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Improvement of the design method of thermal networks: serial connection of heat exchangers

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ABSTRACT

The mathematical model of the system is considered consisting of a series connection of three heating devices. A system of equations based on the energy conservation law is constructed, which turns out to be incomplete. It is shown that, given the known requirements for the system, expressed only in the efficiency of the system, the formalization of design often becomes insoluble. The system of equations is supplemented with expressions in accordance with the hypothesis of the proportionality of the amount of energy in an element and is presented in matrix form. The design task is reduced to determining the elements of the matrix by the value of the determinants. Analysis of the mathematical model made it possible to obtain an expression for the efficiency of the system as a function of energy exchange in its elements. This made it possible to obtain solutions for flows and their relationships in the elements of the system. In addition, the efficiency of inter-network and intra-network energy exchange has been determined, which satisfy the principles of equilibrium and minimum uncertainty in the values of the average parameters of the system. As an application, one of the main parameters, NTU, is considered, which determines the area of heat exchange with the external environment and the mass and dimensional characteristics of the heat exchange system. Models of direct and opposite switching on of flows with variations of flows and the value of the surface of devices when meeting the requirements for the efficiency of the system are considered. The results of comparing the design process with the iterative calculation method are presented and the advantages of the proposed approach are shown.

Keywords: Design; heating devices; matrix elements; system efficiency; values of determinants; intranet and internetwork exchange

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INTRODUCTION

An important component of production projects is heat exchange equipment, which accounts for the bulk of the energy consumption of the equipment. Mathematical models of the interaction of heat devices are based on the law of conservation of energy. Modern design methods based on PINCH analysis use iterative approaches to solve the energy exchange equations of the system. The main problem of which is that the original system of equations turns out to be incomplete, which requires the formation of additional hypotheses for its completion. These hypotheses are usually based on subjective preference, making it extremely difficult to obtain optimized solutions. Pareto optimization, which is the choice of the best solution from predetermined options, does not solve the problem of incompleteness of the system of initial equations.

The relevance of research is associated with the search for an approach that allows you to create a system of initial linear equations with completeness, providing an optimal design solution.

LITERATURE REVIEW

The energy consumption of industrial production is a significant component in their value terms, while half of the energy is represented in the form of heat [1]. The analysis of heat supply networks given in [2] showed that the efficiency of heat transfer depends on the design parameters of the elements, the network topology, and the characteristics of the heat carriers. A separate important area is represented by onboard systems for providing thermal modes of electronic equipment, for which the most important indicators are weight and overall characteristics [3]. The paper [4] presents an analysis of reliability indicators as an integral characteristic of a thermoelectric system for providing thermal conditions and its relationship with design and energy indicators. In [5], the dynamics of a thermoelectric device and its relationship with manufacturing technology, design and energy characteristics and reliability indicators are analyzed. However, these studies were carried out for individual elements, while the system exhibits new properties that are not inherent in the constituent elements. Cost-effective design is achieved by modeling the components of the system using library components [6]. However, the models describe individual devices and do not take into account their interrelationships. The way the coolant moves affects the efficiency of heat transfer, which follows from the comparison of direct-flow and counter-flow heat exchangers. The analysis used a one-dimensional model of thermal conductivity, averaged parameters of temperature difference, heat transfer coefficient and effective contact area of heat carriers [7]. The equivalence of direct and countercurrent heat exchange is achieved only at very large and very small ratios of water numbers [8]. However, the comparison of switching schemes was considered for a separate apparatus, and not for a system with the mutual influence of elements. For a system of heat exchangers, this issue is considered in [9], where a method is presented to reduce a direct-flow system to a counter-current one in order to increase its efficiency. Models of systems with renewable energy sources and buffer storage of the coolant are presented in [10], and microelectronic systems with heat exchange devices in [11]. The most urgent problem of minimizing heat energy losses is manifested in industrial chemical-technological systems, for the implementation of which models of rectification technological schemes for product separation have been developed [12]. When calculating balances, the system of equations is supplemented with equations of functional relationships. To solve them, iterative procedures for optimization problems with constraints have been developed, based on partitioning the domain, taking into account the uncertainty of the initial information [13]. The incompleteness of the initial set of equations required the introduction of additional empirical hypotheses that affect the reliability of the simulation results. In the system of heat exchangers, there are sequential, parallel, bypass and loop equipment combination [14], and the temperature distribution in a multistage cascade is described by nonlinear dependencies [15]. The models use the Newton-Raphson iterative method, which assumes knowledge of the exact values of all variables [16], which is difficult to achieve. Modern mathematical modeling of heat transfer in chemical engineering processes is presented by PINCH analysis [17]. For operating oil refineries, PINCH technologies make it possible to achieve a reduction in energy consumption by up to 30-50 % [18]. The listed modulation methods use iterative solution methods when combining devices in a system into systems, but only analytical models provide optimal solutions. The relevance of changing the concept in the design of thermal systems is substantiated in [19], which shows the possibility of obtaining an exact solution to the problem. The results obtained can also serve as test results when developing complex algorithms.

PURPOSE AND OBJECTIVES OF THE RESEARCH

The aim of the work is to obtain an analytical solution for the distribution of heat exchange efficiencies in a system from a system of connected heat exchangers.

To achieve the goal, the following tasks were set:

1) to develop an analytical model of the serial connection of heat exchangers;

2) to analyze the developed model to determine the main design indicators: efficiency and heat exchange surface.

ANALYTICAL MODEL OF COMBINING DEVICES

In technical applications, there is a serial, parallel and parallel-serial connection of elements [20]. In some cases, when connecting elements in series, such a problem is the problem of sectioning elements, considered in [9]. However, in most cases the arising problem is reduced to the problem of optimal design with all the ensuing problems [21].

Consider the case of a series connection of elements (Fig. 1).

The system of energy conservation equations for the system shown in Fig. 1 will be written in the form:

$$\begin{aligned} &\alpha_1(T_0 - T_1) = T_7 - \Theta_0 \\ &\alpha_2(T_1 - T_2) = T_8 - \Theta_0 \\ &\alpha_3(T_2 - T_3) = T_9 - \Theta_0 \\ &\alpha_4(T_7 - T_{101}) = T_{102} - T_{112} \\ &\alpha_5(T_8 - T_{111}) = T_{112} - T_9 \end{aligned} \quad . \qquad (1)$$

Obviously, such a system of equations is incomplete. Let us supplement the formed system of equations with the hypothesis that the amount of energy in the element is proportional to the applied potential:

$$\begin{aligned} &T_0 - T_1 = \Phi_1(T_0 - \Theta_0) \\ &T_1 - T_2 = \Phi_2(T_1 - \Theta_0) \\ &T_2 - T_3 = \Phi_3(T_2 - \Theta_0) \\ &T_7 - T_{101} = \Phi_4(T_7 - T_{112}) \\ &T_8 - T_{111} = \Phi_5(T_8 - T_9) \end{aligned} \quad . \qquad (2)$$

In matrix notation:

$$\begin{pmatrix} -\alpha_1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ \alpha_2 & -\alpha_2 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & \alpha_3 & -\alpha_3 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -\alpha_4 & 0 & \alpha_4 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & -\alpha_5 & 0 & \alpha_5 & 1 & 0 & -1 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1-\Phi_2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1-\Phi_3 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1-\Phi_4 & 0 & 0 & 0 & \Phi_4 \\ 0 & 0 & 0 & 0 & -1 & 0 & 1-\Phi_5 & \Phi_5 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} T_1 \\ T_2 \\ T_3 \\ T_{101} \\ T_{111} \\ T_7 \\ T_8 \\ T_9 \\ T_{102} \\ T_{112} \end{pmatrix} = \begin{pmatrix} -\alpha_1 T_0 - \Theta_0 \\ -\Theta_0 \\ -\Theta_0 \\ 0 \\ 0 \\ \Phi_1 (T_0 - \Theta_0) - T_0 \\ -\Phi_2 \Theta_0 \\ -\Phi_3 \Theta_0 \\ 0 \\ 0 \end{pmatrix} . \quad (3)$$

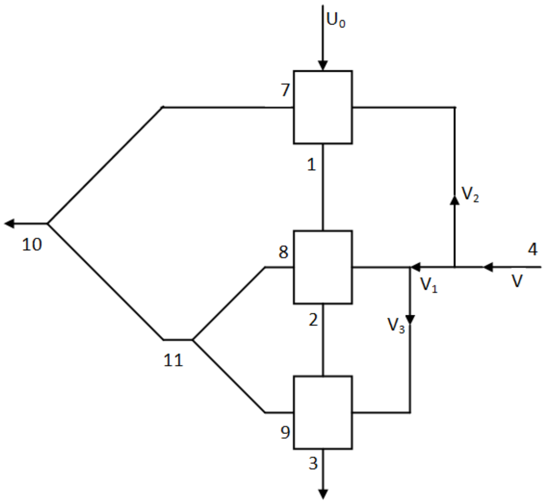

Fig. 1. **Series connection of three elements: U₀ – hot flow inlet; V – cold stream inlet**

Source: **compiled by the authors**

In the problem of modeling with known flow ratios and efficiencies, solving such a system of equations does not pose special problems. In the design problem, with a known only requirement for the system, for example, temperature T3 (requirement for the efficiency of the system), the problem becomes unsolvable. The search for a solution in terms of optimal design [21], due to the problems noted in [19], does not always lead to success.

From a formal point of view, in the most general sense, the system design problem can be formulated as the problem of determining matrix elements by the value of the ratio of determinants:

$$E = \frac{T_0 - T_3}{T_0 - \Theta_0} .$$

The solution of system (3) allows one to obtain an expression for the efficiency of the system E as a function of the efficiency of energy exchange in its elements:

$$E = \begin{pmatrix} \Phi_1 - \Phi_1\Phi_3 - \Phi_2\Phi_3 - \Phi_1\Phi_2 + \\ + \Phi_2 + \Phi_3 + \Phi_1\Phi_2\Phi_3 \end{pmatrix} . \quad (4)$$

Here – the efficiency of the interconnection of energy. Introduced in (3) the efficiency of the on-net energy exchange, together with the equilibrium requirement at the mixing nodes, make it possible to construct a complete system of equations that determines the flows on the branches of the system

$$V_2 + V_1 = V_0$$
$$V_3 + V_4 - V_1 = 0$$
$$V_2\Phi_4 - V_1(1-\Phi_4) = 0$$
$$V_4\Phi_5 - V_3(1-\Phi_5) = 0$$

$$\begin{pmatrix} 1 & 1 & 0 & 0 \\ -1 & 0 & 1 & 1 \\ \Phi_4 - 1 & \Phi_4 & 0 & 0 \\ 0 & 0 & \Phi_5 - 1 & \Phi_5 \end{pmatrix} \cdot \begin{pmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{pmatrix} = \begin{pmatrix} V_0 \\ 0 \\ 0 \\ 0 \end{pmatrix} . \quad (5)$$

In turn, this allows you to get solutions for flows and their relationships in the elements of the system

$$\alpha_1 = \frac{U_0}{V_2}; \qquad \alpha_2 = \frac{U_0}{V_4}; \qquad \alpha_3 = \frac{U_0}{V_3};$$
$$\alpha_4 = \frac{V_2}{V_1}; \qquad \alpha_5 = \frac{V_4}{V_3}; \qquad \alpha_0 = \frac{U_0}{V_0} .$$

$$\begin{pmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{pmatrix} = \begin{bmatrix} V_0\Phi_4 \\ -(\Phi_4 - 1)V_0 \\ V_0\Phi_4\Phi_5 \\ -(\Phi_5 - 1)V_0\Phi_4 \end{bmatrix}$$

$$\begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \\ \alpha_5 \end{pmatrix} = \begin{bmatrix} \dfrac{\alpha_0}{\Phi_4 - 1} \\ \dfrac{\alpha_0}{(\Phi_5 - 1)\Phi_4} \\ \dfrac{\alpha_0}{\Phi_4\Phi_5} \\ \dfrac{\Phi_4 - 1}{\Phi_4} \\ \dfrac{\Phi_5 - 1}{\Phi_5} \end{bmatrix} . \quad (6)$$

It is easy to see that the introduced variables reduce the definitions of the elements of matrix (3) to the problem of determining the efficiency of the inter-network and intra-network energy exchange that satisfy the fundamental physical principles of equilibrium and the minimum uncertainty of the values of the average parameters of the system.

Satisfaction of these principles allows one to obtain from the principle of equilibrium expressions for the efficiency of the intra-network energy exchange:

$$\frac{(\Phi_2 - 1)\,\Phi_3}{\Phi_2 - \Phi_2\Phi_3 + \Phi_3} = \Phi_5; \quad \frac{E - \Phi_1}{E} = \Phi_4 . \quad (7)$$

The principle of minimum uncertainty defines solutions for the efficiencies of the interconnection of energy:

$$\Phi_1 = \Phi_2 = 1 - \sqrt[3]{1 - E}\,, \quad \Phi_3 = \frac{E + (1 - E)^{2/3} - 1}{(1 - E)^{2/3}} \,. \quad (8)$$

As noted above, when solving such problems, there is a need for various types of hypotheses that determine, for example, NTU, when it comes to heat exchangers or the efficiency of system elements.

ANALYSIS OF THE BASIC FEATURES OF AN ANALYTICAL MODEL

When constructing a model for solving the problem shown in Fig. 1, as a rule, assumptions about the uniform distribution of flows along the branches of the “cold” network are used:

$$V_1 = V_2 = V_3 = V_0/3.$$

The second assumption concerns the device structure and processes in it

$$NTU_1 = NTU_2 = NTU_3,$$

which corresponds to the assumption of a uniform distribution of the heat exchange surface and heat transfer coefficients and, as a consequence, the equality of the efficiency of all three devices.

If we further assume that the system uses counterflow heat exchangers, then we can write

$$\Phi = \frac{1 - \exp[-NTU \cdot (1 - \alpha)]}{1 - \alpha \cdot \exp[-NTU \cdot (1 - \alpha)]}\,;$$

$$NTU = \frac{\ln\left(\dfrac{\Phi - 1}{\alpha\,\Phi - 1}\right)}{\alpha - 1} \,. \quad (9)$$

When setting the problem on the minimum NTU sum in the iterative process, both the flows on the network branches and the surface values in the devices vary until the minimum total NTU value is reached with the unconditional fulfillment of the requirement for the efficiency of the system E.

To construct the first approximation of the value of the efficiency of system elements, the hypothesis is adopted

$$\Phi_1 = \Phi_2 = \Phi_3 = 1 - \sqrt[3]{1 - E} = idem\,, \quad (10)$$

satisfying the requirement for system efficiency (E = 0.87). Using (9), it is easy to obtain

$$\begin{aligned} &NTU_1 = NTU_2 = NTU_3 = 1{,}888 \\ &NTU_\Sigma = 5{,}664 \end{aligned} \,,$$

provided that the “cold” flow is divided by three.

More cumbersome iterative schemes arise if the temperatures at the network nodes are specified, as is implemented in the HYSYS or PRO2. In these packages, it is proposed, on the basis of the approximations formulated by the expert for the temperatures at the network nodes, to calculate the devices according to the models considered, for example, in [22].

The exact design scheme, based on the principles set forth in [20, 21], and relations (7.8) allows you to obtain

$$\begin{aligned} &\Phi_5 = \frac{\Phi_2\Phi_3 - \Phi_3}{\Phi_2 - \Phi_2\Phi_3 + \Phi_3} \qquad && \Phi_5 = 0{,}7531 \\ &\Phi_4 = \frac{E - \Phi_1}{E} && \Phi_4 = 0{,}7672 \end{aligned} \,.$$

Since the ratio of flows at the inlet is taken as given α_0 = 0.6, and the efficiency is determined by relations (8), we obtain

$$\begin{pmatrix} \Phi_1 \\ \Phi_2 \\ \Phi_3 \end{pmatrix} = \begin{pmatrix} 0{,}2064 \\ 0{,}2064 \\ 0{,}7935 \end{pmatrix} \qquad \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix} = \begin{pmatrix} 4{,}2145 \cdot \alpha_0 \\ 5{,}3108 \cdot \alpha_0 \\ 1{,}7408 \cdot \alpha_0 \end{pmatrix} .$$

Using (9), we obtain the NTU values for each element of the system

$$\begin{aligned} &NTU_1 = 0{,}3316 \qquad && NTU_2 = 0{,}3846 \\ &NTU_3 = 4{,}2162 && NTU_4 = 4{,}9325 \end{aligned} \,.$$

The error when using the iterative scheme for solving the problem is

$$\Delta_3 = \frac{5{,}6646 - 4{,}9325}{4{,}9325} \cdot 100 = 14{,}84\% \,.$$

Note that in order to achieve better results when designing systems with three or more elements, one should use not a cross flow pattern (Fig. 1), but a counter-flow one, as shown in [9].

The most common structures of industrial heat apparatuses contain a complex connection of heating network elements. An example of such a structure is shown in Fig. 2, where U_0 is the input hot stream; V_0 is the input hot stream, and the derived streams that are formed during their division and interaction.

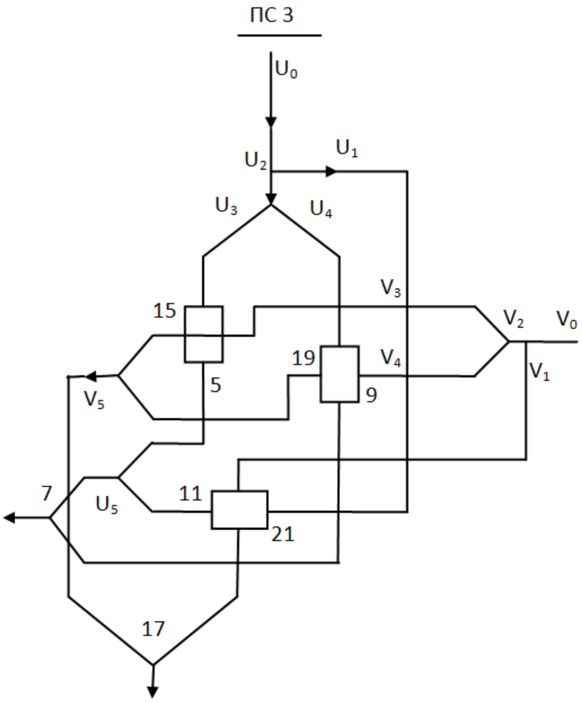

Fig. 2. **System of interaction of streams**

***Source:* compiled by the authors**

The system of energy conservation equations for Fig. 2 will be written in the form:

$$
\begin{aligned}
&\alpha_1(T_0-T_5)=(T_{15}-\Theta_0)\\
&\alpha_2(T_0-T_9)=(T_{19}-\Theta_0)\\
&\alpha_3(T_0-T_{11})=(T_{21}-\Theta_0)\\
&\alpha_4(T_{11}-T_{61})=(T_{62}-T_5)\quad .\\
&\alpha_5(T_9-T_{71})=(T_{72}-T_{61})\\
&\alpha_6(T_{15}-T_{161})=(T_{162}-T_{19})\\
&\alpha_7(T_{21}-T_{171})=(T_{172}-T_{162})
\end{aligned}
$$

As in the previous case, the resulting system of equations is not complete. Let us supplement it with the hypothesis that the amount of energy in the element is proportional to the applied potential:

$$
\begin{aligned}
&(T_0-T_5)=\Phi_1\,(T_0-\Theta_0)\\
&(T_0-T_9)=\Phi_2\,(T_0-\Theta_0)\\
&(T_0-T_{11})=\Phi_3\,(T_0-\Theta_0)\\
&(T_{11}-T_{61})=\Phi_4\,(T_{11}-T_5)\quad .\\
&(T_9-T_{71})=\Phi_5\,(T_9-T_{61})\\
&(T_{15}-T_{161})=\Phi_6\,(T_{15}-T_{19})\\
&(T_{21}-T_{171})=\Phi_7\,(T_{21}-T_{162})
\end{aligned}
$$

Or in matrix notation:

$$
\begin{pmatrix}
-\alpha_1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & -\alpha_2 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & -\alpha_3 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0\\
0 & 0 & \alpha_4 & -\alpha_4 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0\\
0 & \alpha_5 & 0 & 1 & -\alpha_5 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & -\alpha_6 & 0 & \alpha_6 & 1 & 0 & 0 & 0 & -1 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & -\alpha_7 & 0 & 0 & \alpha_7 & 0 & 0 & 1 & -1\\
-1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
\Phi_4 & 0 & 1-\Phi_4 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 1-\Phi_5 & 0 & \Phi_5 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & -1 & 0 & 1-\Phi_6 & \Phi_6 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 1-\Phi_7 & 0 & 0 & \Phi_7 & 0
\end{pmatrix}
\cdot
\begin{pmatrix}
T_5\\ T_9\\ T_{11}\\ T_{61}\\ T_{71}\\ T_{161}\\ T_{171}\\ T_{15}\\ T_{19}\\ T_{21}\\ T_{62}\\ T_{72}\\ T_{162}\\ T_{172}
\end{pmatrix}
=
\begin{pmatrix}
-\alpha_1T_0-\Theta_0\\
-\alpha_2T_0-\Theta_0\\
-\alpha_3T_0-\Theta_0\\
0\\ 0\\ 0\\ 0\\
\Phi_1(T_0-\Theta_0)-T_0\\
\Phi_2(T_0-\Theta_0)-T_0\\
\Phi_3(T_0-\Theta_0)-T_0\\
0\\ 0\\ 0\\ 0
\end{pmatrix}
. \quad (11)
$$

Just as in [20, 21], the system design problem can be formulated as the problem of determining the elements of matrix (11) for a given value of the system's efficiency E:

$$
E=\frac{T_0-T_{171}}{T_0-\Theta_0}=(\Phi_1\Phi_4-\Phi_3\Phi_4-\Phi_2+\Phi_3)\Phi_5+\Phi_2\ . \quad (12)
$$

The requirement for a minimum of uncertainty in the values of average temperatures in the system makes it possible to determine the efficiency of its elements both through the efficiency of the system's energy exchange and the efficiency of the on-grid exchange of energy Φ_4 and Φ_5:

$$
\Phi_1=E \qquad \Phi_2=E \qquad \Phi_3=E\,. \qquad (13)
$$

In this case, the efficiency of the intra-network energy exchange is determined from the solution of the system of equations:

$$2\Phi_5\Phi_4^2-2\Phi_5\Phi_4+2\Phi_5-1=0$$

$$2E^2(2\Phi_4-1)\Phi_5^2=0 \qquad (14)$$

$$\Phi_5={}^2\!/_3 \qquad \Phi_4={}^1\!/_2\,.$$

The equilibrium requirement for the transfer of energy in the elements of the system forms a system of equations for determining the distribution of "cold" flows along the network branches:

$$\frac{E^2(3\Phi_7-2)\alpha_0^2}{9\Phi_7^3-18\Phi_7^2+9\Phi_7}=0.$$

In turn, the results obtained, together with conservation laws, make it possible to formulate complete systems of equations that determine the distribution of flows along the network branches:

$$\frac{3E^2(2\Phi_6-1)\alpha_0^2\left(7\Phi_6^4-4\Phi_6^5-8\Phi_6^3+6\Phi_6^2+2\Phi_6-2\right)}{4(\Phi_6-1)^2\Phi_6^2}=0 \qquad (15)$$

$$\Phi_6={}^1\!/_2 \qquad \Phi_7={}^2\!/_3\,.$$

For a network with a flow with a lower energy potential (receiving network), we get:

$$V_1+V_2=V_0$$
$$V_2-(V_3+V_4)=0$$
$$V_5-(V_1+V_3)=0$$
$$V_3(1-\Phi_6)-V_1\Phi_6=0$$
$$V_5(1-\Phi_7)-V_4\Phi_7=0$$

$$\begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & -1 & 0 \\ -1 & 0 & -1 & 0 & 1 \\ -\Phi_6 & 0 & 1-\Phi_6 & 0 & 0 \\ 0 & 0 & 0 & -\Phi_7 & 1-\Phi_7 \end{pmatrix}\cdot\begin{pmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \end{pmatrix}=\begin{pmatrix} V_0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}.$$

For a network with a flow with a high energy potential (transmission network), we get:

$$U_1+U_2=U_0$$
$$U_2-(U_3+U_4)=0$$
$$U_5-(U_1+U_3)=0$$
$$U_3(1-\Phi_4)-U_1\Phi_4=0$$
$$U_5(1-\Phi_5)-U_4\Phi_5=0$$

$$\begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & -1 & 0 \\ -1 & 0 & -1 & 0 & 1 \\ -\Phi_4 & 0 & 1-\Phi_4 & 0 & 0 \\ 0 & 0 & 0 & -\Phi_5 & 1-\Phi_5 \end{pmatrix}\cdot\begin{pmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \end{pmatrix}=\begin{pmatrix} U_0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}.$$

After simple transformations, we find the flow ratios in the system elements:

$$\begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix}=\begin{bmatrix} \dfrac{\alpha_0\Phi_4\Phi_5}{\Phi_6\Phi_7} \\ \dfrac{(\Phi_5-1)\alpha_0}{\Phi_7-1} \\ \dfrac{(\Phi_4-1)\alpha_0\Phi_5}{(\Phi_6-1)\Phi_7} \end{bmatrix}=\begin{pmatrix} \alpha_0 \\ \alpha_0 \\ \alpha_0 \end{pmatrix}$$

$$\begin{pmatrix} \alpha_4 \\ \alpha_5 \\ \alpha_6 \\ \alpha_7 \end{pmatrix}=\begin{pmatrix} \dfrac{1-\Phi_4}{\Phi_4} \\ \dfrac{1-\Phi_5}{\Phi_5} \\ \dfrac{1-\Phi_6}{\Phi_6} \\ \dfrac{1-\Phi_7}{\Phi_7} \end{pmatrix}. \qquad (16)$$

Relations (13-16) determine the elements of matrix (13) and, therefore, make it possible to obtain the distribution of energy and flux measures at the nodes and on the branches of the networks.

DISCUSSION OF ANALYSIS RESULTS

The results obtained indicate that with a uniform distribution of flows along the lines of networks and heat exchange surfaces in the devices, the system uses the minimum total value of NTU (surface) and at the same time has the efficiency of one device.

This fact is known and used in the design of various heat exchange equipment [9]. The above results indicate that the uneven distribution of flows and surface leads to an increase in the uncertainty in the values of average temperatures, as a result of which the intensity of heat transfer decreases.

Let the system have counterflow heat exchangers:

$$\Phi_n=1-\frac{1-\exp[-NTU_n\cdot(1-\alpha)]}{1-\alpha\cdot\exp[-NTU_n\cdot(1-\alpha)]}$$

$$E=\frac{1-\exp[-NTU\cdot(1-\alpha)]}{1-\alpha\cdot\exp[-NTU\cdot(1-\alpha)]}.$$

According to the proven

$$\Phi_n=E$$

Equality of efficiencies implies equality of NTU:

$$\frac{1-\exp\left[-NTU_n\cdot(1-\alpha_0)\right]}{1-\alpha_0\cdot\exp\left[-NTU_n\cdot(1-\alpha_0)\right]}=$$

$$=\frac{1-\exp\left[-NTU_0\cdot(1-\alpha_0\right])}{1-\alpha_0\cdot\exp\left[-NTU_0\cdot(1-\alpha_0)\right]}$$

$$NTU_n=\frac{\ln\left[\exp(NTU_0\cdot(\alpha_0-1))\right]}{\alpha_0-1}=NTU_0\,.$$

From the uniformity of the distribution of flows over the elements of the system, it follows

$$U_n=\frac{U_0}{N}\qquad NTU_n=\frac{k\cdot F_n}{U_n}$$

$$N\cdot\frac{k\cdot F_n}{U_n}=\frac{k\cdot F_0}{U_0}\qquad F_n=\frac{F_0}{N}$$

In other words, the sum of the surfaces in the elements of the system is equal to the surface in one.

CONCLUSIONS

1. An analytical model has been developed for a system of serial connection of heat exchangers, based on the addition of the system of equations for the conservation of energy by the relationship of the amount of energy in the element to the applied potential.

2. With a matrix representation, the design problem can be formulated as the possibility of determining the efficiency of the system as a function of the efficiency of energy exchange in its elements.

3. The requirement for a minimum of uncertainty in the values of average temperatures in the system makes it possible to determine the efficiency of its elements through the efficiency of the system's energy exchange and the efficiency of the intra-network energy exchange.

4. An analysis of the developed model with respect to the efficiency and heat transfer surface in the system is presented and it is shown that the gain in comparison with iterative design methods for determining the heat transfer area is about an order of magnitude.

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Вдосконалення методу проектування теплових мереж: послідовне з’єднання теплообмінних апаратів

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АНОТАЦІЯ

Розглянуто математичну модель системи, яка складається з послідовного з'єднання трьох теплових пристроїв. Побудована система рівнянь, яка заснована на законі зберігання енергії, яка виявляється неповною. Показано, що при відомих вимогах до системи, які виражені тільки у ефективності системи, формалізація проектування часто стає неможливим. Система рівнянь виконана виразами у відповідності до гіпотези про пропорційність кількості енергії в елементі і представлена у матричній формі. Задача проектування зводиться до визначення елементів матриці по визначенню детермінантів. Аналіз математичної моделі дозволив одержати вираз для ефективності системи як функції енергообміну в її елементах. Це дало можливість одержати рішення для потоків та їх відношень в елементах системи. Крім того, визначені ефективності міжмережового та внутрішньо мережевого обміну енергією, які задовольняють принципам рівноваги і мінімальної невизначеності значень середніх параметрів системи. У якості додатку розглянуто один з основних параметрів – NTU, який визначає площу теплообміну з зовнішнім середовищем, масові та габаритні характеристики системи теплообміну. Розглянуті моделі прямого та зворотного включення потоків і значення поверхні апаратів при виконанні вимог до ефективності системи. Представлені результати порівняння процесу проектування з ітераційним методом розрахунку і показані переваги запропонованого підходу.

Ключові слова: Проектування; теплові апарати; елементи матриці; ефективність системи; значення детермінантів; внутрішньо мережевий і між мережевий обмін

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