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THEORETICAL AND EXPERIMENTAL RECOMMENDATIONS FOR THE ELIMINATION OF GRINDING CRACKS DURING THE PROCESSING OF PERMANENT MAGNETS

А. Усов, М. Куніцин, Ю. Зайчик. **Теоретико-експериментальні рекомендації щодо усунення шліфувальних тріщин при обробці постійних магнітів**. Технологія виробництва постійних магнітів носить точний характер і заснована на крайніх залежностях фізико-механічних властивостей магнітів від складу сплаву і температурно-силових факторів при їх обробці. Шліфування магнітів в сильно коерцитивному стані пов'язана з низькою продуктивністю і відносно великими дефектами тріщин і відколів. Аналіз якості перед- і післявимірної обробки постійних магнітів показує, що основні дефекти магнітів З'являються мікротріщини, поверхневі тріщини, відколи. Їх утворення при шліфуванні магнітів є наслідком відносно низьких механічних характеристик самих сплавів, неправильного вибору шліфувального круга, порушення режимів шліфування, наявності дефектів, що утворилися на попередніх етапах технологічного процесу виготовлення деталей з магнітів. Це розробка теоретичних і експериментальних рекомендацій щодо усунення шліфувальних тріщин при обробці постійними магнітами. Механізм формування тріщин розглядається з позицій впливу геометрії конструктивних компонентів і їх орієнтації по відношенню до напрямку шліфування цих магнітів в сильно коерцитивному стані. Вирішено задачу визначення напружено-деформованого стану поверхневого шару полірованих магнітів у високо коерцитивному стані, ослабленому системою включень. Отримано аналітичні умови рівноваги конструкційних дефектів твердих магнітних сплавів залежно від коефіцієнта тріщиностійкості, а також від значення температури контактного шліфування, яка визначається технологічними параметрами. дозволяє приступити до побудови алгоритму підбору технологічних параметрів, що забезпечують необхідну якість оброблюваних поверхонь. Розроблені технологічні передумови контролю якісних характеристик шліфувальних деталей, за критеріями гранично допустимих температур шліфування, теплового потоку, розмірів конструкційних параметрів, сил різання, коефіцієнтів тріщиностійкості.

Ключові слова: магнітні сплави, шліфування, тріщини, напружено-деформований стан, тріщиностійкість, бездефектна обробка

A. Usov, M. Kunitsyn, Yu. Zaychyk. **Theoretical and experimental recommendations for the elimination of grinding cracks during the processing of permanent magnets**. The manufacturing technology of permanent magnets has high precision characteristics and is based on extreme dependencies of physical and mechanical properties of magnets from the contents of the alloy, crystalline structure and temperature and force factors during their processing. Grinding of magnets in high-coercivity state relates to low productivity and relatively high flaw regarding cracks and scratches. Quality analysis of the preliminary and terminal size processing of permanent magnets shows that the main defects of the magnets are microcracks, surface cracks, chips. Their appearance during the grinding of the magnets is the consequence of relatively low mechanical characteristics of the alloys, wrong choice of characteristics of the grinding wheel, breaking proper grinding modes, presence of the defects that were formed during the previous technological stages of permanent magnets manufacturing. The objective of this research is the development of theoretical and experimental recommendations for eliminating grinding cracks during permanent magnets processing. The mechanism of grinding cracks appearance was considered regarding to the influence of geometry of structural components and their orientation related to the direction of grinding of the magnets in high-coercivity state. The problem of defining stress-strain state of the surface layer of the grinded magnets in high-coercivity state weakened by the system of foreign inclusions was solved. Analytical balance conditions of the structural defects of hard magnet alloys depending on the coefficient of crack resistance and the value of the contact temperature of grinding which is defined by the technological parameters were discovered. These technological parameters make possible the building of the algorithm for choosing technological parameters that provide the required quality of the processed surfaces. Technological backgrounds for controlling quality characteristics of the parts being grinded using the criteria of maximum allowable temperatures of grinding, heat flow, size of structural parameters, cutting forces, crack resistance coefficients were developed.

Keywords: magnet alloys, grinding, cracks, stress-strain state, crack resistance, flawless processing

Introduction

Permanent magnets made of cast high-coercivity anisotropic alloys are widely used in modern instrument making, electrical engineering, machine tool industry, radio electronics. The application area of these magnets continues growing rapidly due to the rise of space industry.

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The manufacturing technology of permanent magnets has high precision characteristics and is based on extreme dependencies of physical and mechanical properties of magnets from the contents of the alloy, crystalline structure and temperature and force factors during their processing. During the production, taking in account large nomenclature of the magnets divided by weight and size characteristics and the variety of manufacturing approaches, it is rather difficult to find optimal technological modes required for the alloy of the compound.

That's why for the specialists who work in production and development of permanent magnets control of the technological processes and their adjustment for different types of magnets are highly important.

Among others, the problem of finding optimal conditions for the high quality (flawless) and productive processing of the magnets during finishing operations, in particular, during grinding.

Studying and analyzing of the quality of preliminary and terminal size processing of permanent magnets from alloys Alnico and Tickonal in a number of industries shows that main defects of the magnets are microcracks, surface cracks, chips.

The research of influence of technological heredity on the process of forming cracks during the grinding of high-coercivity magnets shall be held based on such criteria which can serve as characteristic of the material mechanical properties and reflect the influence of alloy morphology simultaneously. Such an approach let's uncover the auxiliary reserves for increasing mechanical properties of the magnets on each technological operation predeceasing grinding and define the influence of the abrasive processing on cracks appearance.

Analysis of the recent research and publications

Roughness, microhardness, magnitude, and sign of inner technological tensions are the main properties and characteristics of the surface. The presence of foreign fractions and other structural imperfections depend on types and modes of thermal treatment and thermomagnetic treatment and mechanical processing and define the dependency of exploitational qualities of products made from magnets from production technology [1, 2].

Changing of melting methods, modes of thermal treatment and thermomagnetic treatment and draft grinding cause the corresponding change of the separate quality characteristics of the surface which, in its turn, influence on intensity of crack appearance during finishing operations, during terminal grinding, in particular.

In this sense we can say about the existence of the technological heredity of the magnet surface quality and their exploitation properties from the separate technological operations or the whole technological manufacturing process in general [3].

By technological heredity we shall consider the change of the properties of the parts being processed influenced by manufacturing technology [4, 5].

For the proper usage of technological heredity phenomena during magnets production it is required firstly to set up direct connections between the quality of the processed surface and the dominating parameters of the technological process [6, 7].

Physical and mechanical properties peculiarities of the hard magnet alloys like "UNDC35T5" (high fragility, low durability, relatively low values of thermal conductivity and thermal diffusivity) brings them to intractable materials.

That's why structure analysis of the technological process of magnets manufacturing is an important part of increasing the output of valid magnets during the operations of final grinding because the main defects (cracks and scratches) appear precisely during terminal processing of these magnets.

The Fig. 1 presents the structure of the technological process of magnets manufacturing.

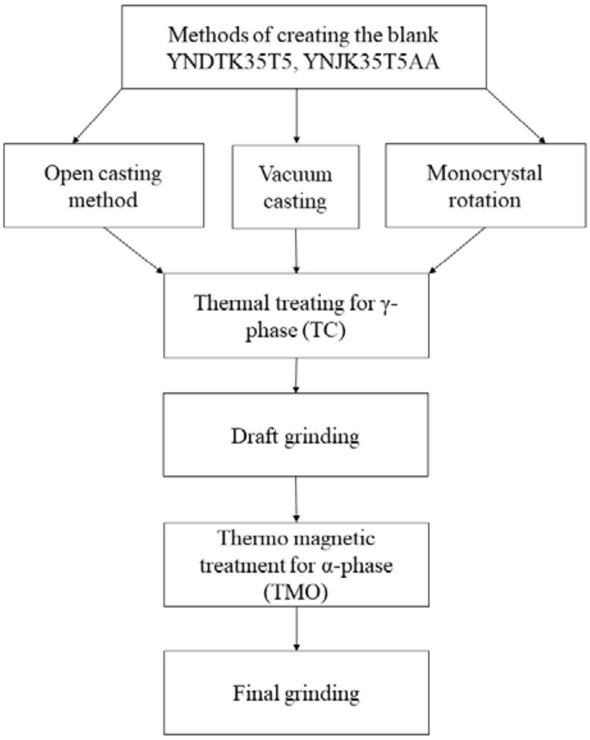


Fig. 1. Structure of the technological process of magnets UNDCT manufacturing

Permanent magnet blanks from Alnico and Tickonal alloys are produced mainly using casting. Currently, *open and vacuum* casting methods are used. The most spread method of open casting is melting in acid crucible. This method is widespread due to its relatively simplicity of technologies and high thermal resistance of the crucibles. One of the disadvantages of acid casting is that the resulting alloy is contaminated.

Melting of the "UNDCT" alloys in vacuum ovens finds more application in the magnets manufacturing technology. This is due to the fact that during the vacuum melting of the "UNDCT35T5" alloy the quantity of the nonmetallic inclusions is 70% less and predisposition to cracks and chips for these magnets during processing is significantly lower than in case with open melting in acid crucible.

The growing of the monocrystal is a more progressive method for creating blanks. Theoretically this process is reduced to the creation of conditions in which atoms of the alloy are being "frosted" on the monocrystalline seed building the crystal cell. The typical technology of this monocrystal production method includes the following operations:

1. The monocrystalline seed is set on the bottom of the crucible with crystallographic orientation /100/ parallel to the axis of the crucible together with chemical contents corresponding to the required alloy;
2. The crucible with its contents, graphite heater and thermal insulation shield are set into the vacuum chamber. The graphite crucible is being heated and moved. The resulting monocrystal is cooled to room temperature. The seed and the top part of the ingot are removed.

The production of such magnets is connected with the development of anisotropic alloys like "UNDCT" for the products where high magnet properties with simple geometry are required.

For the further mechanical processing (draft grinding) blanks from the "UNDCT" alloys are heat treated (hardening and tempering). The main purpose of this operation is to increase viscosity and ductility of the alloy. Otherwise during draft grinding almost all of the blanks are prone to crack and chip appearance due to the high fragility [8].

The main objection of the thermomagnetic treating of the UNDCT alloys is to obtain optimal structure that defines the required level of the magnetic properties. Meantime, in addition to all types of anisotropy external or aimed single axis anisotropy is formed due to the texture of the products ($\alpha+\alpha'$) – transformations. Here it is important to keep optimal temperature of high-coercivity transformation. For the UNDCT alloys it lies in the range [860...800 °C]. In this range the conditional Curie point is located. UNDCT alloys are sensitive to the influence of external magnetic field at the temperature just near the Curie point [9].

The purpose and objections of the research

Development of theoretic and experimental recommendations regarding technological methods for reasonable decreasing of grinding cracks during processing of products made from materials and alloys which surface layer has heredity defects of structural or technological origin.

Grinding of magnets in high-coercivity state relates to low productivity and relatively high flaw from cracks and chips. The influence of the morphology and forming of the non-metallic inclusions is significant for the appearance of grinding cracks during the processing of the magnets.

Morphology, sic mechanism of kinetics $\alpha\rightarrow\sigma+\gamma$[TT]$\rightarrow\alpha+\alpha'$[TMT] – transformation, significantly influences on the structure of the UNDCT alloys, hence defines technological conditions of the flawless grinding of the products from magnets (TT – thermal treating, TMT – thermos magnetic treatment).

For achieving the objective, the following problems shall be solved:

1. Conduct the analysis of the reasons for crack appearance during grinding of the hard magnet alloys depending on the manufacturing technology and the structural heterogeneity accompanying the technological process on the mechanism of microcrack appearance and their transformation into magistral cracks. Also, to define the influence of the morphology and non-metallic inclusions on the intensity of grinding cracks appearance;
2. Obtain the dependencies together with experimental research that would theoretically determine the areas of combinations of technological parameters which provide the required quality of the processed surfaces according to the condition of maximum efficiency of grinding.

Indeed, ($\alpha\rightarrow\alpha+\gamma$) – transformation corresponds to the growth of the new phase that differs from α-matrix by contents (enriched with iron) and by the type of parameters of crustal cells (GCK). Meantime, γ-phase has definite crystallographic connection with α-matrix. Moreover, γ-phase gives the alloys higher ductility due to the fact that it has higher ductility than the base – α_γ-phase. The heterogeneity of the alloy structure in the state of α_γ-phase defines the alloy's physical and mechanical characteristics. Particles of the α_γ-phase of the UNDCT alloy vary by size from 1...4 microns (at an annealing temperature of 900 °C) to 25...30 microns (at 1200 °C). Existence of these particles in the conditions of complex forced and thermal strain state of the surface layer on the stage of draft grinding can be the reason of microcrack creation on the boundaries of γ-phase with α-matrix. And the probability of microcrack appearance will be as high as the size of the particles of the α_γ-phase and the magnitude of the dynamic strains that are formed by thermal and mechanical processes accompanying the grinding operation. The particles of the α'-phase after the thermal treating without magnetic field influence the surface quality during the draft grinding insignificantly due to the small sizes (2 times less than sizes of α_γ particle) and their multi-orientation in α-matrix.

During the thermo magnetic treating of the alloys the particles of α'-phase are oriented along the direction which has the lowest angle with the direction of the magnetic field. As a result, induced single axis magnetic anisotropy appears. The peculiarity of the structure of high-coercivity state of the given alloys is periodic alternation of stem-like particles of α'-phase surrounded by the matrix of α-phase.

Magnetic anisotropy, which is common for high-coercivity state forms mechanical and thermophysical anisotropies for the given alloys.

This, in its turn, contributes to the quality of the surface layer depending on the direction of grinding of magnets in this state. Definition of the relative processing direction on selected criteria which characterize the quality of the grinded surfaces can decrease the flaw on the main defects like cracks and burns and increase the output of good products.

Appearance of macro and micro cracks on the grinded surfaces of magnets is the consequence of the concentration of thermomechanical strains in the points of accumulation of different non-metallic inclusions like sulfides, nitrides, carbides and pores and micro pits. The coefficient of strain intensity reflects the technological heredity for hard magnet alloys.

The research of authors [8, 9] proves that mechanical properties of magnets like UNDC, UNDCT are influenced by various factors from casting methods, quantity of non-metallic inclusions, modes of thermal treating, modes and methods of draft grinding. The influence of most of them on such indicators of mechanical durability like tension durability limits $[\sigma]_p$, or bending durability limits which were chosen as criteria for technological heredity was insignificant when, at the same time, magnet was prone for crack appearance. This is explained by the fact that rather rude criteria were chosen as initial, and they reflect only integral properties of magnets during the technological process and respond to changes in the surface layer to a lesser extent.

Materials and methods of the research

To estimate the technological heredity which appears not only by the changes in physical and mechanical properties of the material on the whole cut but also on the surface, more sensitive criteria that reflect changes with the material in differential form which appear during product manufacturing. Taking into account the fact that for heredity quality estimation for hard magnet alloys one shall consider the variable fragility of magnets from operation to operation, then it becomes obvious that more acceptable is crack resistance criteria. Moreover, the methods for estimating these criteria are more advanced and this criterion is more "fine-tuned" for determining the properties of magnet surface. The main advantage of this criterion is the fact that it is sensitive to various defects in the material which appear during each operation and its value depends on geometry of these defects.

Let's consider the construction of the theoretical value of this criterion – the coefficient of strain intensity [10]:

$$K_1(P_*, a_i, l) = \lim_{S \to 0} \sqrt{2\pi S}\,\sigma_y(x, y, \tau),$$

where, S – normal distance to the defect's contour in its plane Z=0; $\sigma_y(x, y, 0)$ – normal strains in the plane of defect's location Z=0.

As mentioned, the coefficient is a function of external load P, size of defect l and geometry parameters of the product a_i which origins from solving thermomechanical problem. Condition of the local destruction on the defect contour makes possible to define the spreading of this defect and particularly find the combination of external loads which from one side is determined by the technological process parameters and from the other side divides the stability and instability areas of the product's state with crack-like defects. In other words, if magnet has gone through the whole cycle of technological process till the operation of final grinding and on the way acquired some structure defects like non-metallic inclusions on the melting stage, microcracks (during thermal treating and draft grinding), anisotropy and heterogeneity (during TMT), then crack appearance intensity during final grinding of magnet will be determined by defects size $2l$, value of the thermomechanical strains which appear during grinding and value of crack resistance coefficient K_{1C} of this magnet. The value of crack resistance characteristics K_{1C} reflects all the accumulated defectivity of the structure [11].

For the quantity analysis of crack resistance of UNDCT alloys in dependency from blanks creation methods and further thermal treating at γ-phase, thermomagnetic treating at $\alpha+\alpha'$ phase the following work was done.

Prismatic examples from monocrystals UNDCT 35T5AA and polycrystals UNDCT35T5 derived from open melting in acid crucible were exposed to thermal treating at γ-phase, thermomagnetic treating at $\alpha+\alpha'$ while varying the modes of TT and TMO, modes of tempering and cooling speed, also were exposed to mechanical testing for magnetic properties.

The controlled parameters during this were crack resistance coefficient values K_{1C}, bending strains σ_{bend}, tension strains σ_t and magnet properties characteristics (coercivity force of the material by magnetization H_C and the remaining induction B_r).

Experimental data given in [12] shows that values of the crack resistance coefficient are different for mono and polycrystals UNDCT in stage of blank product. This is explained by the fact that percental contents of non-metallic inclusions in the alloy varies in rather wide range in dependency on the material production method.

Research results

The phase contents of the given alloys change the value of the crack resistance coefficient in wide range. For polycrystals in the initial state K_{1C}=30 MPa$\times\sqrt{\text{m}}$ while the existence of γ-phase increases the crack resistance coefficient up to K_{1C}=97 MPa$\times\sqrt{\text{m}}$. This can be explained by the fact that γ-phase while being more ductile than main matrix of this alloy promotes the slowdown of microcracks. But the presence of this phase in UNDCT alloy decreases the coercivity force by 40%, the remaining magnet induction by 15…20% and maximum magnet energy more than 60%. That's why the presence of the γ-phase in finished magnet products is not allowed.

Due to this magnet were exposed to thermomagnetic treatment (TMT) in permanent magnet field.

The impact of the thermomagnetic fields significantly influences the mechanical properties of studied magnets. Thus, the main parameter of crack resistance K_{1C} varies relative to K_{1C} of magnets at γ-phase in 2...3 times. Polycrystals in dependency on the modes of thermomagnetic treatment (time of aging during heating τ_2 and during tempering τ' and type of cooling on the air) have values of the crack resistance coefficient from 35 MPa$\times\sqrt{\text{m}}$ to 65 MPa$\times\sqrt{\text{m}}$. Their magnet properties during this vary in lesser range. Other mechanical characteristics have rather small dispersion of values (σ_{nr} – 20%, σ_{bend} – 35%).

The favorable influence of repetitive TMT on mechanical properties of magnets shall be highlighted. But in the same time the magnet properties of given alloys are decreasing and their production prime cost is increasing.

Analyzing the experimental research results for defining the influence of direction of thermomagnetic treatment on crack resistance of magnets, it is required to mention the tangible differences in values of K_{1C} depending on the direction of magnetic field. Thus, in case of examples testing in lateral direction of TMT, coefficient K_{1C} took the following values: for polycrystals 53 MPa$\times\sqrt{\text{m}}$, and monocrystals 145 MPa$\times\sqrt{\text{m}}$. While in the same time transverse direction of TMT decreases K_{1C} for the alloys to 37 MPa$\times\sqrt{\text{m}}$ and 100 MPa$\times\sqrt{\text{m}}$ correspondingly. This is explained by the fact that direction of the domains in base Fe matrix in sum creates anisotropy of mechanical properties of these alloys. During the grinding of these magnets the direction of TMT shall be considered, and the direction of the processing shall be coordinated with the direction that creates the maximum resistance to destruction of magnets.

For detecting grinding direction when magnets have the maximum crack resistance let's consider the influence of thermomagnetic treatment on destruction resistance after TMT.

During the thermal treating without magnetic field particles of α' phase, are oriented by long axis along the crystallographic directions of type <100> which creates the lowest angle with magnet field direction. As a result, the single axis induced magnet anisotropy appears.

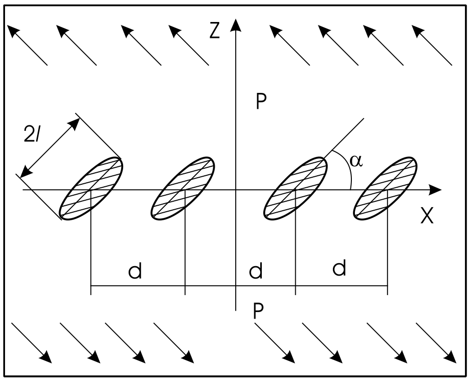


Fig. 2. Calculation scheme for analyzing the influence of TMT and its direction on grinding cracks appearance

One of the peculiarities of the structure of high-coercivity state in the given alloys is periodic shifting of ellipse-like particles of α′ phase, surrounded by the matrix of α-phase.

Further grinding of the given alloys leads to forming crack-like defects in surface layer [12]. Intensity of crack appearance is connected with the modes of thermal treating which, in their turn, influence the size of particles of α′ phase and the direction of TMT relative to the lateral supply during grinding.

Let’s consider the mechanism of grinding crack appearance from the point of influence of α‘ phase geometry and its orientation relative to the direction of grinding of magnets in high-coercivity state.

To do this, we formalize the process of grinding magnets, considering them as fragile solid objects with a periodic system of defects with sharp edges of the same length and angle to the *X* axis, which passes through the centers of these defects (Fig. 2).

The distance between neighboring defects is constant and equals to $d[d_{nk}e^{i\beta nk}=(n-K)d]$. Taking in assumption that to all defects self-balanced load is applied ($PK(xK)=P(xK)$) and conjugation break function $<U_k(x)>=U(xK)$. To define them it is enough to consider only one integral equation of type [10]:

$$\int_{-l}^{l}\left[\left\langle U_k'(t)\right\rangle K(t-x)+\left\langle \overline{U_k'(t)}\right\rangle L(t-x)\right]dt=\pi P(x),|x|<1, \tag{1}$$

where

$$\begin{gathered}K(x)=\frac{1}{x}+\frac{1}{2}\sum_{-\infty}^{\infty}\left(\frac{1}{x+kde^{i\alpha}}+\frac{1}{x+kde^{-i\alpha}}\right);\\ L(x)=\frac{1}{2}\sum_{-\infty}^{\infty}\left(\frac{1}{x+kde^{i\alpha}}-\frac{x+kde^{-i\alpha}}{(x+kde^{i\alpha})^2}\right).\end{gathered} \tag{2}$$

Let’s use the well-known relations [12]:

$$\begin{gathered}\mathrm{ctgz}=\frac{1}{z}+\sum_{k=-\infty}^{\infty}\left(\frac{1}{z-\pi k}+\frac{1}{\pi k}\right);\\ \cos\mathrm{ec}(z)^2=\frac{1}{z^2}+\sum_{k=-\infty}^{\infty}{}'\frac{1}{(z-\pi k)^2}.\end{gathered} \tag{3}$$

Symbol Σ′ means that when k=0, then the addend equals to zero.

After the simple transformations we will get:

$$\begin{gathered}K(x)=\frac{\pi}{2d}\left(e^{i\alpha}\mathrm{ctg}\frac{\pi xe^{i\alpha}}{d}+e^{-i\alpha}\mathrm{ctg}\frac{\pi xe^{-i\alpha}}{d}\right);\\ K(x)=\frac{\pi}{2d}(e^{-i\alpha}-e^{-3i\alpha})\left(\mathrm{ctg}\frac{\pi xe^{-i\alpha}}{d}-\frac{\pi xe^{-i\alpha}}{d}\mathrm{cosec}^2\left(\frac{\pi xe^{-i\alpha}}{d}\right)\right).\end{gathered} \tag{4}$$

Thus, the problem of providing a quantum-deformed state of the surface layer of polished magnets in a high-coercive state is reduced to measuring the quantum-deformed state of an elastic plane weakened by a periodic inclusion system. The solution of the latter is reduced to finding the function $<U'_K(x)>$ that determines the shape of the emerging crack from the singular integral equation (1), the kernels of which are determined by formulas (4).

In the equation (1) let’s pass to unidimensional variables $\xi=t/l$ and $\eta=x/l$:

$$\int_{-l}^{l}\left[\left\langle U_k'(\xi)\right\rangle K(l\xi-lr)+\left\langle \overline{U_k'(\xi)}\right\rangle Ll(\xi-r)\right]d\xi=\pi P(r)\,. \tag{5}$$

For the kernels $K(l\xi)$, L(lξ) when $\lambda=2l/d<1$ the following assumptions are true:

$$\begin{gathered} lK(l\xi)=\frac{1}{\xi}+m(\xi); \\ m(\xi)=\lambda\sum_{k=1}^{\infty}a_k(\lambda\xi)^{2k-1};\ lL(l\xi)=\lambda\sum_{k=1}^{\infty}b_k(\lambda\xi)^{2k-1}, \end{gathered}\tag{6}$$

where:

$$a_k=-\frac{\pi^{2k}B_k\cos 2k\alpha}{(2k)!};\ b_k=-\frac{k\pi^{2k}(1-e^{-2i\alpha})e^{-2ik\alpha}B_k}{(2k)!},\tag{7}$$

B_k – Bernoulli numbers [13].

Then the equation (5) takes the following form:

$$\int_{-1}^{1}\frac{\langle U'(\xi)d\xi\rangle}{\xi-r}=\pi P(r)-\int_{-1}^{1}\Big[\langle U'(\xi)\rangle m(\xi-r)+l\big\langle\overline{U'(\xi)}\big\rangle L(l\xi-lr)\Big]d\xi.\tag{8}$$

The resulting Fredholm equation of second kind after rearranging the order of integration takes the form:

$$\langle U'(r)\rangle=-\frac{1}{\pi\sqrt{1-r^2}}\int_{-1}^{2}\frac{P(\xi)\sqrt{1-\xi^2d\xi}}{\xi-r}+\frac{1}{\pi\sqrt{1-r^2}}\int_{-1}^{2}\Big[\langle U'(\xi)\rangle M(\xi,r)+\big\langle\overline{U'(\xi)}\big\rangle N(\xi,r)\Big]d\xi,\tag{9}$$

where:

$$\begin{gathered} M(\xi,r)=\frac{1}{\pi}\int_{-1}^{1}\frac{\sqrt{1-\tau^2}m(\xi-\tau)}{\tau-r}; \\ N(\xi,r)=\frac{l}{\pi}\int_{-1}^{1}\frac{\sqrt{1-\tau^2}L(l\xi-l\tau)}{\tau-r}dr. \end{gathered}\tag{10}$$

Solution of the equation (9) let's find in a form of series [10]:

$$\langle U'(r)\rangle=\sum_{k=0}^{\infty}U_k^{'}(r)\lambda^{2k}.\tag{11}$$

Putting expressions (6), (11) in (9) and equating the expressions for similar powers of λ we obtain the system of equations for defining functions $U'_k(\eta)$:

$$U'(r)=-\frac{1}{\pi\sqrt{1-r^2}}\int_{-1}^{1}\frac{\sqrt{1-\xi^2}P(\xi)}{\xi-r}d\xi,\tag{12}$$

$$U_k^{'}(r)=\frac{1}{\pi\sqrt{1-r^2}}\sum_{n=1}^{k}\int_{-1}^{1}H_n(\xi,r)\Big[a_nU_{k-n}^{'}(\xi)+b_n\overline{U_{k-n}^{'}(\xi)}\Big]d\xi,\tag{13}$$

where:

$$H_n(\xi,r)=\frac{1}{\pi}\int_{-1}^{1}\frac{\sqrt{1-\tau^2}}{\tau-r}(\xi-\tau)^{2n-1}f\tau.\tag{14}$$

Integrals (14) can be easily calculated [10].

Finding the solution $<U_k(\eta)>$, sic define the functions $U_k(\eta)$, for example with the precision till $O(\lambda^6)$:

$$U_1^{'}(r)=\frac{1}{\pi\sqrt{1-r^2}}\int_{-1}^{1}H_1(\xi,r)\Big[a_1U_0^{'}(\xi)+b_1\overline{U_0^{'}(\xi)}\Big]d\xi;$$

$$U_2^{'}(r)=\frac{1}{\pi\sqrt{1-r^2}}\int_{-1}^{1}\left\{\begin{matrix}H_1(\xi,r)\left[a_1U_0^{'}(\xi)+b_1\overline{U_0^{'}(\xi)}\right]+\\+H_2(\xi,r)\left[a_2U_0^{'}(\xi)+b_2\overline{U_0^{'}(\xi)}\right]\end{matrix}\right\}d\xi;$$

$$U_2^{'}(r)=\frac{1}{\pi\sqrt{1-r^2}}\times\int_{-1}^{1}\left\{\begin{matrix}H_1(\xi,r)\left[a_1U_0^{'}(\xi)+b_1\overline{U_0^{'}(\xi)}\right]+\\+H_2(\xi,r)\left[a_2U_0^{'}(\xi)+b_2\overline{U_0^{'}(\xi)}\right]+\\+H_3(\xi,r)\left[a_3U_0^{'}(\xi)+b_3\overline{U_0^{'}(\xi)}\right]\end{matrix}\right\}. \tag{15}$$

Using the values of the integrals from (12) and (16) we can obtain sequentially, $U'_1(\eta)$, $U'_2(\eta)$ and $U'_3(\eta)$.

Having solutions of the integral equation (9) in form of (11) with the algorithm for calculating functions $U'_i(\eta)$, complex potentials $\Phi(Z)$ and $\Psi(Z)$ can be found using formulas [14]:

$$\Phi(z)=\frac{e^{i\alpha}}{2d}\int_{-l}^{l}\operatorname{ctg}\frac{\pi}{d}(te^{i\alpha}-z)U'(t)dt;$$

$$\Psi(z)=\frac{e^{i\alpha}}{2d}\int_{-l}^{l}\left\{U'(t)e^{-2i\alpha}ctg\frac{\pi}{d}(te^{i\alpha}-z)\left[\begin{matrix}\operatorname{ctg}\frac{\pi}{d}(te^{i\alpha}-z)+\\+\frac{\pi}{d}e^{-i\alpha}(t-te^{i\alpha}+ze^{i\alpha})\operatorname{cosec}^2\frac{\pi}{d}(te^{i\alpha}-z)\end{matrix}\right]U'(t)\right\}dt. \tag{16}$$

Formulas for calculating strain intensity coefficient for given case can be written in form:

$$K_1^{\pm}-iK_2^{\pm}=\mp\sqrt{l}\lim_{r\to\pm1}\left[\sqrt{1-r^2}U'(r)\right]. \tag{17}$$

Putting here the value $U'(\eta)$ from (11) considering (15) we can find:

$$K_1^{\pm}-iK_2^{\pm}=\mp\sqrt{l}\left\{\begin{matrix}-\frac{1}{\pi}\int_{-1}^{1}\sqrt{\frac{1\pm\xi}{1\pm\xi}}P(\xi)d\xi+\lambda^2(a_1G_0+b_1\overline{G_0})+\\+\lambda^4\left[\begin{matrix}G_0\left(2a_2-\frac{1}{2}a_1^2-\frac{1}{2}b_1\overline{b_1}\right)+\overline{G_0}(2b_2-a_1b_1)+\\+a_2G_2+b_2\overline{G_2}\pm\frac{3}{2}(a_2G_1+b_2\overline{G_1})\end{matrix}\right]+0(\lambda^6)\end{matrix}\right\}; \tag{18}$$

$$G_n=\frac{1}{\pi}\int_{-1}^{1}\xi\sqrt[n]{1-\xi^2}P(\xi)d\xi.$$

This relationship defines strain intensity coefficients for arbitrary load $P(x_k)$ on the banks of crack-like defects.

For the analysis of the conditions for grinding cracks occurrence during magnet processing, we can assume in first approximation that to the banks of α'-phase in the points $x_k=\xi$ concentrated tangential P_Z and shear P_Y forces and thermoelastic tensile stresses balancing them are applied. Considering this, we have the following:

$$G_n=-\frac{P_z-iP_y}{\pi l}\xi^n\sqrt{1-\xi^2}\ . \tag{19}$$

When surface layer is subject to prevailing loads Pz and balancing $\tau_{XY}(y)$ perpendicular to α'-phase lines we have:

$$P(y)=-S=-(\sigma-i\tau)=-\frac{-P}{2}(1-e^{2i(\phi-\alpha)});$$

$$G_0=-\frac{1}{2}S;\ \ G_1=0;\ \ G_2=-\frac{1}{8}S; \tag{20}$$

$$K_{1}^{\pm}-iK_{\Pi}^{\pm}=\sqrt{l}\left\{S-\frac{\lambda^{2}}{2}(a_{1}S+b_{1}\overline{S})-\frac{\lambda^{4}}{S}\left[\begin{matrix}S(9a_{2}-2a_{1}^{2}-2b_{1}\overline{b_{2}})+\\+\overline{S}(9b_{2}-4a_{1}b_{1})\end{matrix}\right]\right\}+0(\lambda^{6}). \quad (21)$$

Putting the coefficients a_k, b_k from (7) in (21) and separating the real and imaginary parts we conclude to the formulas convenient for the calculation:

$$K_{I}=\sqrt{l}\left\{\sigma-\frac{\pi^{2}\lambda^{2}}{3\cdot 2^{3}}(\sigma U_{1}+\tau V_{1})-\frac{\pi^{4}\lambda^{4}}{2^{7}}\left[\sigma(U_{2}-W_{2})+\tau V_{2}\right]\right\}+0(\lambda^{6});$$

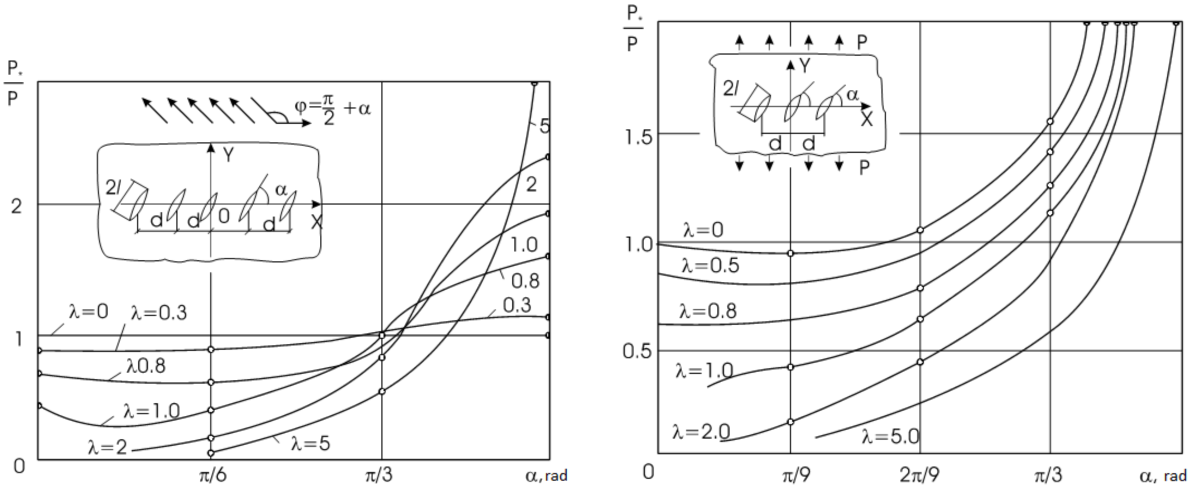
$$K_{II}=\sqrt{l}\left\{\tau+\frac{\pi^{2}\lambda^{2}}{3\cdot 2^{3}}(\sigma V_{1}+\tau W_{1})+\frac{\pi^{4}\lambda^{4}}{2^{7}}\left[\sigma V_{2}+\tau(W_{2}-U_{2})\right]\right\}+0(\lambda^{6}), \quad (22)$$

where:

$$U_{1}=2\cos 2\alpha-4\cos 4\alpha;\ V_{1}=\sin 2\alpha-\sin 4\alpha;\ W_{1}=\cos 4\alpha;\ U_{2}=\frac{4}{9}\sin^{2}\alpha+\frac{28}{45}(\cos 4\alpha-\cos 6\alpha);$$

$$V_{2}=-\frac{2}{9}\sin 2\alpha+\frac{28}{45}(\sin 4\alpha-\sin 6\alpha);\ W_{2}=\frac{5}{9}-\frac{4}{9}\cos 2\alpha+\frac{14}{45}\cos 4\alpha.$$

There the dependency P_*/P_0 from the orientation angle α of crack-like defects with different values of the parameter λ on the Figure 3(*a*). When λ<1 the interaction of the neighboring defects tilted to the axis x under the angle α<π/3 leads to surface layer durability decrease. The minimum of the destruction load is achieved with different α. Collinear defects become more dangerous only with λ→1. When the values of α are close to π/3 the ultimate load is the same as in case of single crack-like defect (for 0<λ<1). For λ≥1 all curves with increasing of α ascend from zero to some maximum value with α=π/2. For the angles α close to π/2 with all values λ>0 the interaction of the defects leads to strengthening of surface layer and besides the maximum strengthening is observed with α<π/2.



a *b*

Fig. 3. Influence of the geometry and orientation angle α of crack-like defects on the condition of their evolving into main cracks: *a* – dependancy P_*/P_0 from angle α of crack-like defects on parameter lambda; *b* – dependancy of the critical load on α angle for different values of λ

On the Figure 3(*b*) is presented the dependency of critical load from angle d for the different values of λ with uniaxial tension (in case when we ignore the action of the shear load P_Y) is perpendicular to x axis. Decrease of the distance between the centers of the defects leads to durability decrease for all values of α. The minimum which is present on the curve λ=0 (Fig. 3) for a single defect with α=11 π/90 with decrease of distance between the defects is shifted to the direction of the value α=0. For λ≥1 the most dangerous defects are those which are close to collinear defects.

The mechanism of technological cracks occurrence can also be studied from the position of the hypothesis about the most “weak link” by which the structural parameter shall be considered. Its size is chosen as the criterion of the flawless processing using the formula [15]:

$$l_0 < \frac{K_C^2}{\pi[GT_k(1+\nu)\alpha_t]^2}\,. \tag{23}$$

Formula (23) gives simple sufficient criterion. When it is achieved the crack-like defect won't be transformed to main crack.

If the inclusions are in form of ellipse, then because of immediate local heating of the surface layer of the magnet in contact zone the disk-like crack can appear. Indeed, when grinding under the influence of thermoelastic strains and cutting forces, on the banks of disk-like defect of radius *R*, the forces along the defect axis appear:

$$P = G(1+\nu)\alpha_t T_k \iint\limits_{(S)} (\overrightarrow{n_z}, \overline{ds}) = G(1+\nu)\alpha T_k S_0\,, \tag{24}$$

where S_0 – square of the projection of the defect border on crack plane.

Strain intensity coefficient for this case is determined in the following way:

$$K_I = \frac{P}{(\pi R)^{3/2}}\,. \tag{25}$$

Using viscosity, destructions of magnet the radius of disk-like defect can be determined. If the following conditions are met:

$$R \le \frac{1}{\pi}\left[\frac{G(1+\nu)\alpha_t T_k S_0}{K_{1C}}\right]^{2/3}, \tag{26}$$

then the defect won't evolve to main crack. And in case of ellipsoidal form we have:

$$S = 4ab, \quad R = \frac{1}{\pi}\left[\frac{4G(1+\nu)\alpha_1 abT_k}{K_{1C}}\right]^{2/3}. \tag{27}$$

Here *a* and *b* are main axes of the ellipse in the cut of ellipsoid of disk-like crack.

Obtained analytical conditions (23), (26), (27) of the structural defect balance with size 21 (in case of most "weak link") depend on the coefficient of crack resistance K_{1C}, coefficients v, G, α_t and the value of contact temperature T_K which is determined by mode part.

The dominating factor in forming of contact temperature is grinding depth. This factor is decisive in the choice of flawless conditions of grinding. The correlation between crack resistance coefficient and grinding depth was studied on the examples of polycrystals and monocrystals in high-coercive state (POG 121A, MG121A) was defined. Experimental data is given in Table 1. Research results comparison shows that crack resistance of magnet alloys falls with increase of the depth. Decrease intensity depends on the characteristics of grinding wheels.

Thus, in case of grinding using diamond wheels, the intensity of crack resistance is insignificant, because these wheels have high thermal conductivity and grinding contact temperature T_K is lower than using wheels 24A25CM18K5. The insignificant decrease of crack resistance coefficient using wheels on 10…20% indicates their serial usage for grinding operations.

The grit of the wheels in the range from r=12 microns to r=400 microns influences crack resistance of the given alloys insignificantly. While the hardness of grinding wheels significantly affects the intensity of crack resistance and the value of K_1. In the work [15] during the research of kinetics of strain formation in the surface layer of grinded products was mentioned the ability to reduce temporal tangential strains by choosing grinding wheels which have minimal friction coefficient ρ in the contact with the material being processed. Since the values tangential strains τ_{XY} aids to the evolving of the defects to main cracks, one of the available ways for reducing flaw by grinding cracks are the ways of reducing the component of cutting force – P_Z friction force.

On the Figure 4 are presented theoretical and experimental studies of crack appearance phenomena during the grinding of magnets UNDCT35T5 with various cutting depths. To develop the technological criteria for controlling the process of flawless grinding it was considered that this is a multifactor process. Physical and mechanical properties of the processed metal, its structure, grinding modes and grinding wheels characteristics, conditions of the preliminary processing cutting fluids and characteristics of the instruments influence the quality of the surface layer during grinding.

Table 1

Correlation of the crack resistance coefficient with grinding parameters on the examples with monocrystals and polycrystals in high-coercivity state (POG 121A, MG121A)

№	Example sizes				$P_{\max}$	Exam. state	Grinding parameters					K_C
	L^*	t^*	b^*	l^*			V_g	V_{kp}	T_{gr}	Cutting fluid	Wheel	
	mm	mm	mm	mm	H		m/s	m/s	m/s			MPa $\sqrt{M}$
11	28	7.5	15.1	7.0	1440	M	–	–	–	–	–	83
22	28	7.5	15.1	7.4	940	PO	–	–	–	–	–	59
33	28	7.6	15.1	7.5	2340	MG	–	–	–	–	–	146
44	28	7.5	15.1	7.1	1110	POG	0.17	35	0.04	–	LO 20 100%	65
55	28	7.5	15.5	7.1	790	MG	0.17	35	0.04	HLC*	24A25 CM18K5	46
66	28	7.8	15.5	8.0	320	MG	0.17	35	0.03	No coolant	24A25 CM18K5	20
77	28	7.6	15.5	7.3	850	POG	0.17	35	0.03	HLC*	24A25 CM18K5	33
88	28	7.6	15.5	7.6	340	POG	0.17	35	0.03	No coolant	24A25 CM18K5	20

*HLC – hard lubrication coolant from polyamide group having high heat capacity. Is used for decreasing grinding temperature when applied to cutting edge of the wheel;
L^*, t^*, b^*, l^* – geometry sizes of the sample;
M – monocrystal magnets;
MG – monocrystals exposed to thermal treating at γ-phase;
PO – polycrystal magnets obtained by open melting;
POG – polycrystals exposed to thermal treating at γ-phase.

That's why to provide the quality of the processed surfaces it is required to select the processing modes, cutting fluids and instrument characteristics by functional connections between physical and mechanical properties of the materials and grinding process parameters in a way when current values of grinding temperature $T(x, y, \tau)$ and heating flux $q(y, \tau)$ of the strains $\sigma_{p\,\max}$ and forces P_Y, P_Z, intensity coefficient $K_1(S, \alpha, \sigma_{p\,\max})$ do not exceed their limit values which guarantee the required quality of the surface layer.

Let's consider the following system of limiting inequalities that let us build the algorithm for selecting technological parameters which provide the required quality of the processed surfaces.

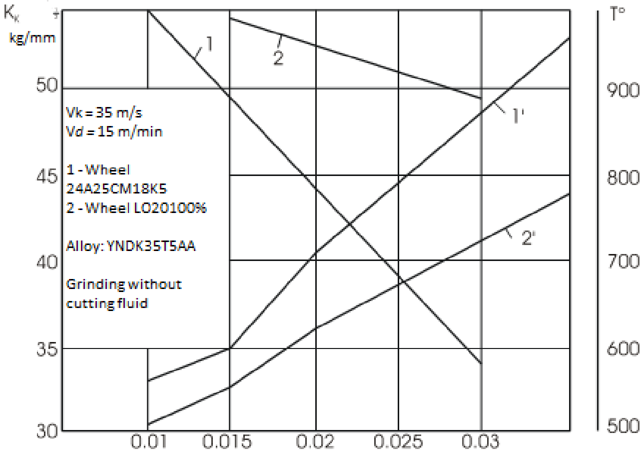


Fig. 4. Influence of grinding depth onthe intensity of crack occurance during YNDK magnets processing in the state of α-phase (final grinding)

When studying the kinetics of the temperature field of the product considering the peculiarities of cutting the single grains it was found that the grain consists of regular (constant) and instant (impulse) components. Impulse component – T_M describes the temperature state of the metal just under the grain. Constant component T_K characterizes heating of the metal in the processing zone as a result of aggregate affect by multiple grains of the instrument.

Despite the short-time action of the instant temperature on the metal and its rapid attenuation along the depth it takes part in forming of structurally strained state of thin surface layer of the product. That's why the bounding inequalities by the values of the temperature and its spreading depth will correspondingly be equal to:

$$T(x,y,\tau)=\frac{C}{2\pi\lambda}\sum_{k=0}^{n}H\left(\tau-\frac{kl}{V_w}\right)H\left(\frac{L+kl}{V_w}\right)\int_{\gamma_1}^{\gamma_2} f(\tau,\tau')d\tau'\le[T]_M\ ; \tag{28}$$

$$T([h],0,\tau)=\frac{C}{2\pi\lambda}\sum_{k=0}^{n}H\left(\tau-\frac{kl}{V_w}\right)H\left(\frac{L+kl}{V_w}\right)\int_{\gamma_1}^{\gamma_2}\psi(\tau,\tau')d\tau'\le[T]_{\mathrm{ST}}\ , \tag{29}$$

where:

$$\psi(\tau,\tau')=\exp\left[-\frac{V_d(kl-V_w\tau')}{2a}-\frac{V_d^2(\tau-\tau')}{4a}-\frac{(kl-V_w\tau')^2+[h]^2}{4a(\tau-\tau')}\right]; \tag{30}$$

$[T]_{\mathrm{ST}}$ – acceptable temperature of structural transformations of given metal;

$[h]$ – ultimately acceptable depth of structural transformations.

In some cases the quality loss of the surface layer becomes significant only with spreading of structural transformations at certain depth which value is determined by the product's operating conditions and additionally by technical conditions. Limit values of such depth are determined by deep heating zone, sic by constant component of the temperature field. The bounding inequalities in this case have the following form:

$$T_k(o,y,\tau)=\frac{CV_{kp}}{\pi\lambda l\sqrt{V_g}}\int_0^{\tau}\int_{-l}^{l}\frac{x(r,t)e^{-\frac{(y-r)^2}{4(\tau-t)}}}{2\sqrt{\pi(t-\tau)}}\left\{\frac{1}{\sqrt{\pi(t-\tau)}}+\gamma e^{y^2(\tau-t)}\left[1+\Phi(\gamma\sqrt{\tau-t})\right]\right\}drdt\,; \tag{31}$$

$$T_k([h],0)=\frac{CV_w}{2\pi\lambda l\sqrt{V_d}}\int_0^{\sqrt{Dt_{gr}}}\sqrt{[h]^2+y'^2}\,e^{\frac{V_d y'}{2a}}K_{1/2}\left(\frac{V_d}{2a}\sqrt{y'^2+[h]^2}\right)dy'\le[T]_L\,; \tag{32}$$

$$T_k^{\max}(L,0)\frac{CV_w a}{\lambda l V_d^2}\sqrt{\frac{a}{\pi}}\left[1-\exp\left(-\frac{V_d\sqrt{Dt_{gr}}}{a}\right)\right]\le[T]_{\mathrm{ST}}\,. \tag{33}$$

In the last inequality the ultimate temperature on a surface (X=0) was used as bounding factor.

The occurrence of grinding cracks depends on the values of temporary strains which are formed in the surface layer under the effect of thermomechanical phenomena guiding this process. Maximum strains appear in the zone of intensive cooling. That's why the structure of controlling inequality in this case will be the following:

$$\sigma_{\max}(x,\tau)=2G\frac{1+\nu}{1-\nu}\alpha_t T_k^{\max}\,erf\left(\frac{x}{2\sqrt{\alpha\tau}}\right)\le[\sigma]_{ic}\,. \tag{34}$$

Phenomenological approach in estimation of crack appearance in the surface layer of the metals during grinding doesn't take into account many technological factors, in particular the influence of thermal treating modes of this metals and defectivity of their structure connected with previous mechanical processing. That's why more "sensitive" bounding criterion is required. Its structure shall contain functional connections of technological parameters of diamond abrasive processing and technological heredity shall be considered. In this role we can use the limitation of intensity strains coefficient with established relations to technological parameters and main criterion of metal crack resistance – coefficient K_{1C}:

$$K_1=\frac{1}{\pi\sqrt{l}}\int_{-l}^{l}\sqrt{\frac{l+t}{l-t}}\left\{\sigma_{xx},\sigma_{yy}\right\}dt\le K_{1C}\,, \tag{35}$$

where $2l$ – typical linear size of structural defect.

Flawless grinding of the materials having low mechanical characteristics is possible if we limit the cutting forces, in particular, the tangential component P_Z and decrease the friction coefficient of abrasive with processed metal – ρ.

Thus, from the research about the influence of cutting forces on strained state of the surface layer we can build another one auxiliary condition for flawless grinding:

$$P_z\le\frac{\pi\sqrt{Dt_{gr}}}{KP^2\sin\pi\theta}\left[[\tau]_c-\frac{E\rho\sqrt{Rt}}{2(1-\nu^2)\sqrt{R}}\right], \tag{36}$$

where $[\tau]$ – limit value of the shear stress;

$$\theta = \frac{1}{\pi}\text{arctg}\frac{1-2\nu}{2\rho(1-\nu)};$$

ρ – minimal acceptable value of the friction coefficient between the abrasive and processed metal which is provided by using of the cutting fluid with impregnating compound;

K – relation coefficient P_y / P_z.

If we know the value of the crack resistance coefficient K_{1C}, sizes of the most “weak link” of structural l, we can define the range of the technological parameters which provide the limit of heating flux during which structural defects balance is saved:

$$q^* = \frac{P_z V_w \alpha_b}{\sqrt{D t_{gr}}} \le \frac{\sqrt{3}\lambda K_{1C}}{H l \sqrt{\pi l}\,\delta^*}. \quad (37)$$

The conditions of the flawless grinding can be implemented using reliable information about the processed metal’s structure. Thus in case of prevailing character of the structural imperfections with length 2l their regular location relative to the contact zones of the instrument with the product we can use the condition of defect’s balance as criterial relationship in the following form:

$$l_0 < \frac{K_C^2}{\pi[G T_k (1+\nu)\alpha_t]^2}. \quad (38)$$

In this formula the technological part lies in in the connection of contact temperature value TK with grinding modes and instrument characteristics.

The obtained inequalities give the connection between the limit characteristics of temperature and force fields with the controlling technological parameters. They set up the area of combinations of technological parameters (modes, cutting fluids, grinding wheel characteristics) which provide the required quality of working surfaces of the products from hard-to-cut materials.

Conclusions

1. The elimination of grinding cracks during magnets processing which have low mechanical properties and anisotropy depends on the right choice of grinding wheel characteristics, following the modes which provide flawless processing considering the technological process of their manufacturing and morphology.
2. Technological criteria, proposed in a form of bounding inequalities by the limit values of grinding temperatures, burn depth, cutting forces, heating flux, strain intensity let us find the area of combinations of technological parameters – modes, cutting fluids, grinding wheel characteristics which provide the required quality of the products.
3. Technological background for controlling quality characteristics of the grinded details by the ultimately acceptable grinding temperature values, heating flux, heredity defects, cutting forces, crack resistance coefficients which provide the choice of optimal technological conditions for flawless processing.
4. We obtained the methodology for reasonable selection of technological parameters for flawless grinding of the materials and metals especially prone for crack occurrence.

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