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DEVELOPMENT OF THE MARKOVIAN MODEL FOR THE LIFE CYCLE OF A PROJECT'S BENEFITS

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В даному дослідженні виконано розвиток стандарту P5 (Персонал, Планета, Прибуток, Процес, Продукт), який вже набув в світі широкого розповсюдження і застосування в практиці проектного управління. Однак в стандарті P5 представлена схема життєвого циклу переваг проектів, яка дозволяє тільки на якісному рівні відобразити процеси життєвого циклу. Для переходу до кількісних оцінок запропоновано використати ланцюг Маркова, який являє собою феноменологічне відображення складних систем без урахування фізичної їх природи.

Побудована когнітивна марківська модель життєвого циклу переваг проекту у вигляді комунікацій між станами проектної системи. Когнітивна структура життєвого циклу являє собою подобу орієнтованого графа, де вершини графа позначають стани системи, а зв'язки комунікації між ними. Запропоновано підхід щодо визначення перехідних ймовірностей за на основі оцінки комунікацій з урахуванням витрат ресурсів часу на виконання операцій у формі правил. Характер комунікацій між станами $s \to j$ ***ланцюга Маркова визначає величину перехідних ймовірностей*** π_{sj}. ***Ресурсні витрати часу для кожного стану поділені на п'ять інтервалів*** π_{sj}: ***{0} – витрати відсутні; {0.01–0.1} – незначні витрати часу; {0.1–0.3} – нижній рівень витрати часу; {0.3–0.7} – середні витрати часу; {0.7–1.0} – найбільше часу. Логіка вибору значень умовних перехідних ймовірностей ланцюга Маркова дозволяє визначити дані для моделювання траєкторії життєвого циклу переваг проектів в координатах ймовірностей станів системи і кроків.***

Показано, що застосування ланцюгів Маркова є раціональним для відображення життєвого циклу переваг проектів. Проведено аналіз для певного рівня технологічної зрілості проектного середовища (організації), що відповідає сукупності значень перехідних ймовірностей. Досліджено вплив рівня технологічної зрілості проектного середовища (організації) на результативність проектів.

Інший приклад щодо виконання проекту по формуванню іміджу навчальної установи шляхом застосування фронтальних інформаційних комунікацій через телебачення, пресу, участь в масових політичних заходах також виявив позитивні оцінки. Розподіл ймовірностей на початку проекту (V1) і по його завершенні (V2) суттєво відрізняються. У наслідок виконання проекту збільшились величини ймовірністей станів p7 (Вигода) та p8 (Додаткова вигода). На початку проекту: $p7^{(V1)}+p8^{(V2)}=0.14+0.05=0.19$. ***Після виконання проекту:*** $p7^{(V1)}+p8^{(V2)}=0,22+0.08=0.30$. ***Оцінка даних проекту формування іміджу навчальної установи показала, що отримані результати не суперечать гіпотезі про можливість застосування ланцюгів Маркова для визначенння характеристик життєвого циклу переваг проекту***

Ключові слова: когнітивна схема, ланцюг Маркова, рівень технологічної зрілості, життєвий цикл, траєкторії розвитку життєвого циклу

1. Introduction

Since the time when the methodology of PRINCE2 (Projects in a Controlled Environment) [1] and MSP (Managing Successful Programs) [2] had been developed, the focus of goal setting in project activity has substantially shifted towards results and benefits, obtained upon completion of the project, rather than time, money, quality.

The standard GPM® Global P5TM (personnel, planet, prosperity, processes, products) sets the requirements for projects in line with the initiative of the UN – the United Nations Global Compact. The UN initiative aims to promote social responsibility of business and to support solving the problems on globalization and creation of conditions for sustainable development of the mankind. In each project, one can trace a relationship with the global UN context – 17 goals and 169 objectives of human development to the year 2030 [3].

The contradiction between the needs of practice concerning the implementation of projects in the field of sustainable human development and the lack of acceptable models for the quantitative evaluation of the life cycle of projects' benefits defines the relevance of research in terms of ensuring the effectiveness of project activity [4]. Development of mathematical models that would implement the paradigm of processes of the life cycle of projects' benefits could make it possible to achieve proactive (predicted in advance) improvement of project management [5].

2. Literature review and problem statement

According to a 2016 survey of nearly 15 thousand specialists in project management from 147 countries, the website of the publisher of the standard GPM® Global P5TM gives the following data about those who applies the specified standard [3]: 39 % are the project managers; 19 % are the scientists from academic institutions; 17 % are professionals on the issues of sustainable development; 11 % are executives; 8 % are the providers of trainings; 6 % are government officials. In this case, 76 % of them use the standard P5 in practice. It is indicated that 93 % of the people who make decisions have seen a substantial increase in the effectiveness of a project. At the same time, 94 % of executives said that projects and project management are an integral part of sustainable development in the fields of personnel, planet, prosperity, processes, products [3].

The organization and management of interaction between the components of a project system predetermine the success of projects implementation [5]. In order to construct a cognitive structure of the life cycle model of projects' benefits, we shall employ a known pattern that is included in the standard GPM® Global P5TM (Fig. 1).

The complexities of interaction between the states of a project system during project management are predetermined by the existence of a set of external and internal factors [6]. The uncertainty of the project environment and the uniqueness of project tasks make it impossible to isolate and thoroughly study the system based on separate elements [7]. Project systems are characterized by the emergent properties – certain characteristics that are not peculiar to individual elements, as well as the sums of elements' properties [8]. Such features necessitate studying processes within the project systems, based not on the properties of individual components, but considering the project in general [9]. Therefore, in order to represent the trajectory of project development in the phase space of state probabilities, it is proposed to apply phenomenological models [10]. The class of such models includes the Markovian chains, which make it possible to represent a connection between the input and output parameters, without taking the physical essence of processes into consideration [11]. The Markovian chains reflect the topological structure of relationships between the elements of the system. At the same time, setting the model parameters for specific projects is implemented based on practical data by determining conditional probabilities of transitions between elements of the system [12].

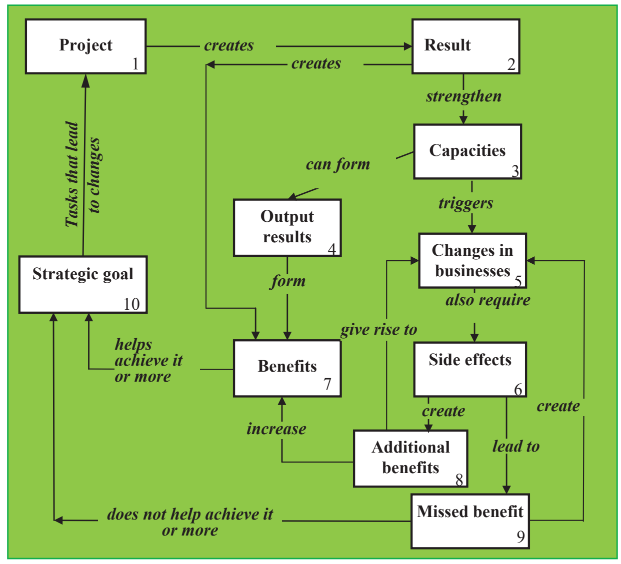

Fig. 1. Cognitive structure of the life cycle of a project's benefits: 1 – project; 2 – result; 3 – capacity; 4 – result at the output; 5 – changes in business; 6 – side effects; 7 – benefit; 8 – additional benefit; 9 – missed benefit; 10 – strategic goal

The properties of project management have been investigated when solving a task on initiating the projects on labor protection using the Markovian model [12]. Owing to the cognitive properties of the Markovian chains, the features of implementation of a «lifelong education» paradigm were described [13]. In paper [14], the Markovian model reflects the system of change in the conditions of athletes involved in training to compete. The Markovian model is used to assess the role of communication within a project team [15]. Construction of the specified models is performed through the decomposition of projects into discrete states showing the transitions between these states applying a directed graph [16]. The complexity of the identification of the specified models manifests itself in the variations of approaches to determining conditional probabilities of transitions between discrete states. That is, the specificity of representation of different objects through the homogeneous Markovian chains with discrete states and discrete time is defined by the techniques to estimate the probabilities of transitions [17].

An analysis of publications reveals that the main problem that prevents the wide application of the Markovian chains in order to represent complex systems is the development of specific structures for particular projects and the transformation of unique graphical diagrams into directed graphs. In this case, there are always difficulties in determining the probabilities of transitions between the states of the system. It is the totality of probabilities of transitions between the states of the system makes it possible to phenomenologically represent the systems properties of interaction between the totality of specific processes within a project. Thus, it is a promising task to construct the Markovian models that would represent significant properties of projects and which would make it possible to move from qualitative assessments to determining the quantitative estimates of trajectories in project development [18]. Communicative interaction between the states of projects is the most important condition for the successful implementation of project activity [19], which is typically regarded as a characteristic of the level of technological maturity of an organization that depends on the competence and skills of project performers [20].

3. The aim and objectives of the study

The aim of this study is:

– to devise a method for the transformation of a cognitive scheme of the life cycle of projects' benefits, which is presented in the standard [3], into the Markovian chain;

– to formalize determining the transition probabilities of the Markovian model taking into consideration the time cost of communication implementation between the system's elements;

– to explore the features of practical determination of the development trajectory of the life cycle of projects' benefits under conditions of various characteristics of the technological maturity of the system.

4. Construction of the life cycle model of a project's benefits

We shall represent a life cycle model of a project's benefits, given in the standard [3], using ten discrete states. In order to construct a cognitive model of the life-cycle of a project's benefits as the communication between the states of the project system, we point to the basic transitions between these states (Fig. 1). These links represent transitions between the states of the system when it is represented by a directed graph.

A cognitive structure of the life cycle is similar to a directed graph with vertices that correspond to the states of a project, and with arcs that represent the communication links between the states of a project [11]. We accept the assumption on that the sum of probabilities of all states is equal to unity; we shall also define that transitions from each state are the incompatible events. Under such conditions, the graph can be transformed into the Markovian chain with discrete states [14]. To this end, we shall add to the directed graph the delay links to each of the 10 states. The result of such addition is the Markovian chain graph (Fig. 2).

We shall describe the Markovian chain by applying a method of state probability [15]. The probabilities of transitions are shown in the marked graph (Fig. 2). A step is to be to understood as a cycle of the execution of operations, which include a set of certain operations [13]. In the first approximation, we accept the assumption on that all the steps are equivalent and are carried out at random.

Identifiers S_i, $\{i=1,...,10\}$ denote the possible states of the system. They form the Markovian chain with 10 states, which are a complete group of incompatible events. A given model represents the Markovian chain, since both the processes of project management and the Markovian chains could undergo changes in the probabilities of the system's states in steps k. There are probabilities of transitions to other states, the sum of transition probabilities from a certain state is equal to unity. The sum of the probabilities of all states at each step is also equal to unity [12]. There occurs the similarity of the topological structure of transitions [11].

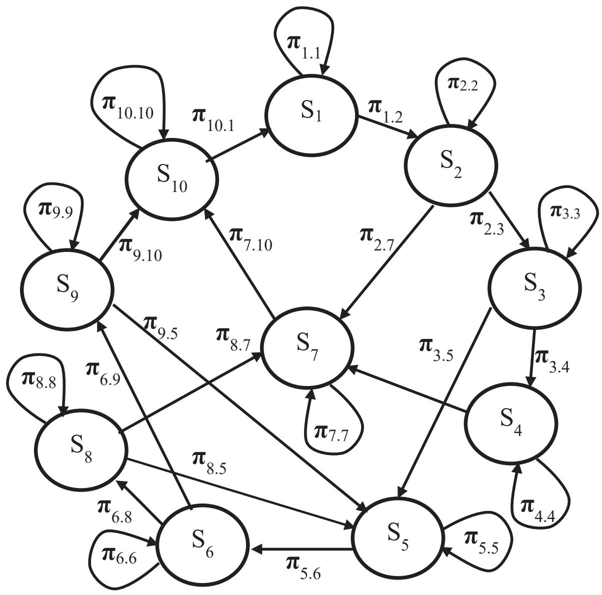

Fig. 2. Marked graph of the Markovian chain that represents the structure of the life-cycle of projects' benefits: S_1 – project; S_2 – result; S_s – capacity; S_4 – result at the output; S_5 – changes in business; S_6 – side effects; S_7 – benefit; S_8 – additional benefit; S_9 – missed benefit; S_{10} – strategic goal

The sum of transition probabilities π_{ij} from a certain state $i\in(1, 2,..., m)$ into other states $j\in(1, 2,..., m)$ is equal to unity [14]:

$$\sum_{j=1}^{m}\pi_{ij}=1, \quad \{i=1,2,\cdots,m\}, \tag{1}$$

where $m=10$ is the number of possible states of the system.

A transition matrix that describes the graph contains a value of «0» in the case of the missing link between certain states or in the presence of the link the value for a conditional probability π_{ij} of the transition [11]. General solution to a system of equations that describe the Markovian chain, shown in Fig. 2, can be written in the following form:

$$\left\|\begin{matrix} p_1(k+1)\\ p_2(k+1)\\ p_3(k+1)\\ p_4(k+1)\\ p_5(k+1)\\ p_6(k+1)\\ p_7(k+1)\\ p_8(k+1)\\ p_9(k+1)\\ p_{10}(k+1)\end{matrix}\right\|^{T}=\left\|\begin{matrix} p_1(k)\\ p_2(k)\\ p_3(k)\\ p_4(k)\\ p_5(k)\\ p_6(k)\\ p_7(k)\\ p_8(k)\\ p_9(k)\\ p_{10}(k)\end{matrix}\right\|^{T}\cdot\left\|\begin{matrix} \pi_{1.1} & \pi_{1.2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & \pi_{2.2} & \pi_{2.3} & 0 & 0 & 0 & \pi_{2.7} & 0 & 0 & 0\\ 0 & 0 & \pi_{3.3} & \pi_{3.4} & \pi_{3.5} & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & \pi_{4.4} & 0 & 0 & \pi_{4.7} & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & \pi_{5.5} & \pi_{5.6} & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & \pi_{6.6} & 0 & \pi_{6.8} & \pi_{6.9} & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & \pi_{7.7} & 0 & 0 & \pi_{7.10}\\ 0 & 0 & 0 & 0 & \pi_{8.5} & 0 & \pi_{8.7} & \pi_{8.8} & 0 & 0\\ 0 & 0 & 0 & 0 & \pi_{9.5} & 0 & 0 & 0 & \pi_{9.9} & \pi_{9.10}\\ \pi_{10.1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \pi_{10.10}\end{matrix}\right\|, \tag{2}$$

where π_{ij} is the transition probabilities.

The sum of the probabilities of all states $p_i(k)$ at each step k is equal to unity [12]:

$$\sum_{i=1}^{m} p_i(k) = 1, \tag{3}$$

where $p_i(k)$ is the probability of the i-th state in step k; $i \in (1, 2,..., m=10)$.

Step k is to be understood as a certain controlling influence that changes the distribution of probabilities $p_i(k)$ in the next step $(k+1)$ for all $i \in (1, 2, ..., m=10)$ [13].

Determining in (1) all elements π_{ij} and the initial state probabilities $\{p_1(k),\ p_2(k), ...\ p_{10}(k)\}$ makes it possible to calculate the magnitudes of $\{p_1(k+1), p_2(k+1), ...\ p_{10}(k+1)\}$.

5. Identification of the Markovian chain based on the characteristics of communications

The interactions within a system under project management are shown in the directed graph (Fig. 2). For any discrete state $s\{s \in 1, ..., 10\}$ the total time T_s of communications with other states can be determined as the sum of duration of the interaction with these states $t_{sj}\{s \in 1, ..., 10;\ j \in 1, ..., 10\}$ [12]:

$$T_s = \sum_{j=1}^{n=10} t_{sj}, \tag{4}$$

where t_{sj} is the duration of project in communication $s \to j$ from state s.

Each communication $s \to j$ of the system requires certain time cost t_{sj} in a project. The value of $\pi_{sj}=t_{sj}/T_s$ means the probability of a transition from state $s \to j$.

The sum of all transition probabilities for a certain state s is equal to:

$$\sum_{j=1}^{n=10} \pi_{sj} = \sum_{j=1}^{n=10} \frac{t_{sj}}{T_s} = \frac{1}{T_s} \sum_{j=1}^{n=10} t_{sj} = 1. \tag{5}$$

Thus, the transition probabilities π_{sj} to any state from $s\{s \in 1, ..., 10\}$, which are represented in each row of the matrix of transition probabilities, form an incompatible group of events.

This property $\pi_{sj}\{s \in 1, ..., 10; j \in 1, ..., 10\}$ makes it possible to explore the behavior of the system at different variants of input data on a project. Changing π_{sj} can alter characteristics of the system.

In the general case, transition probabilities $\pi_{sj}\{s \in 1, ..., 10;\ j \in 1, ..., 10\}$ «set up» the Markovian models for an actual object. In this case, in order to determine π_{sj} the two approaches are typically used. The first one implies an expert assessment of the values for transition probabilities [11]. The second one employs a questionnaire that makes it possible to determine the probabilities of states $p_1(k)$, $p_2(k)$,..., $p_n(k)$, which then serves as the basis for finding the values for transition probabilities [12]. We propose an approach that is based on that the values for transition probabilities are determined based on planning the communications within a system taking into account the cost of a time resource required to execute operations (Table 1).

The rules for determining the values for transition probabilities, given in Table 1, make it possible to find data in order to model the changes in the probabilities of states of the system for projects at any combinations of resourcing. We shall determine the values for transition probabilities $\pi_{i,j}$ from Table 1. Based on the matrix of transition probabilities, provided that the original state of the system is known, it is possible to find all the probabilities of states of the system $p_1(k)$, $p_2(k)$,..., $p_{10}(k)$ after any of the k-th step from dependence (2).

Table 1

Determining the values for transition probabilities π_{sj}

Character of communication in terms of the time resource cost	Transition probabilities π_{sj}
Maximum time	0.7–1.0
Average resource use	0.3–0.7
Low level of time cost	0.1–0.3
Insignificant time cost	0.01–0.1
No time cost	0

6. Solving applied problems based on modeling the life cycle of a project's benefits

6. 1. The impact of the level of technological maturity of the system

The general solution to the accepted transition probability matrix that represents the basic variant of the structure of a life cycle of a project's benefits takes the iterative form:

$$\begin{Vmatrix} p_1(k+1) \\ p_2(k+1) \\ p_3(k+1) \\ p_4(k+1) \\ p_5(k+1) \\ p_6(k+1) \\ p_7(k+1) \\ p_8(k+1) \\ p_9(k+1) \\ p_{10}(k+1) \end{Vmatrix}^{\mathrm{T}} = \begin{Vmatrix} p_1(k) \\ p_2(k) \\ p_3(k) \\ p_4(k) \\ p_5(k) \\ p_6(k) \\ p_7(k) \\ p_8(k) \\ p_9(k) \\ p_{10}(k) \end{Vmatrix}^{\mathrm{T}} \cdot \begin{Vmatrix} 0.4 & 0.6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.4 & 0.3 & 0 & 0 & 0 & 0.3 & 0 & 0 & 0 \\ 0 & 0 & 0.4 & 0.3 & 0.3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.4 & 0 & 0 & 0.6 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.4 & 0.6 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.4 & 0 & 0.3 & 0.3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.4 & 0 & 0 & 0.6 \\ 0 & 0 & 0 & 0 & 0{,}4 & 0 & 0.2 & 0.4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.4 & 0 & 0 & 0 & 0.4 & 0.2 \\ 0.6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.4 \end{Vmatrix}. \tag{6}$$

The elements of the probability matrix, given in (6), are chosen based on data from Table 1. Based on (4) and (5), we note that rows of the transition probability matrix are independent. Each row describes a characteristic of a particular state in terms of communications with other states. Thus, for example, for state 2 – «Result», which means that communications with state 7 – «Benefits» ($\pi_{2.7}=0.3$), as well as with state 3 – «Capacities» ($\pi_{2.7}=0.3$), can be related in terms of time cost to the level at the limit of average time costs and at the lower level. As shown by the example given, transition probabilities for each state are conditional characteristics that represent the allocation of time cost among all communications from a given state. In this case, the allocation of time costs depends on the general level of maturity of the project environment. Based on the specified conditions, we have determined for the basic variant of the system similar conditional transition probabilities for all $\pi_{ii}=0.4$, which corresponds, according to Table 1, to the average costs of a time resource.

Fig. 3 shows results of modeling a change in the states of the system for the matrix of transition probabilities (6), which is the base in this study.

For a certain level of technological maturity of the project environment (organization), which corresponds to the totality of values for transition probabilities $\pi_{ii}\{i\in 1, ..., 10\}=0.4$, derived from Table 1, one can make the following conclusions. The largest probabilities of states after step 25 are demonstrated by states 1 – «Project», 2 – «Result», 7 – «Benefit», 10 – «Strategic goal» (Fig. 4). Next, the most significant are states 5 – «Changes in business» and 6 – «Side effects». The probabilities of the remaining states 3, 4, 8, 9 at step 25 are almost identical at the level of insignificant time costs.

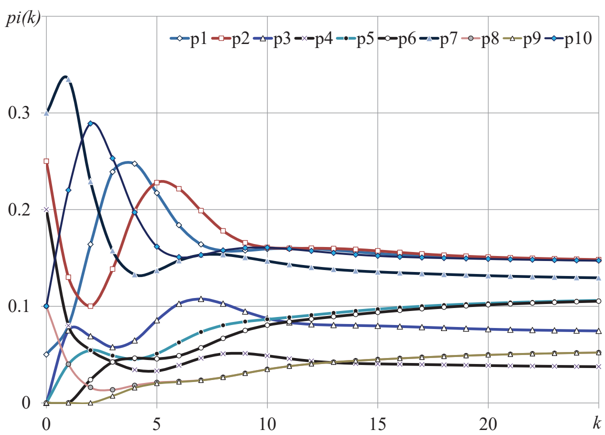

Fig. 3. Change in the probabilities of states of the system in line with the basic variant (π_{ii}=0.4): *p*1 – project; *p*2 – result; *p*3 – capacity; *p*4 – result at the output; *p*5 – changes in business; *p*6 – side effects; *p*7 – benefit; *p*8 – additional benefit; *p*9 – missed benefit; *p*10 – strategic goal

In order to estimate the probability distribution of states of the system for other characteristics of the system, we shall make certain changes to the matrix of transition probabilities. We shall examine an influence of the level of technological maturity of the project environment (organization), which corresponds to the totality of values for transition probabilities π_{ii}=0.2, derived from Table 1. Such probabilistic characteristics represent the states being related to the relatively low time cost for personal needs. That is, owing to the high qualification of staff, almost the entire resource of time is used for work on the project. The modified Markovian model will have the following solution:

$$\left\|\begin{matrix} p_1(k+1)\\ p_2(k+1)\\ p_3(k+1)\\ p_4(k+1)\\ p_5(k+1)\\ p_6(k+1)\\ p_7(k+1)\\ p_8(k+1)\\ p_9(k+1)\\ p_{10}(k+1)\end{matrix}\right\|^T = \left\|\begin{matrix} p_1(k)\\ p_2(k)\\ p_3(k)\\ p_4(k)\\ p_5(k)\\ p_6(k)\\ p_7(k)\\ p_8(k)\\ p_9(k)\\ p_{10}(k)\end{matrix}\right\|^T \cdot \left\|\begin{matrix} 0.2 & 0.8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0.2 & 0.6 & 0 & 0 & 0 & 0.2 & 0 & 0 & 0\\ 0 & 0 & 0.2 & 0.3 & 0.5 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0.2 & 0 & 0 & 0.8 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0.2 & 0.8 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0.2 & 0 & 0.6 & 0.2 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0.2 & 0 & 0 & 0.8\\ 0 & 0 & 0 & 0 & 0,4 & 0 & 0.4 & 0.2 & 0 & 0\\ 0 & 0 & 0 & 0 & 0.4 & 0 & 0 & 0 & 0.2 & 0.4\\ 0.8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.2 \end{matrix}\right\|. \quad (7)$$

For the changed data (7), we obtain the results shown in Fig. 4.

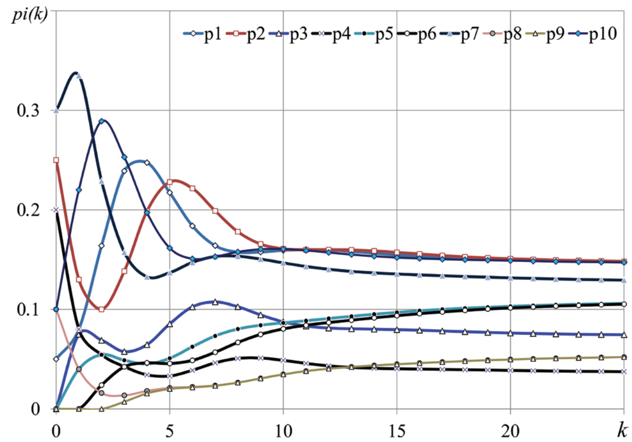

Fig. 4. Change in the probabilities of states of the system in line with the changed variant of the level of technological maturity (π_{ii}=0.2): *p*1 – project; *p*2 – result; *p*3 – capacity; *p*4 – result at the output; *p*5 – changes in business; *p*6 – side effects; *p*7 – benefit; *p*8 – additional benefit; *p*9 – missed benefit; *p*10 – strategic goal

Under these conditions, the influence of states 1 – «Project», 2 – «Result», 7 – «Benefit», 10 – «Strategic goal» of the life cycle of a project's benefits becomes crucial in the implementation of successful project activities. In this case, the character of change in other probabilities of states of the system also becomes different from the basic variant shown in Fig. 3.

Because the probabilities of states $p_s(k)$, $s\{s\in 1, ..., 10\}$ are derived from (3), then, with respect to (8) and (9), they form an incompatible group of events. That makes it possible to relate magnitudes $p_s(k)$ to the duration of execution of project's operations.

Thus, the duration of work in state 6 in the basic project is monotonically changing to the magnitude of $p_6(k)=0.09$ in step $k=25$ (Fig. 3). This is equivalent to 9 % of the total time costs for the basic variant of the project. In this case, the highest probability in step 25 is correlated with state 2 («Result») – 13.8 % of the total time cost of the project. States 1 («Project») and 2 («Strategic goal») would require, accordingly, 13.6 % and 13.3 % of the time allocated to the project. The general pattern of differences in the simulation results of the basic (variant 1) and changed (variant 2) Markovian model with a sample of state probability $p_s(k)$ $\{s\in 1, ..., 10;$ $k=25\}$ is shown in Fig. 5.

Simulation results show that a change in the level of technological maturity represented by variations in the matrix of transition probabilities leads to a change in the overall pattern of probability distribution of states of the system (Fig. 5). For variant 2 of the model, we adopted for all states of the system $\pi_{ij}=0.2$ $\{i=j;\ i\in 1, ..., 10;\ j\in 1, ..., 10\}$ in contrast to values $\pi_{ij}=0.4$ $\{i=j;\ i\in 1, ..., 10;$ $j\in 1, ..., 10\}$. Given such input data, we obtained results that differ from basic variant 1. The results of a given simulation experiment reflect the substantial property of project management, implying that effectiveness of projects depends on the level of technological maturity of the project system.

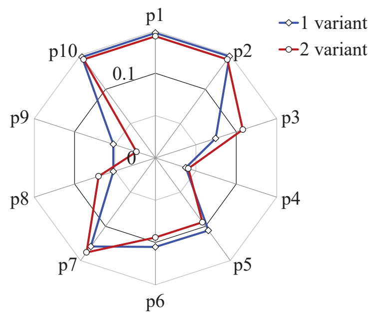

Fig. 5. Influence of the level of technological maturity of a project system on the probability of states of the system in step 25

6. 2. Estimation of the project of positive image formation for an educational establishment

Existing approaches to making up the tactic and strategy aimed at positive image formation are aimed at obtaining certain benefits and value through the exploration of relations and patterns at the level of organizational project management, rather than operational activity. Controversies in the management of projects aimed at positive image formation in the organizational-technical and social systems are predetermined by several features:

– the presence of a set of factors and their interrelations, which does not make it possible to isolate and examine in detail the individual elements of the system, which is why anything that happens within them should be considered as a totality;

– the lack of sufficient information about the dynamics of processes, which often forces to pass to a qualitative analysis of such processes;

– the turbulent environment and variability of the character of processes over time;

– corporative interests of the organization and state importance of results of positive image formation.

Because of these features, such projects can be attributed to a class of weakly structured systems, in which random processes are typically implemented. The set of factors of the system creates a complex system of reciprocal direct and inverse relationships that change over time. It is rather difficult to see and understand the logic of image construction within such a multifactor system. The procedure of decision making could be based on modeling complex processes employing the Markovian chains [21].

Managing a positive image of an educational establishment is the process that involves the formation of persistent representations among clients, partners, and the public about the prestige of the establishment, quality of its educational services, reputation of managers. Positive image makes it possible for educational institutions to ensure the rise in loyalty and support from partners, consumers, staff, regional and municipal authorities, and the media. The positive image of an educational establishment consists of a set of components. Each component of the positive image of an educational establishment, in turn, affects image formation of the institution in general. However, in order to construction an adequate model to manage the positive image of an institution, it is necessary to know which components, given the specifics of this institution, should be prioritized. What are the components of the positive image of an educational establishment? What are the criteria for choosing a given educational institution?

All of the above makes it a relevant task to manage a positive image of educational institutions, with one of the possible methods to estimate the efficiency being the application of the developed Markovian model.

The practical implementation of the system of positive image management of an educational establishment (using the College of Tsing as an example) revealed the following contradictions [21]:

– the result of positive image formation is determined by the external environment and it is not localized within the educational institution;

– the system, which is composed of an educational establishment and the external environment, is loosely structured; in it, there are many relationships, norms, structural features, community rules, customs and traditions, which are rather difficult to be accounted for in their entirety;

– it is almost impossible to receive a feedback, in order to evaluate the effectiveness of activities that are aimed at positive image formation of an educational establishment;

– substantial uncertainty arises because of the complexity of selecting the target beneficiaries – who should be the target of communication and information activities.

The College of Informatics and Computer Technologies (district of Tsing), in which the project was implemented, is in an industrial area. The total number of students is 7,200. There are 420 college instructors. Part of the college graduates continue their education at the universities in China and abroad (about 10 %). Main places of employment of the college graduates include industry, banking, transport, services, educational establishments.

Because it is quite difficult to select a target group among population, the method of the frontal information interaction with audience was accepted. The properties of the frontal information contacts are inherent to the following media: television, newspapers, participation in massive political actions. The planned activities are 3,640 tv spots, 2–3 publications per month in each of 43 newspapers, as well as intensifying public activities with mass attendance and high-quality political events. These measures are aimed at improving the attitude of all categories of people, and especially those who expresses indifference and rejection of the positive image of the college.

The matrix of transition probabilities $\|\pi_{ij}\|$ at the beginning of the project is based on Table 1.

$$\|\pi_{ij}\| = \begin{Vmatrix} 0.3 & 0.7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.3 & 0.4 & 0 & 0 & 0 & 0.3 & 0 & 0 & 0 \\ 0 & 0 & 0.4 & 0.3 & 0.3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.4 & 0 & 0 & 0.6 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.4 & 0.6 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.3 & 0 & 0.3 & 0.4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.5 & 0 & 0 & 0.5 \\ 0 & 0 & 0 & 0 & 0.4 & 0 & 0.2 & 0.4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.4 & 0 & 0 & 0 & 0.4 & 0.2 \\ 0.4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.6 \end{Vmatrix}.$$

Data on the existing probability of states of the life cycle of projects' benefits prior to the project are shown in Fig. 6.

In the course of one year, all planned activities under the project of forming a positive image of the Tsing College were fulfilled. Taking into consideration the fact that a significant change in the probability of states of the life cycle of projects' benefits was expected, the residents of the province were asked to fill in a questionnaire in order to determine the new values for transition probabilities.

$$\left\|\pi_{ij}\right\| = \left\| \begin{matrix} 0.2 & 0.8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.3 & 0.3 & 0 & 0 & 0 & 0.4 & 0 & 0 & 0 \\ 0 & 0 & 0.2 & 0.5 & 0.3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.3 & 0 & 0 & 0.7 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.2 & 0.8 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.3 & 0 & 0.5 & 0.2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.5 & 0 & 0 & 0.5 \\ 0 & 0 & 0 & 0 & 0,1 & 0 & 0.2 & 0.7 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.5 & 0 & 0 & 0 & 0.3 & 0.2 \\ 0.6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.4 \end{matrix} \right\|.$$

The results were used to evaluate the new distribution of states of the system applying the Markovian model (Fig. 7). The initial probability distribution was taken to be a probability distribution of states prior to the start of the project, corresponding to step 25 in Fig. 6.

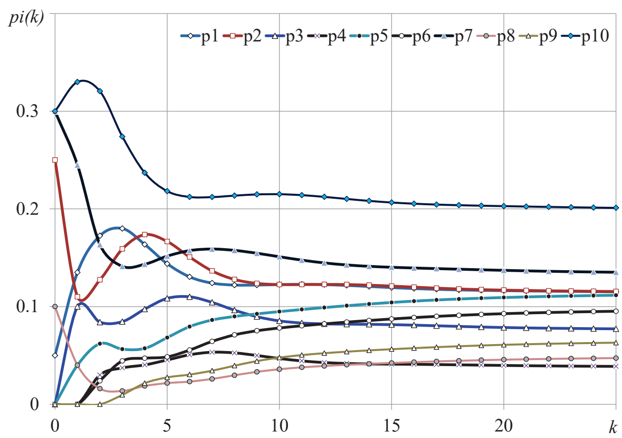

Fig. 6. Change in the probabilities of states of the system for a project of positive image formation for an educational institution: $p1$ – project; $p2$ – result; $p3$ – capacity; $p4$ – result at the output; $p5$ – changes in business; $p6$ – side effects; $p7$ – benefit; $p8$ – additional benefit; $p9$ – missed benefits; $p10$ – strategic goal

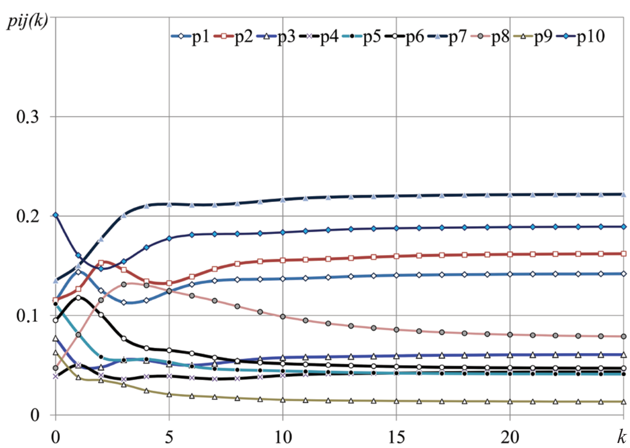

Fig. 7. Change in the probability of states of the system aimed to form a positive image of the educational establishment following the implementation of the project: $p1$ – project; $p2$ – result; $p3$ – capacity; $p4$ – result at the output; $p5$ – changes in business; $p6$ – side effects; $p7$ – benefit; $p8$ – additional benefit; $p9$ – missed benefit; $p10$ – strategic goal

Representation of the probability distribution of states of the system at the beginning of the project and after one year of work to improve the image of the Tsing College is shown in Fig. 8.

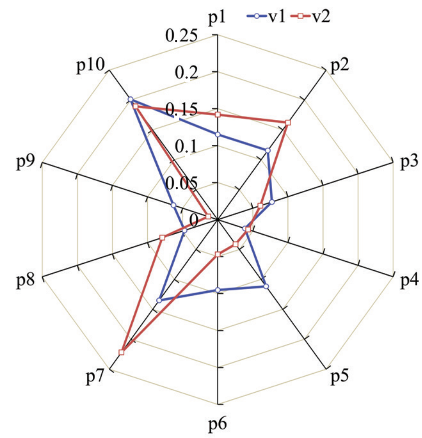

Fig. 8. Distribution of the probability of states in the life cycle of benefits from the project on positive image formation of the Tsing College at the beginning of the project (V1) and after its completion (V2): $p1$ – project; $p2$ – result; $p3$ – capacity; $p4$ – result at the output; $p5$ – changes in business; $p6$ – side effects; $p7$ – benefit; $p8$ – additional benefits; $p9$ – missed benefit; $p10$ – strategic goal

Simulation results reveal that the implementation of the project changes the results of the assessment of the project system (Fig. 8). Variations in the matrix of transition probabilities lead to a change in the overall distribution pattern of the probabilities of state of the system. The probability distribution at the beginning of the project (V1) and upon its completion (V2) differs significantly. The result of the project implementation is the increased magnitudes of the probability of states $p7$ (Benefit) and $p8$ (Additional benefit). At the beginning of the project: $p7^{(V1)}+p8^{(V2)}=0.14+0.05=0.19$. Upon completion of the project: $p7^{(V1)}+p8^{(V2)}=0.22+0.08=0.30$. In addition, the implementation of the project to form a positive image led to a decrease in the probability $p9$ (Missed benefit): from $p9^{(V1)}=0.15$ to $p9^{(V2)}=0.01$ (Fig. 8). An analysis and estimation of data on the project's implementation reveal that the findings obtained do not contradict the hypothesis about the possibility of applying the Markovian chains for determining the characteristics of the life cycle of a project's benefits.

7. Discussion of results regarding the life cycle of a project's benefits

The approach to the transformation of the life cycle of a project's benefits into the Markovian chain, presented here, provides a tool for the analysis and justification of project management strategies not only in terms of gaining benefits, but it also makes it possible to assess the contribution of each state to the final result [18]. To this end, it is necessary to identify the values for the transition probabilities of each state in a life cycle by employing any appropriate method [20]. That will make it possible to adjust the Markovian chain to represent an actual system [19].

When interpreting data on the development of a project's trajectory in the coordinates of the life-cycle of a project's benefits under different conditions, one should take into consideration that the probabilities of states of the system $\{p_1(k), p_2(k), ..., p_{10}(k)\}$ represent probabilities of conflicting events from a complete group. For all states $s\{s\in 1, ..., 10\}$, the total time Θ of the project's implementation in each step k is the sum of duration of communications in these states $\tau_s(k)\{s\in 1, ..., 10\}$:

$$\Theta = \sum_{s=1}^{n=10} \tau_s(k), \tag{8}$$

where Θ is the total duration of the project; $\tau_s(k)$ is the time it takes for the project to be in state $s\{s\in 1, ..., 10\}$ in step k.

The system can spend a certain time $\tau_s(k)$ in each of the states $s\{s\in 1, ..., 10\}$ in step k. The value of $p_s(k)=\tau_s(k)/\Theta$ means the probability of the project's being in state $s\{s\in 1, ..., 10\}$ in step k.

The sum of all probabilities of states in accordance with (3) is equal to unity:

$$\sum_{s=1}^{n=10} p_s(k) = \sum_{j=1}^{n=10} \frac{\tau_s(k)}{\Theta} = \frac{1}{\Theta}\sum_{j=1}^{n=10} \tau_s(k) = 1. \tag{9}$$

Thus, the probabilities of state of the system originate from the time that the project is in state $s\{s\in 1, ..., 10\}$ in step k. Such a property of the probabilities of states of the system makes it possible, for particular projects under the specified duration, to determine the expenditure of time resources in certain processes (states).

In this study, we have improved the standard P5 (Personnel, Planet, Profit, Process, Product) [3]. The standard P5 have already been widely applied in the practice of project management. However, the standard P5 provides a scheme of the life cycle of projects' benefits, which reflects the concept of qualitative approach to the assessment of a life cycle.

The transformation of the life cycle of projects' benefits into the Markovian chain, proposed in this paper, has for the first time made it possible to move from the qualitative assessments of the progress of projects to the quantitative characteristics of the system's trajectory in the coordinates of the probabilities of states of the system. In this case, the quantitative estimates of probabilities make up a multivariate pattern of changes in the possible states of the system. As is known, projects are related to the field of science that explores the phenomena and essence, relations and patterns in the processes of project management throughout the life cycle of managed social or organizational-technical systems with attributes of uniqueness, limited in time, quality, and resources, focused on the achievement of certain useful results and their values based on product creation. Such an approach is productive for solving applied tasks on project management.

The obtained data for modeling a life cycle of projects' benefits for a variety of real-world project systems allow us to draw a conclusion about the possibility of assessing the trajectory of projects.

A comparative analysis to other approaches is not provided because the available publications do not report methods for the quantitative estimation of projects' trajectories over the life cycle of projects' benefits. However, the paper gives a comparative analysis for projects under conditions of various characteristics of technological maturity of the system. This comparison is based on the hypothesis on that there is an influence of the level of technological maturity of the project environment (organization) on the effectiveness of projects. And the level of technological maturity (qualification) of the project environment can be compared with a set of values for transition probabilities π_{ii}, derived from Table 1. Thus, totalities $\pi_{ii}=0.2$ represent states in relation to the relatively low time cost for personal needs. That is, given the high qualification of staff, almost the entire resource of time is used for work on the project. It is characteristic of totalities $\pi_{ii}=0.4$ that nearly half of the time resource is used for personal needs – the preparation of projects and search for solutions during project implementation. After all, as you know, projects are related to the field of science that explores the phenomena and essence, relationships and patterns in the processes of management of projects/programs/portfolios over their life cycles as the managed social or organizational-technical systems with attributes of uniqueness, limited in time, quality, and resources, focused on the attainment of the specified useful results and their values based on product creation. Since projects are a unique activity, this is exactly what predetermines different time costs in projects teams to search for project solutions.

Another example relates to the assessment of the life cycle of a project's benefits in the field of education. The project that was implemented implied the formation of a positive image of an educational institution through the use of frontal information communications through television, newspapers, participation in mass political activities. The plan included stories on television, the preparation of publications in the press, as well as the intensification of public activities. The project's implementation was reflected by a change in the elements of the matrix of transition probabilities, which led to a change in the overall pattern of probability distribution of states of the system. The probability distribution at the beginning of the project (V1) and upon its completion (V2) differs significantly. The result of the project's implementation is the increased magnitudes of the probabilities of states $p7$ (Benefit) and $p8$ (Additional benefit). In addition, the implementation of the project on the positive image formation led to a decrease in the probability $p9$ (Missed benefit): from $p9^{(V1)}$ 0.15 to $p9^{(V2)}=0.01$. The evaluation of data on the project of the positive image formation of an educational establishment showed that the results obtained do not contradict the hypothesis about the possibility of applying the Markovian chains to determine the characteristics of the life cycle of a project's benefits.

8. Conclusions

1. We have constructed a cognitive Markovian model of the life cycle of a project's benefits in the form of communications between the states of the project system. The cognitive structure of a life cycle is similar to the directed graph where vertices of the graph represent the states of the system, and links are the communications between them.

2. An approach has been proposed for determining the transition probabilities based on the evaluation of communications taking into consideration the cost of time to execute operations in the form of rules. The character of communications between states $s\to j$ in the Markovian chain defines the magnitude of transition probabilities π_{sj}. The costs of time for each state is divided into five intervals π_{sj}: {0} – no costs; {0.01–0.1} – insignificant time cost; {0.1–0.3} – the lowest

level of time cost; {0.3–0.7} – average time cost; {0.7–1.0} – the largest time cost. The logic of choosing values for conditional transition probabilities in the Markovian chain makes it possible to determine data for the simulation of the trajectory of the life cycle of projects' benefits in the coordinates of the probabilities of states of the system and steps.

3. It has been shown that the application of the Markovian chains is rational in order to represent the life-cycle of projects' benefits. We have investigated the influence of the level of technological maturity of the project environment (organization) on the effectiveness of projects. We have also assessed the life cycle of benefits of the project aimed at the formation of positive image for an educational establishment through the frontal information communications using television, newspapers, participation in mass events. The probability distribution at the beginning of the project (V1) and upon its completion (V2) differs significantly. The implementation of the project increased the probabilities of states $p7$ (Benefit) and $p8$ (Additional benefit). At the beginning of the project: $p7^{(V1)}+p8^{(V2)}=0.14+0.05=0.19$. Upon implementation of the project: $p7^{(V1)}+p8^{(V2)}=0.22+0.08=0.30$. The evaluation of data on the project aimed at positive image formation of an educational establishment showed that the results obtained do not contradict the hypothesis about the possibility of applying the Markovian chains to determine the characteristics of the life cycle of a project's benefits.

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RAY TRACING SYNTHESIS OF IMAGES OF TRIANGULATED SURFACES SMOOTHED BY THE SPHERICAL INTERPOLATION METHOD

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Вирішується завдання візуалізації зворотним трасуванням (Ray Tracing) тріангульованих поверхонь, згладжених методом сферичної інтерполяції. Метод сферичної інтерполяції в основному був розроблений для інтерполяції тріангульованої поверхні з подальшою метою візуалізації цієї поверхні методом зворотного трасування. Такий підхід дозволяє поєднати метод зворотного трасування з накопиченою базою моделей з триангульованою поверхнею. Метод сферичної інтерполяції є універсальним і дозволяє також будувати плоскі і просторові гладкі криві, проведені через довільно задані точки. Пропонований алгоритм інтерполяції заснований на простій алгебраїчній поверхні – сфері і не використовує алгебраїчні поліноми третього і більш високих ступенів. Наведені аналітичні співвідношення для реалізації кожного етапу побудови інтерполюючої поверхні цим методом. Для візуалізації інтерполюючої поверхні розроблений ітераційний алгоритм (ІТА) обчислення точки перетину проекційного променя з цією поверхнею. Пропонований ІТА має можливість широкого розпаралелювання обчислень. Розроблено алгоритм побудови точок інтерполюючої поверхні, крок якого збігається з кроком ітераційного процесу обчислень, що дозволяє виконувати алгоритм візуалізації та побудови точки поверхні за один прохід ІТА. Результати досліджень підтверджені моделюванням процесу візуалізації в пакеті Wolfram Mathematica. Таким чином, виконано рішення задачі суміщення нових методів побудови гладких геометричних форм тріангульованих поверхонь і методу зворотного трасування, що в цілому дозволить підвищити реалістичність синтезованих сцен в комп'ютерній графіці

Ключові слова: зворотне трасування, проекційний промінь, моделювання кривих і поверхонь, квадрік, сферична інтерполяція

1. Introduction

In modern computer graphics, methods of rasterization and ray tracing are applied to synthesize images of 3D objects [1, 2]. The main line of studies in computer graphics consists in improving realism of synthesized scenes. When synthesized by rasterizing, any surface must necessarily be triangulated (be approximated by triangles) [1]. Such approach leads to distortion of the surface shape. To reduce negative effect of perception of a triangulated surface, various methods of illumination interpolation within a triangle are applied in the process of its synthesis. Gouraud shading and Phong shading are the most common methods [1, 2]. To date, studies are underway to improve these methods [3]. However, all these methods do not eliminate distortion of the surface geometry arising in the process of its triangulation. One of the possible ways to solve this problem consists in application of the ray tracing method for solving the image synthesis problems [2].

The ray tracing method has been actively developed in recent years. It enables synthesis of images of analytically described surfaces without their prior triangulation which significantly improves realism of the synthesized object images. At the same time, the database of object models and software products accumulated in computer graphics is focused on triangulated surfaces. Such representation is necessary in