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MATHEMATICAL MODELING OF THE STRESSED-DEFORMED STATE OF CIRCULAR ARCHES OF SPECIALIZED CRANES

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Представлена методика розрахунку напружено-деформованого стану (НДС) кругових арок із урахуванням деформацій вигину і розтягування-стиснення, зосереджених та розподілених зовнішніх навантажень. Врахування умов обпирання опорних перетинів виконаний в рамках алгоритму чисельно-аналітичного варіанту методу граничних елементів (МГЕ). Розглянуто приклад розрахунку НДС арки, де представлено повне рішення для кінематичних і статичних параметрів стану арки

Ключові слова: метод граничних елементів, фундаментальна система функцій, арочні системи, спеціальний кран, MATLAB

Представлена методика расчета напряженно-деформированного состояния (НДС) круговых арок с учетом деформаций изгиба и растяжения-сжатия, сосредоточенных и распределенных внешних нагрузок. Учитывание условий опирания опорных сечений выполнено в рамках алгоритма численно-аналитического варианта метода граничных элементов (МГЭ). Рассмотрен пример расчета НДС арки, где представлено полное решение для кинематических и статических параметров состояния арки

Ключевые слова: метод граничных элементов, фундаментальная система функций, арочные системы, специальный кран, MATLAB

1. Introduction

Rods in the shape of an arc of a circle (circular arches) are widely used in construction and industrial machine-building, particularly, in crane building when designing specialized cranes. Their application is predetermined by advantages in comparison with the structures that consist of straight rods, given the higher load-bearing capacity and rigidity [1]. In the present study, in order to calculate circular arches, we applied a numerical-analytical variant of the boundary element method. The use of this method is predetermined by advantages over the existing methods of structural mechanics. In particular, the methods of force, displacements, initial parameters, finite elements and others do not have the possibility to accurately account for the concentrated moment in the form of external load and exact solution to a problem on the planar deformation of a circular arch. These shortcomings are missing in the variant of BEM, developed in the papers of the authors of present work. Results of the numerical-analytical variant of the boundary element method are distinguished by high accuracy and reliability.

The scientific literature typically reports solutions to SDS of circular arches, obtained by known methods. It is rather difficult, however, to estimate accuracy of the obtained results and their reliability. Given this, a task of applying BEM to the calculation of SDS of circular arches is relevant for the following reasons:

1. There appears a possibility to obtain accurate results for SDS of arches at different loads and boundary conditions of support, which is problematic when employing the known methods of structural mechanics.

2. It is necessary to make sure that BEM actually allows obtaining accurate results for the calculation of SDS.

3. It is extremely useful to have resolving equations for the boundary value problems of flat deformation of circular arches under various supporting conditions.

2. Literature review and problem statement

Accurate calculation of SDS of circular arches causes certain difficulties because one should take into account bending, stretching-compression, and shear deformations at the same time [2]. This is especially true for determining displacements in the arches, to which a Vereshchagin formula is not applicable since the load and single curves are nonlinear, whereby one has to employ the Mohr's integral, Castiglianos theorem, and numerical methods [3].

There are known solutions to different problems on the flat deformation of circular arches that consider bending deformations only [4]. There is also a solution derived to the problem on flat deformation of a circular rod taking into account deformations of bending and stretching-compression only for the case of loading $q_y(\alpha)=q=\text{const}$ [5]. The absence of sufficiently accurate analytical solution to the problem of flat deformation of arches contributed to the fact that authors of a number of papers recommend replacing curved rods with a set of straight rods. Article [6] shows that the benefit of such an approach is the smaller number of algebraic equations to which the resolved problem is reduced. It should be noted, however, that the given model yields error not exceeding 1 % under condition that the straight rod tightens the arc of the curved rod less than 5°. This means that the ring can be represented by a regular polygon consisting of 72 rods, arch in 180° – by 36 rods, etc.

The arched elements of construction can also be calculated using the classical methods of structural mechanics (displacements, initial parameters, forces) or numerical methods. In the first case, classical methods do not belong to computer methods as they possess a complex logic and a small number of arithmetic operations. For this reason, they do not have professional packages of applied programs [7].

At present, the most common numerical method is a finite element method (FEM) [8]. This method corrects shortcomings of the classical methods, it has a relatively simple algorithm logic and a large number of arithmetic operations. The method is implemented in a variety of professional packages of applied programs. It should be noted, however, that FEM allows obtaining accurate and reliable results of the estimation of arches' SDS only if there is an exact matrix of the rigidity of a circular arch. Regrettably, due to the complexity of its formation, no exact rigidity matrix of the flat deformation of a circular rod has been built up to now. A major problem for FEM is the problem of convergence of the obtained solution, error estimation, associated with discretization of the original geometric model, artificial limit for a computation area, etc.

The search for alternative approaches resulted in the development of BEM [9]. In it, not the entire examined region undergoes discretization, as is the case for FEM, but its boundary only. BEM qualitatively outperforms FEM. It employs exact solution to the flat deformation of a circular rod with respect to bending and stretching. A new variant of BEM was devised [10], numerical-analytical, which has a number of advantages in comparison with the classical variants of BEM. The method implies working out a fundamental system of solutions (analytically) and the Green's functions (also analytically) for each considered problem. In order to account for certain boundary conditions, or conditions of contact between separate modules, a small system of linear algebraic equations is constructed, which must be solved numerically. The value of the method is predetermined by several reasons. Discretization of only the bound of the region occupied by an object drastically reduces an order of the system of solving equations; there is a possibility to reduce dimensionality of the problem to be solved. In addition, as shown in paper [11], this method is strictly substantiated mathematically as it employs fundamental solutions to differential equations. This means that within a framework of the accepted hypotheses it makes it possible to obtain exact values for parameters of the problem (efforts, displacements, strains, frequencies of native oscillations, critical forces of stability loss, etc.) inside the region. At the same time, the method possesses simplicity of the algorithm logic, good convergence of the solution, high stability, and insignificant accumulation of errors in numerical operations.

However, in terms of calculating arch structures for specialized cranes, there are no at present solving equations of boundary value problems for determining the unknown initial parameters of arches under existing conditions for bearing the boundary sections.

3. The aim and objectives of the study

The aim of present work is to identify conditions for accounting the concentrated moments when solving analytically a state of arch structures, as well as present a calculation of kinematic parameters of circular arches, conducting of which is based on the exact solution of flat deformation.

To accomplish the set aim, the following tasks have been considered in the study:

– to obtain solving equations of boundary value problems for determining the unknown initial parameters of circular arches under existing conditions for bearing the boundary sections;

– to develop a procedure for calculating SDS of circular arches;

– to perform calculation of a circular arch, taking into account the action of distributed and concentrated loads.

4. Development of a mathematical model

We shall represent mathematical modeling of SDS of circular arches in the MATLAB programming environment in a static statement.

4. 1. Calculation procedure and basic equations

Consider the calculation scheme of a circular arch (Fig. 1) with an arbitrary combination of static load.

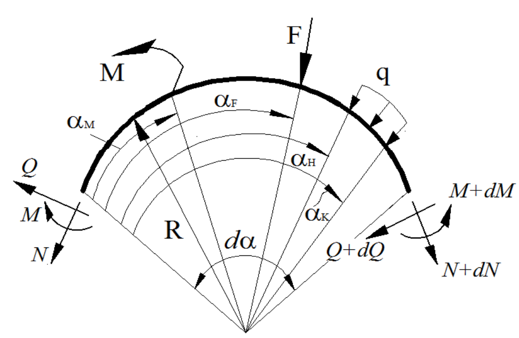

Fig. 1. Calculation scheme of the circular arch

A system of differential equations for the flat deformation of a circular rod considering the deformations of bending and stretching relative to the radial $\vartheta(\alpha)$ and tangential $u(\alpha)$ displacements takes the form:

$$\begin{cases} \vartheta^{IV}(\alpha)+\dfrac{EAR^2}{EI}\vartheta(\alpha)+u'''(\alpha)-\dfrac{EAR^2}{EI}u'(\alpha)=\dfrac{R^4}{EI}q_y(\alpha); \\ \left(1+\dfrac{EAR^2}{EI}\right)u''(\alpha)+\vartheta'''(\alpha)-\dfrac{EAR^2}{EI}\vartheta'(\alpha)=-\dfrac{R^4}{EI}q_x(\alpha), \end{cases} \qquad (1)$$

where EA is the arch cross-section rigidity under stretching-compression, kN; EI is the arch cross-section rigidity at bending, kNm[2]; R is the arch axis radius, m; α is the angular coordinate, rad.; $q_y(\alpha)$ is the radial load of the arch (along a normal to the axis), kN/m; $q_x(\alpha)$ is the tangential load of the arch, kN/m.

Solution to the Cauchy problem on the flat deformation of an arch can be represented in a matrix form as follows [12]:

$$\begin{pmatrix} EI\nu(\alpha) \\ EI\phi(\alpha) \\ M(\alpha) \\ Q(\alpha) \\ EAu(\alpha) \\ N(\alpha) \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} & -A_{13} & -A_{14} & A_{15} & A_{16} \\ & A_{22} & -A_{23} & -A_{13} & & A_{26} \\ & & A_{22} & A_{12} & & -A_{36} \\ & & & A_{11} & & -A_{46} \\ A_{51} & A_{52} & -A_{53} & -A_{54} & A_{11} & A_{56} \\ & & & -A_{64} & & A_{11} \end{pmatrix} \begin{pmatrix} EI\nu(0) \\ EI\phi(0) \\ M(0) \\ Q(0) \\ EAu(0) \\ N(0) \end{pmatrix} + \begin{pmatrix} B_{11} \\ B_{21} \\ -B_{31} \\ -B_{41} \\ -B_{51} \\ -B_{61} \end{pmatrix}, \qquad (2)$$

where $EI\phi(\alpha)$ is the rotation angle of the arch cross-section, kNm[2]; $M(\alpha)$ is the bending moment in a cross-section, kNm; $Q(\alpha)$ is the shear force in a cross-section, kN; $N(\alpha)$ is the normal force in a cross-section, kN.

Fundamental orthonormalized functions and terms from the radial load (Fig. 1) after integrating take the form [10, 11]:

$$A_{11}=\cos\alpha; \quad A_{12}=R\sin\alpha;$$

$$A_{13}=R^2(1-\cos\alpha);$$

$$A_{14}=\left(R^3+\frac{EIR}{EA}\right)\frac{\sin\alpha-\alpha*\cos\alpha}{2};$$

$$A_{15}=-\frac{EI}{EA}\sin\alpha;$$

$$A_{16}=R^3\left(1-\frac{1}{2}\alpha*\sin\alpha-\cos\alpha\right)-\frac{EIR}{EA}*\frac{1}{2}\alpha*\sin\alpha;$$

$$A_{22}=1; \quad A_{23}=R*\alpha;$$

$$A_{26}=R^2(\alpha-\sin\alpha); \quad A_{36}=R(1-\cos\alpha);$$

$$A_{46}=\sin\alpha; \quad A_{51}=\frac{EA}{EI}\sin\alpha;$$

$$A_{52}=\frac{EAR}{EI}(1-\cos\alpha); \quad A_{53}=\frac{EAR^2}{EI}(\alpha-\sin\alpha);$$

$$A_{54}=\frac{EAR^3}{EI}\left(1-\cos\alpha-\frac{1}{2}\alpha\cdot\sin\alpha\right)-R\frac{1}{2}\alpha\cdot\sin\alpha;$$

$$A_{56}=R\cdot\frac{1}{2}\cdot(\sin\alpha+\alpha\cdot\cos\alpha)+ +\frac{EAR^2}{EI}\left(\alpha+\frac{1}{2}\alpha\cdot\cos\alpha-\frac{3}{2}\sin\alpha\right);$$

$$A_{64}=-\sin\alpha; \qquad (3)$$

$$B_{11}=\left(R^4+\frac{EIR^2}{EA}\right)\times \times\left\{\frac{M}{R^2}\frac{(\alpha-\alpha_M)_+\cdot\sin(\alpha-\alpha_M)_+}{2}+\frac{F_y}{R}\frac{\sin(\alpha-\alpha_F)_+-(\alpha-\alpha_F)_+}{2}+ +q_y\left[H(\alpha-\alpha_H)-\cos(\alpha-\alpha_H)_+-\frac{1}{2}(\alpha-\alpha_H)_+\cdot\sin(\alpha-\alpha_H)_+- -H(\alpha-\alpha_K)+\cos(\alpha-\alpha_K)_+\cdot\sin(\alpha-\alpha_K)_+\right]\right\};$$

$$B_{21}=M\cdot R\cdot\sin(\alpha-\alpha_M)_++F_y\cdot R^2\cdot\left[H(\alpha-\alpha_F)-\cos(\alpha-\alpha_F)_+\right]+ +q_yR^3\left[(\alpha-\alpha_H)_+-\sin(\alpha-\alpha_H)_+-(\alpha-\alpha_H)_++\sin(\alpha-\alpha_F)_+\right];$$

$$B_{31}=M\cdot\cos(\alpha-\alpha_M)_++F_y\cdot R\cdot\sin(\alpha-\alpha_F)_++ +q_yR^2\left[H(\alpha-\alpha_H)-\cos(\alpha-\alpha_H)_+- -H(\alpha-\alpha_K)+\cos(\alpha-\alpha_K)_+\right];$$

$$B_{41}=\frac{M}{R}\cdot\left[-\sin(\alpha-\alpha_M)_+\right]+F_y\cdot\cos(\alpha-\alpha_H)_++ +q_y\cdot R\cdot\left[\sin(\alpha-\alpha_H)_+-\sin(\alpha-\alpha_K)_+\right];$$

$$B_{51}=M\cdot\begin{bmatrix}\left(1+\dfrac{EIR^2}{EI}\right)\dfrac{\sin(V-\alpha_m)_++(\alpha-\alpha_m)_+\cdot\cos(\alpha-\alpha_m)_+}{2}- \\ -\dfrac{EAR^2}{EI}\cdot\sin(\alpha-\alpha_M)_+\end{bmatrix}+F_y\times \times\left\{\left(1+\frac{EAR^2}{EI}\right)R^2\cdot\frac{\sin(\alpha-\alpha_H)_+-(\alpha-\alpha_H)_+\cdot}{2}-\frac{EAR^3}{EI}\left[H(\alpha-\alpha_F)-\cos(\alpha-\alpha_F)\right]\right\}+q_y\times \times\begin{Bmatrix}\left(1+\dfrac{EAR^2}{EI}\right)R^2\cdot\dfrac{\sin(\alpha-\alpha_H)_+-(\alpha-\alpha_H)_+\cdot\cos(\alpha-\alpha_H)-\sin(\alpha-\alpha_K)_++(\alpha-\alpha_K)_+\cdot\cos(\alpha-\alpha_K)_+}{2}- \\ -\dfrac{EAR^4}{EI}\left[(\alpha-\alpha_H)_+-\sin(\alpha-\alpha_H)_+-(\alpha-\alpha_K)_++\sin(\alpha-\alpha_K)_+\right]\end{Bmatrix};$$

$$B_{61}=\frac{M}{R}\cos(\alpha-\alpha_M)_++ +F_y\cdot\sin(\alpha-\alpha_F)_++q_y\cdot R\times \times\left[H(\alpha-\alpha_H)_+-\cos(\alpha-\alpha_H)_+-H(\alpha-\alpha_K)+\cos(\alpha-\alpha_K)_+\right].$$

4. 2. Accounting for boundary conditions of bearing the arches

In order to determine parameters of the arch SDS, it is necessary to construct and solve a boundary value problem, taking into account conditions for bearing the boundary points. In BEM, this equation takes the form:

$$A_*\cdot X_*=-B, \qquad (4)$$

where A_* is the matrix of boundary values of fundamental functions that takes into consideration boundary conditions of the arch; X_* is the matrix of initial and final parameters of bending and stretching of the arch; B is the load matrix at a boundary value of variable α.

We shall consider different conditions for bearing an arch.

4. 2. 1. Hinged bearing

Calculation scheme of the arch with a hinged bearing is shown in Fig. 2.

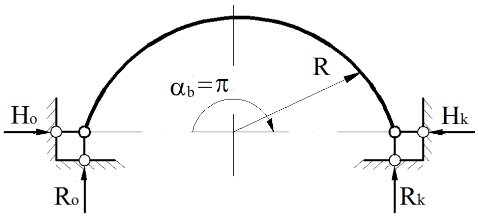

Fig. 2. Hinged fastening of the arch

We shall construct matrices of initial X and final Y parameters of the arch, which take into account a hinged bearing of two boundary cross-sections

$$X_* = \begin{pmatrix} EI\vartheta(0)=0; EI\phi(\alpha_r) \\ EI\phi(0) \\ M(0)=0; Q(\alpha_r) \\ Q(0) \\ EAu(0)=0; N(\alpha_r) \\ N(0) \end{pmatrix}; \quad Y = \begin{pmatrix} EI\vartheta(\alpha_r)=0 \\ EI\phi(\alpha_r) \\ M(\alpha_r)=0 \\ Q(\alpha_r) \\ EAu(\alpha_r)=0 \\ N(\alpha_r) \end{pmatrix}. \qquad (5)$$

Matrix X shows that at hinged bearing, rows 1, 3, 5 are equal to zero. Since these rows are associated with the columns of matrix A with the same numbers, it is necessary to reset to zero columns 1, 3, and 5. The zero rows of matrix X can be substituted with non-zero parameters of matrix Y. Such a shift should be compensated for by introducing nonzero elements to matrix A. According to the theory of a numerical-analytical variant of BEM, such elements will comprise A (2.1)=–1; A (4.3)=–1; A (6.5)=–1.

Matrix equation for a boundary value problem for the arch in Fig. 2 will take the form

$$\begin{pmatrix} & A_{12} & & -A_{14} & & A_{16} \\ -1 & A_{22} & & -A_{13} & & A_{26} \\ & & & -A_{12} & & -A_{36} \\ & & -1 & -A_{11} & & -A_{46} \\ & A_{52} & & A_{54} & & -A_{56} \\ & & & A_{64} & -1 & -A_{64} \end{pmatrix} \begin{pmatrix} EI\phi(\pi) \\ EI\phi(0) \\ Q(\pi) \\ Q(0) \\ N(\pi) \\ N(0) \end{pmatrix} = \begin{pmatrix} -B_{11}(\pi) \\ -B_{21}(\pi) \\ B_{31}(\pi) \\ B_{41}(\pi) \\ B_{51}(\pi) \\ B_{61}(\pi) \end{pmatrix}, \quad (6)$$

After solving equation (6), reactions of the arch will be equal to $H_0 = Q_{(0)}$; $R_0 = N_{(0)}$; $M_0 = M_{(0)}$; $H_K = Q_{(\pi)}$; $R_K = N_{(\pi)}$, while SDS in the inner points is determined from equation (2).

4. 2. 2. Rigid pinching and hinged bearing

By analogy, constructing matrices X_*, Y, we proceed to the equation of the right problem for this case of bearing (Fig. 3).

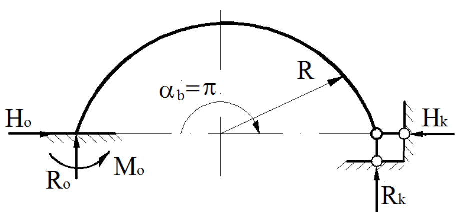

Fig. 3. Rigid pinching and hinged bearing of the arch

$$\begin{pmatrix} & & -A_{13} & -A_{14} & & A_{16} \\ -1 & & -A_{23} & -A_{13} & & A_{26} \\ & & A_{22} & A_{12} & & -A_{36} \\ & -1 & & A_{11} & & -A_{46} \\ & & -A_{53} & -A_{54} & & A_{56} \\ & & & -A_{64} & -1 & A_{11} \end{pmatrix} \begin{pmatrix} EI\phi(\pi) \\ Q(\pi) \\ M(0) \\ Q(0) \\ N(\pi) \\ N(0) \end{pmatrix} = \begin{pmatrix} -B_{11}(\pi) \\ -B_{21}(\pi) \\ B_{31}(\pi) \\ B_{41}(\pi) \\ B_{51}(\pi) \\ B_{61}(\pi) \end{pmatrix}, \quad (7)$$

After solving (7), reactions of the arch will be equal to

$$H_0 = Q_{(0)}; \; R_0 = N_{(0)}; \; H_K = Q_{(\pi)}; \; R_K = N_{(\pi)}.$$

4. 2. 3. Rigid pinching of two boundary points

Matrix equation for a boundary value problem of the given case of bearing takes the form (Fig. 4).

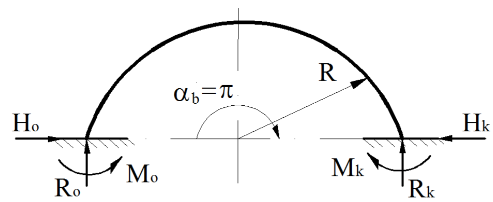

Fig. 4. Rigid pinching of the arch

$$\begin{pmatrix} & & -A_{13} & -A_{14} & & A_{16} \\ -1 & & -A_{23} & -A_{13} & & A_{26} \\ & & A_{22} & A_{12} & & -A_{36} \\ & -1 & & A_{11} & & -A_{46} \\ & & -A_{53} & -A_{54} & & A_{56} \\ & & & & -1 & A_{11} \end{pmatrix} \begin{pmatrix} M(\pi) \\ Q(\pi) \\ M(0) \\ Q(0) \\ N(\pi) \\ N(0) \end{pmatrix} = \begin{pmatrix} -B_{11}(\pi) \\ -B_{21}(\pi) \\ B_{31}(\pi) \\ B_{41}(\pi) \\ B_{51}(\pi) \\ B_{61}(\pi) \end{pmatrix}, \quad (8)$$

The arch reactions will equal

$$H_0 = Q_{(0)}; \; R_0 = N_{(0)}; \; M_0 = M_{(0)};$$

$$H_K = Q_{(\pi)}; \; R_K = N_{(\pi)}; \; M_k = M_{(\pi)}.$$

We shall draw an example of applying BEM technology to calculate a state of the arch under the action of concentrated moments, force, and distributed load.

5. Results of determining the SDS parameters of arches

We shall consider example of determining the SDS parameters of the arch (Fig. 5) with an arbitrary loading and fastening.

The results will be presented numerically (in a tabular form) and visually (in the form of diagrams).

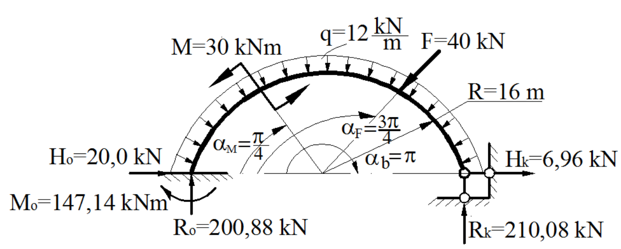

Fig. 5. Calculation scheme of the arch

Angular coordinates for the concentrated time and concentrated force

$$\alpha_M = \frac{\pi}{4}; \quad \alpha_F = \frac{3}{4}\pi.$$

Angular coordinates of the uniformly distributed load.

$$\alpha_H = 0; \quad \alpha_K = \pi.$$

At the specified initial data, values of the boundary parameters after solving equation (7) will be equal to (Table 1)

Reactions of the arch supports will equal, respectively, H_o=20.00 kN, R_o=200.88 kN, M_o=147.14 kNm, H_k=6.96 kN, R_k=210.08 kN.

The SDS parameters' values are summarized in Table 2. Data in Table 2 indicate that the obtained results are accurate. This conclusion is justified by that the arch SDS was calculated by equations from the method of initial parameters (2), in which initial parameters were taken from Table 1. If the solution is exact, boundary conditions for bearing must hold at the right support, which is confirmed by data in Table 2.

Table 1

Value of boundary parameters of the arch in Fig. 5

No. of entry	Parameter	Value
1	$EI\phi(\pi)$, kNm^2	–958.34
2	$Q(\pi)$, kN	–6.96
3	$M(o)$, kNm	147.14
4	$Q(o)$, kN	–20.00
5	$N(\pi)$, kN	–210.08
6	$N(o)$, kN	–200.88

Diagrams of the arch SDS parameters in Cartesian coordinates are shown in Fig. 6–11.

Table 2

Results of calculating the arch in a numerical form

Angular coordinate		Parameters of the stressed-deformed state					
Rad.	Degrees	$EAv \cdot 10^{-3}$, kNm^2	$EA\phi \cdot 10^{-2}$, kNm^2	M, kNm	Q, kN	$Eau \cdot 10^{-4}$, kNm	N, kN
0.0	0.0	0.0	0.0	147.14	–20.0	0.0	–200.88
0.09	5	0.133	1.86	119.79	–19.15	–0.033	–202.59
0.17	10	0.497	3.35	93.72	–18.16	–0.092	–204.22
0.26	15	1.038	4.48	6915	–17.02	–0.200	–205.76
0.35	20	1.703	5.28	46.24	–15.76	–0.371	–207.19
0.44	25	2.445	5.78	25.19	–14.38	–0.617	–208.50
0.52	30	3.214	6.00	6.15	–12.88	–0.943	–209.69
0.61	35	3.970	5.96	–10.74	–11.29	–1.348	–210.75
0.70	40	4.673	5.71	–25.34	–9.61	–1.831	–211.66
0.79	45	5.289	5.27	–67.55	–7.86	–2.383	–214.30
0.87	50	5.760	4.25	–77.16	–5.89	–2.993	–214.90
0.96	55	6.036	3.12	–83.98	–3.87	–3.643	–215.33
1.05	60	6.100	1.92	–87.96	–1.83	–4.310	–215.57
1.13	65	5.945	0.68	–89.07	0.24	–4.973	–215.64
1.22	70	5.570	–0.54	–87.30	2.30	–5.608	–215.53
1.31	75	4.980	–1.73	–82.67	4.34	–6.192	–214.24
1.40	80	4.189	–2.84	–75.20	6.35	–6.704	–214.78
1.48	85	3.219	–3.82	–64.9	8.31	–7.124	–214.14
1.57	90	2.095	–4.64	–52.03	10.21	–7.433	–213.33
1.66	95	0.853	–5.26	–36.50	12.03	–7.618	–212.36
1.75	100	–0.468	–5.65	–18.48	13.76	–7.668	–211.23
1.83	105	–1.824	–5.77	1.87	15.38	–7.578	–209.96
1.92	110	–3.165	–5.59	24.41	16.89	–7.345	–208.55
2.01	115	–4.435	–5.08	48.96	18.26	–6.975	–207.02
2.09	120	–5.578	–4.21	–75.35	19.50	–6.478	–205.37
2.18	125	–6.534	–2.96	103.36	20.59	–5.871	–203.62
2.27	130	–7.240	–1.32	132.79	21.53	–5.175	–201.78
2.36	135	–7.635	0.74	163.40	22.30	–4.421	–199.86
2.44	140	–7.673	2.85	139.20	–16.95	–3.645	–201.38
2.53	145	–7.383	4.63	116.14	–16.07	–2.882	–202.82
2.62	150	–6.813	6.10	94.39	–15.06	–2.165	–204.18
2.71	155	–6.008	7.28	74.13	–1394	–1.520	–205.44
2.79	160	–2.016	8.18	55.50	–12.72	–0.970	–206.61
2.88	165	–3.879	8.83	38.66	–11.40	–0.532	–207.66
2.97	170	–2.638	9.27	23.72	–9.99	–0.219	–208.59
3.05	175	–1.334	9.51	10.80	–8.50	–0.040	–209.40
3.14	180	0.0	9.58	0.0	–6.96	0.0	–210.08

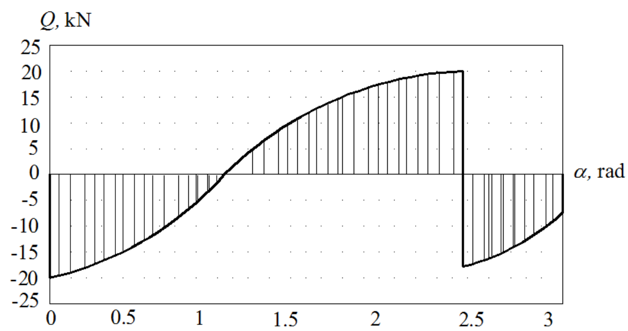

Fig. 6. Diagram of transverse forces Q, kN

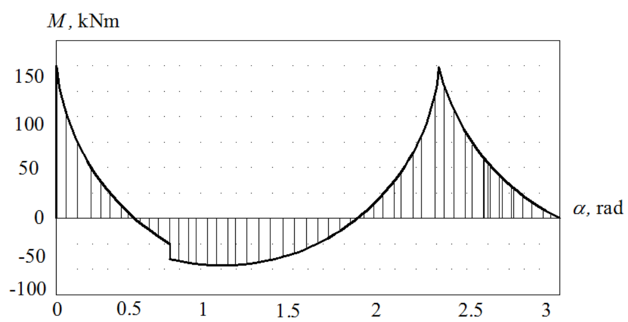

Fig. 7. Diagram of bending moments M, kNm

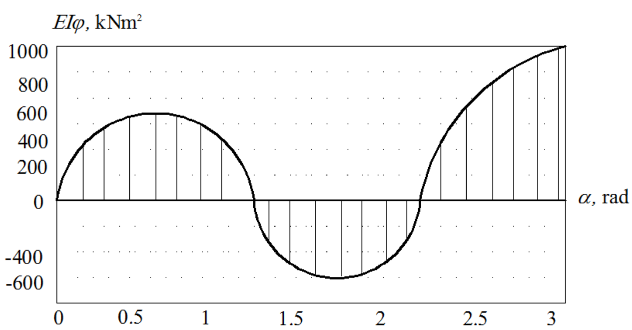

Fig. 8. Diagram of angles of rotation $EI\varphi$, kNm^2

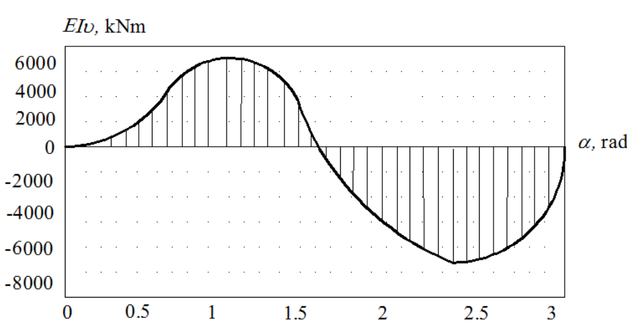

Fig. 9. Diagram of deflection $EI\upsilon$, kNm^3

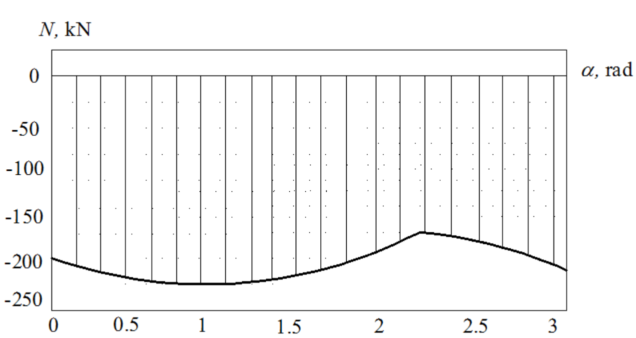

Fig. 10. Diagram of normal forces N, kN

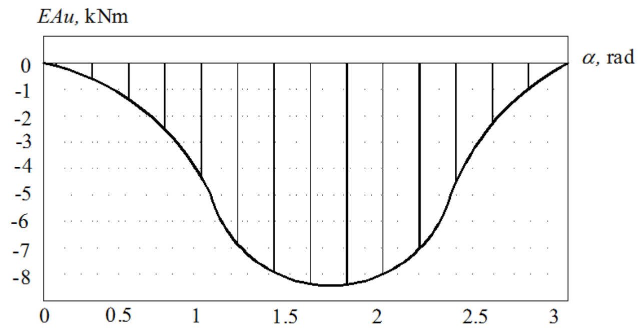

Fig. 11. Diagram of tangential displacements EAu, kNm

5. Discussion of results of mathematical modeling of the stressed-deformed state of circular arches

It follows from Table 2 and diagrams in Fig. 6–11 that SDS of the arch obtained using a calculation technique is accurate. The calculation employed initial parameters of the arch from Table 1. Boundary conditions for the hinged bearing must be fulfilled in the final cross-section, which is confirmed by data in Table 2 and diagrams in Fig. 6–11. This outcome is explained by the fact that we applied a technology for solving boundary value problems for arches using a numerical-analytical variant of BEM and exact solution to the problem on flat deformation of a circular rod considering the deformations of bending and stretching-compression. This indicates a large advantage of the proposed procedure over the methods of forces, displacements, and FEM. Typically, a circular rod is replaced with a polygonal model of rectilinear rods; the accuracy of such a model does not exceed 1 % if a straight rod tightens the arc of 5 degrees. For the considered problem, it is necessary to calculate a system of 36 rods. Should FEM be used, it would require solving a system of 105 linear algebraic equations, while in the case of BEM, it will be needed to solve a system of 6 equations only. To calculate the ring applying FEM, one has to solve 213 equations, when employing BEM – only 6 equations again. An even bigger difference in the number of equations will emerge when estimating the arched systems.

As an improvement of the proposed approach to the calculation of arch structures, it is possible to point out the applicability of BEM to solving boundary value problems for arches with variable parameters (for instance, rigidity and radius of curvature). Such arches have differential equations with variable coefficients. In this case, a body of the arch is split into n parts; all parameters of the arch are considered constant within the limits of every part. That is why the equation of the stressed-deformed state will hold in each of the n parts (2). Next, a BEM technique may be applied to the system of n parts. The practice of solving such problems demonstrates that at $n \geq 30$, results of BEM almost coincide with the known precise solutions (error is less than 1 %).

6. Conclusions

1. We obtained solving equations of the boundary value problems for determining the unknown initial parameters of circular arches under existing conditions for bearing the boundary cross-sections. They represent systems of linear algebraic equations that take into account different bound-

ary conditions for bearing and were obtained in accordance with the theory of BEM.

2. A procedure for calculating SDS of circular arches was devised. Its special feature is in the fact that one applies an exact solution to the problem on planar deformation of a circular rod considering the deformations of bending and stretching-compression. For this reason, it makes it possible, in contrast to the existing methods (forces, displacements, FEM, initial settings), to obtain exact solutions of SDS at a minimum number of solving equations.

3. We performed calculation of a circular arch, taking into account the action of distributed and concentrated loads. The calculation showed that using BEM can help obtain exact solutions to the problems of flat deformation of arches at a minimal number of solving equations. It is also demonstrated that BEM makes it relatively simple to take into account the concentrated moments in the form of an external load and to have a complete solution of SDS in the form of power and kinematic parameters. These issues cause difficulties in the existing methods and they are typically not reflected in the relevant publications.

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