaTiO_3$; material ***m1***), for the ceramics B ($BaTiO_3 + 5\%CaTiO_3$; material ***m2***), for yttrium (material ***m3***), for magnesium (material ***m4***), for beryl (material ***m5***); for cobalt (material ***m6***); for beryllium (material ***m7***), and for zinc (material ***m8***).

Table 1. Values of the Coefficient $A^{+}_{p,n}$, $n = 1,2$

Combination of materials	$A^{+}_{1,n}$	$A^{+}_{2,n}$	$A^{+}_{3,n} \cdot 10^{-11}$	$A^{+}_{4,n} \cdot 10^{-11}$
m1–m2	0.365 | –0.72	–0.470 | 0.566	–0.543 | 0.541	0.354 | –0.842
m3–m2	0.061 | –0.182	–0.094 | 0.121	–0.113 | 0.103	0.053 | –0.222
m5–m2	–0.456 ± *i*0.488	0.142 ∓ *i*0.657	0.025 ∓ *i*0.765	–0.621 ± *i*0.462
m3–m4	0.095 | –0.26	–0.132 | 0.191	–0.413 | 0.4574	0.233 | –0.799
m6–m5	0.524 | –2.43	–0.864 | 1.65	–0.914 | 1.09	0.368 | –2.787
m1–m8	0.561 | –1.062	–0.900 | 1.117	–1.223 | 1.267	0.990 | –2.278
m3–m8	0.103 | –0.288	–0.184 | 0.252	–0.258 | 0.252	0.169 | –0.654
m6–m2	0.490 | –2.345	–0.800 | 1.487	–0.970 | 1.24	0.408 | –2.887
m8–m4	–0.105 ± *i*0.121	0.028 ∓ *i*0.147	0.0267 ∓ *i*0.448	–0.373 ± *i*0.302

If the force $\mathbf{P}$ is located on the Z-axis, i.e.,

$$\mathbf{P} = (0,0,P_3),$$

then we can write representation (14) in the form

$$\sigma_z = -P_3 \frac{A_{10}}{z_0^2}, \quad \tau_{xz} = P_3 \frac{A_{20}x}{|z_0^3|}, \quad \tau_{yz} = P_3 \frac{A_{20}y}{|z_0^3|}, \tag{15}$$

Table 2. Values of the Coefficient $A^{-}_{p,n}$, $n = 1, 2$

Combinations of materials	$A^{-}_{1,n}$	$A^{-}_{2,n}$	$A^{-}_{3,n} \cdot 10^{-11}$	$A^{-}_{4,n} \cdot 10^{-11}$
m1–m2	$0.195 \mp i0.875$	$0.0549 \mp i0.9$	$-0.001 \pm i1.01$	$-0.281 \pm i0.977$
m3–m2	$0.142 \mp i0.694$	$0.0564 \mp i0.71$	$-0.02 \mp i1.362$	$-0.36 \pm i1.321$
m5–m2	$0.406 \mp i1.831$	$0.122 \mp i1.952$	$0.022 \pm i1.768$	$-0.535 \pm i1.774$
m3–m4	–0.062 | 0.178	–0.09 | 0.123	0.201 | –1.77	0.09 | –0.398
m6–m5	$0.6 \mp i0.586$	$0.135 \mp i0.811$	$0.03 \pm i0.515$	$-0.414 \pm i0.248$
m1–m8	$0.156 \mp i0.156$	$0.038 \mp i0.227$	$0.007 \pm i0.282$	$-0.227 \pm i0.147$
m3–m8	$0.123 \mp i0.139$	$0.044 \mp i0.178$	$-0.004 \pm i0.38$	$-0.318 \pm i0.287$
m6–m2	$0.525 \mp i2.2$	$0.101 \mp i2.2$	$0.0394 \pm i1.258$	$-0.366 \pm i1.127$
m8–m4	–0.057 | 0.173	–0.105 | 0.137	0.148 | –0.126	0.0923 | –0.0402

$$u = P_3 \frac{A_{30} x}{z_0^2}, \qquad v = P_3 \frac{A_3^0 y}{z_0^2}, \qquad w = -P_3 \frac{A_{40}}{|z_0|}, \tag{16}$$

where

$$A_{k0} = \sum_{n=1}^{2} \frac{A_{k,n}}{\xi_n^3}, \quad k = 1, 2, \qquad A_{30} = \sum_{n=1}^{2} \frac{A_{3,n}}{2\xi_n^2}, \qquad A_{40} = \sum_{n=1}^{2} \frac{A_{4,n}}{\xi_n}.$$

Suppose that two oppositely directed concentrated forces

$$\mathbf{P}^{\pm} = (0, 0, \pm P_3^{\pm})$$

act along the Z-axis in different half spaces at the points $M^{\pm}\left(0, 0, z_0^{\pm}\right)$, respectively. In this case, it is possible to represent normal displacements for $z = 0$ in the following form:

$$w = -P_3^{+} A_{40}^{+} \frac{1}{z_0^{+}} + P_3^{-} A_{40}^{-} \frac{1}{\left|z_0^{-}\right|}.$$

This enables us to determine the condition under which normal displacements w in the plane of joint of the half spaces are equal to zero

$$\frac{P_3^+}{P_3^-}\frac{|z_0^-|}{z_0^+} = \kappa_0, \qquad \kappa_0 = \frac{A_{40}^-}{A_{40}^+}. \tag{17}$$

In Table 3, we present the values of the coefficient κ_0 for some combinations of the materials.

Table 3. Values of the Coefficient κ_0

Combinations of materials	***m1–m2***	***m5–m2***	***m8–m4***	***m3–m8***	***m3–m4***
κ_0	0.9159	0.90887	1.4720	1.246	1.81397

Conclusions

In the present work, we obtain, in a simple explicit form, fundamental solutions for a piecewise-homogeneous transversely isotropic space, which enable us to determine the conditions imposed on interface defects in the presence of volume loads. The loads may be applied both over the volume and over the surfaces of measure zero in the three-dimensional space. In particular, we obtain simple dependences of stresses and displacements in the plane of joint of the half spaces on the values of concentrated forces acting at arbitrary points of the space. It is established that, unlike the case of an isotropic piecewise homogeneous space, under the conditions of symmetric normal loading in the plane of joint of the half spaces, we observe the formation of strains. We establish conditions (17) under which strains are absent in the plane of joint of the half spaces.

The obtained results are of independent interest and make it possible to improve the formulation of problems posed for interface defects.

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