

APPLICATION OF THE METHOD FOR THE CALCULATION OF STATICALLY INDETERMINATE SYSTEMS

Khomyak Yu., Naumenko Y., Abu Shena O.

As mentioned above, the dependence of the distribution voltage from element stiffness design exists only in statically indeterminate systems [1, 2].

Therefore, the example of applying the method of virtual objects [3] to the use of an object with virtual stiffness consider the example of loading of a rod, whose scheme is shown in Fig. 5.

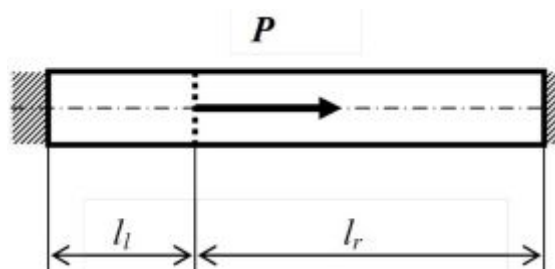


Figure 5 – An example of a statically indeterminate loading horizontal rod

Horizontal rod, rigidly clamped at the ends, loaded the current along its axis by a force P applied at a distance l_l from its left edge and l_r – in from the right and the dividing plane for your application the rod into two parts – left and right.

The effect of gravity is neglected. It is known that in this case there are three functions: $\sigma_l, \sigma_r, \Delta\sigma$ two variables: F_l, F_r , which specified relationships:

$$\sigma_l = P \frac{l_r E_l}{l_l E_r F_r + l_r E_l F_l}; \quad (1)$$

$$\sigma_r = P \frac{l_l E_r}{l_l E_r F_r + l_r E_l F_l}; \quad (2)$$

$$\Delta\sigma = \sigma_l - \sigma_r = P \frac{l_r E_l - l_l E_r}{l_l E_r F_r + l_r E_l F_l}, \quad (3)$$

where σ_l, σ_r – accordingly, the stress in the left and right parts of the rod; F_l, F_r – section of the left and right sides; E_l, E_r – the stiffness of the rod parts.

Analysis of equations (1) – (3) shows that, unfortunately, the condition $\Delta\sigma = 0$ is impossible at finite F_l, F_r . This corresponds to the statement that in statically indeterminate structures in the general case all elements at the same time it is impossible to obtain equal stress.

Therefore speech can go only about minimizing $\Delta\sigma$, and the whole problem reduces to the optimization with three optimization criteria – $\sigma_l, \sigma_r, \Delta\sigma$, two variables – F_l, F_r and restrictions.

Its formulation for our example is follows.

To find the optimal values F_l^* and F_r^* , where $|\Delta\sigma| \rightarrow \min, \sigma_l \rightarrow \max, \sigma_r \rightarrow \max$ and are the following limitations:

$$\sigma_l \leq [\sigma]; \quad (4)$$

$$\sigma_r \leq [\sigma]; \quad (5)$$

$$F_{l \min} \leq F_l \leq F_{l \max}; \quad (6)$$

$$F_{r \min} \leq F_r \leq F_{r \max}. \quad (7)$$

The solution of the optimization problem with three optimization criteria presents significant challenges and does not provide a clear end result. Therefore, as the only optimization criterion chosen was the total mass m of the design part (host).

In our example, the total mass of the rod m or, equivalently, its metal content can be determined according to the formula:

$$m = m_l + m_r = \rho(l_l F_l + l_r F_r), \quad (8)$$

and its optimal (lowest) value respectively equal to:

$$m^* = \rho(l_l F_l^* + l_r F_r^*). \quad (9)$$

Let us analyze the expression (9). As mentioned above, the value $\Delta\sigma = 0$ is unattainable. The geometric meaning of the constraints (4) and (5) is that they are cut from the rectangle of the argument region in which the condition of strength.

For each pair of arguments in (8) can be calculated from the value of the mass of the rod. A pair of values F_l^* and F_r^* , which providing the minimum value of the mass m^* , will solve the problem, that is the space of sections of parts of the rod the least mass that meets the requirements of (4) to (7).

Problem was solved by numerical method, for which the field of argument was covered with mesh, the nodes which were calculated pairs of values F_l, F_r . For each node was calculated values of stresses in both parts of the rod. If these values are not exceeded, the received for this node the value of m compared with m -values in other nodes and choosing the minimum.

In the transition to a virtual method of the rigidity value of the cross-sectional area of the whole stem were recorded at baseline, and the sorting in the grid nodes were subjected to values of moduli of elasticity is now different in the left and right sides.

In this algorithm, minimization subjected to the absolute value of the difference between stresses in various parts of the rod:

$$\Delta E = |E_l - E_r|. \quad (10)$$

The objective of the calculation is to find the minimum cross – section, wherein the selection of the E_l and E_r still manages to ensure implementation of the inequalities (4) and (5).

Here is a concrete example of applying the method of virtual stiffness to calculate the optimal core designs loaded according to the scheme (Fig. 5).

Initial data for calculation: the length of the left part of the rod – $l_l = 0,4$ m; the length of the right part of the rod – $l_r = 0,6$ m; the external force – $P = 10000$ N; allowable stress is – 120 MPa; the nominal value of the elastic modulus of real material rod – $E_{\text{nom}} = 200000$ MPa; the density of the material of the rod – $\rho = 7800$ kg/m³.

Step 0. Believe $\sigma_l = [\sigma]$ and by the formula (1) defined the value of F_l , that provides the left part tension equal to $[\sigma]$:

$$F_l = \frac{P}{[\sigma]} \cdot \frac{l_l}{l_l + l_r} = 0,5 \cdot 10^{-4} \text{ m}^2. \quad (11)$$

As in step 0 the rod has a uniform cross-sectional area $F_r = F_l = F_0^*$, the tension in the right side:

$$\sigma_r = \frac{P}{F_r} \cdot \frac{l_l}{l_l + l_r} = 80 \text{ MPa}. \quad (12)$$

Thus, all of the rods of equal cross section has the smallest mass m_0^* :

$$m_0^* = \rho \cdot F_0^* (l_l + l_r) = 0,39 \text{ kg}. \quad (13)$$

Step 1. From design considerations to select the boundaries of variation: $F_{l \min} = 0,000035 \text{ m}^2$; $F_{l \max} = 0,0002 \text{ m}^2$; $F_{r \min} = 0,000035 \text{ m}^2$; $F_{r \max} = 0,0002 \text{ m}^2$.

Define square sections F_{l1}^* and F_{r1}^* , that providing m_1^* :

$$\begin{aligned} F_{l1}^* &= 0,60 \cdot 10^{-4} \text{ m}^2; \\ F_{r1}^* &= 0,36 \cdot 10^{-4} \text{ m}^2; \\ m_1^* &= 0,3559 \text{ kg} \end{aligned} \quad (14)$$

at stresses equal to 119,0476 MPa in left and 79,36507 MPa in right parts.

Calculated the relative change in the cross-sectional area in the transition from step 0 to step 1:

$$\Delta F_l = \frac{F_{l1}^*}{F_0^*} = 1,54; \quad (15)$$

$$\Delta F_r = \frac{F_{r1}^*}{F_0^*} = 0,22. \quad (16)$$

Step 2. Select the border of the variation of the stiffness: $E_{l \min} = 100 \text{ 000 MPa}$; $E_{l \max} = 250 \text{ 000 MPa}$; $E_{r \min} = 100 \text{ 000 MPa}$; $E_{r \max} = 250 \text{ 000 MPa}$.

Virtual determined values of moduli of elasticity E_{l2}^* and E_{r2}^* , that providing m_2^* :

$$\begin{aligned} E_{l2}^* &= 1,66 \cdot 10^{11} \text{ MPa}; \\ E_{r2}^* &= 2,49 \cdot 10^{11} \text{ MPa}; \\ m_2^* &= 0,3276 \text{ kg}. \end{aligned} \quad (17)$$

Calculated the elastic modulus relative change in the transition from step 1 to step 2:

$$\Delta E_l = \frac{E_{l2}^*}{E_{\text{nom}}} = 0,83; \quad (18)$$

$$\Delta E_r = \frac{E_{r2}^*}{E_{\text{nom}}} = 1,245. \quad (19)$$

Step 3. Count the rod area by the method of virtual models, based on the ratios:

$$\frac{F_{l3}^*}{F_0^*} = \frac{E_{\text{nom}}}{E_{l2}^*}; \quad (20)$$

$$\frac{F_{r3}^*}{F_0^*} = \frac{E_{\text{nom}}}{E_{r2}^*}. \quad (21)$$

Using (20) and (21), obtain:

$$F_{l3}^* = \frac{F_0^* \cdot E_{\text{nom}}}{E_{l2}^*} = 0,602 \cdot 10^{-4} \text{ m}^2; \quad (22)$$

$$F_{r3}^* = \frac{F_0^* \cdot E_{\text{nom}}}{E_{r2}^*} = 0,4 \cdot 10^{-4} \text{ m}^2, \quad (23)$$

what provides the stress in the left part 115,119 MPa and right – 76,746 MPa at weight $m_3^* = 0,375$ kg, that is within the normal range. Comparing F_{l1}^* and F_{r1}^* , then F_{r1}^* and F_{r3}^* , then m_1^* and m_3^* , we find that these values are exceedingly close.

Especially it should be noted that the final result (22) and (23) were obtained without using the data of step 1, only where the rod is composed of two parts of unequal cross sections.

For design stages using does not recognize the asymmetry of CAD, detail of the left symmetrical in form, allowing other factors – properties and the external loading to make very exotic (virtual) values.

And only at the last stage, these factors acquire the features of reality through shape recovery [4]. Thus the «whimsical» CAD manages to fool on the information level and result-set to design asymmetric, complex shapes, and thus optimal, parts and components.

A method is proposed in which the model of the future design object transformation, some intermediate States which can exist only in the virtual view. This way it helps to find such intermediate state, in which the optimization of geometric characteristics can be performed most efficiently. The method involved when you design a symmetrical variable thickness bottoms of tanks [5, 6], working under pressure, with positive technical and economic effect.

Under asymmetric mechanical loading (that is most often found in machine) is optimal asymmetric design details, the design of which, as a rule, very hardly. Modern computer software means for calculation of stress-strain state of parts and components sharply increase its effectiveness only when given the symmetry of the structure. In this case a real significant gain in conflict with the requirement of optimality of the latter, – «symmetric» CAD can't design asymmetrical shape of the part.

There is a vicious circle: then detail farther from the symmetry, so it is «better», but the harder in modeling and design.

Method of virtual object allows to overcome these contradictions. For this for design stages using does not recognize the asymmetry of CAD, detail of the left symmetrical in form, allowing other factors – properties and the external loading to make very exotic (virtual) values. And only at the last stage, these factors acquire the features of reality through shape recovery. Thus the «whimsical» CAD manages to fool on the information level and result-set to design asymmetric, complex shapes, and thus optimal, parts and components.

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design a symmetrical variable thickness bottoms of tanks, working under pressure, with positive technical and economic effect.

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