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THE EFFECT OF VARIABLE CROSS-SECTION OF PRESTRESSED BEAMS ON THE LOAD-BEARING CAPACITY OF THE STRUCTURE

І. Прокопович, А. Ткачьов, О. Ткачьов, П. Прокопович. **Вплив змінного поперечного перерізу попередньо напружених балок на несучу здатність конструкції.** В даній роботі розглядаються питання, які пов'язані з підвищенням несучої здатності прольотних балок кранів мостового типу методом попереднього напруження. Розроблена нова математична модель мостового крана із заздалегідь напруженими балками, яка базується на загальній теорії стійкості пружних систем та дозволяє враховувати реальні умови конструктивного виконання прольотної будов. В даній роботі були отримані рівняння кривої прогинів цієї балки, які дозволяють розглянути та проаналізувати вплив ексцентрично подовжніх сил на напружено-деформований стан прольотної балки з урахуванням змінного моменту інерції її поперечного перерізу по довжині прольоту при небезпечної комбінації навантаження. В роботі наведено уточнюючий розрахунковий коефіцієнт для критичної поздовжньої сили, який залежить від відношень моментів інерції поперечних перерізів прольотної головної балки, а також довжин опорних дільниць балки до довжини самої балки. За результатами отриманих рівнянь було проведено дослідження статичної жорсткості головної балки в залежності від відношення повздовжніх та поперечних сил, що діють на балку. Аналіз отриманих результатів виявив рекомендовані співвідношення довжин опорної та середньої частин моста з врахуванням їх геометричних характеристик перерізу. Отримані у даній роботі результати можуть бути у подальшому використані для модернізації кранів з метою підвищення їх вантажопідйомності, поширення терміна їх служби без демонтажу, а також для вдосконалення існуючих конструкцій та інженерних методів розрахунку як на стадіях їх проектування, так і в умовах реальної експлуатації.
Ключові слова: прольотні балки, попереднє напруження, прогин моста, деформаційний стан, статична жорсткість

I. Prokopovych, A. Tkachev, O. Tkachev, P. Prokopovych. **The effect of variable cross-section of prestressed beams on the load-bearing capacity of the structure.** This paper examines issues related to increasing the load-bearing capacity of span beams of bridge-type cranes by the method of prestressing. A new mathematical model of a bridge crane with prestressed beams has been developed, which is based on the general theory of the stability of elastic systems and allows taking into account the real conditions of the construction of span structures. In this work, the equations of the deflection curve of this beam were obtained, which allow us to consider and analyze the influence of eccentric longitudinal forces on the stressed-deformed state of the span beam, taking into account the variable moment of inertia of its cross-section along the length of the span with a dangerous load combination. The work provides a more detailed calculation coefficient for the critical longitudinal force, which depends on the ratios of the moments of inertia of the cross sections of the span main beam, as well as the length of the support sections of the beam to the length of the beam itself. Based on the results of the obtained equations, a study of the static stiffness of the main beam was conducted depending on the ratio of longitudinal and transverse forces acting on the beam. The analysis of the obtained results revealed the recommended ratio of the lengths of the supporting and middle parts of the bridge, taking into account their geometric cross-section characteristics. The results obtained in this work can be used in the future for the modernization of cranes in order to increase their load capacity, extend their service life without dismantling, as well as for the improvement of existing structures and engineering calculation methods both at the stages of their design and in the conditions of real operation.
Keywords: span beams, prestressing, bridge deflection, deformation state, static stiffness

Introduction

The subject of consideration in this work is the overhead structures of cargo-lifting machines. Currently, load-bearing metal structures with prestressing are widely used in modern production, these are span [1] and crane beams [2], towers [3], crane booms [4] and others. On the one hand, the metal structures of the listed devices have an increased bearing capacity, as they are exposed to loads in different planes, they are much lighter than ordinary structures, and, as a rule, much cheaper [5]. On the other hand, pre-stressed crane bridges have less rigidity due to the smaller moment of inertia of the sections. This is because during the design and calculation of such bridges, simplifications were allowed in the calculation schemes of span beams. They were considered as a prismatic rod with a constant cross-section along the entire length [6]. Moreover, all the necessary geometric characteristics of the beam sections were determined on the basis of the initial straight shape of the bridge [7]. In real conditions, where the condition – high bearing capacity with low metal capacity – is necessarily met, the main beams are made with a variable cross-section along the length of the span (Fig. 1).

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Fig. 1. Overhead crane with a main beam of variable cross-section

In prestressed span beams, the determination of deformations of bridges in an arbitrary cross-section, as well as any other questions related to the stiffness of the structure, require special research and analysis. This is because the eccentric application of the axial load and the deviation of the beam from the straight shape have a significant effect on its deflections and induced stresses [8]. Thus, when the span length of crane bridges is increased, the pre-stressed beams are more deformable than ordinary ones. As a result, this leads to a violation of the performance of the structure and the nodes and mechanisms associated with it [9].

Reasoning in a similar way, we come to the fact that in spanned metal structures with a variable moment of inertia of the section, under similar loads, due to the reduction of the placed material in the investigated sections, the bending stiffness of the bridge should be significantly greater. We note that of special interest is the case when the moving transverse load is located near the support or above it. For such a combination of loads, the deflections of the span beam reach their maximum value and can be commensurate with the working deflections of the bridge, and in some cases even exceed the maximum allowable values [10].

Thus, it is relevant to carry out studies of the stress-strain state of span structures, in which the geometric characteristics of the cross-sections correspond to the real conditions of its construction and operation. The specified analysis is possible with a more accurate determination of beam deformation values, based on the developed new refined mathematical model of the span structure.

Analysis of publications on the topic of research

The analysis of publications shows that for crane span beams, studies were conducted on the main criteria of workability, when it is bent in two main planes – vertical [11] and horizontal [12]. Numerical simulations [13, 14] were also carried out, on the basis of which the analysis [15, 16] of the stress-strain behavior of the mathematical models under study was carried out. A number of publications were devoted to the calculation of the deflection of a directly prestressed beam [17] and studies of its static stiffness. Depending on the different ratio of transverse and longitudinal loads acting on the beam, optimal deflections and bends in the vertical plane were determined [18].

As a result, it was established:

1) the deformed state of the bridges was obtained under the condition that in the calculation schemes of mathematical models, span beams were considered as elastic prismatic rods with a constant section along the entire length of the rod;

2) there were no publications and studies related to the work of a prestressed beam with different geometric cross-section characteristics;

3) for our case, the above-mentioned approach will be difficult to use, as it is not correct and can only give approximate results.

This, in turn, requires the development and consideration of another mathematical model, which will take into account and consider issues where the maximum approximation of the calculation scheme to the real structural form of the beam is put forward.

The purpose and tasks of the research

Thus, the purpose of this paper is to further study the stress-strain state of the prestressed main beam, taking into account different values of the moments of inertia of its sections, and the issues considered in it are those in which the maximum approximation of the calculation scheme to the real structural form is put forward span beam.

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To achieve the goal, it is necessary to solve the following tasks: to analyze already known types of calculation schemes and mathematical models of span structures with preliminary tension; taking into account the conducted analysis, develop a new mathematical model of the pre-stressed beam, which would take into account the change in the geometric characteristics of the cross sections of the crane bridge; investigate the stress-strain behavior of the specified span structure and conduct an analysis of the obtained results.

Presentation of the main material

In machine-building constructions, the performance of overhead cranes is, as a rule, performed in such a way that part of the beam material is removed in the places of attachment to the supports and near them. In the middle part, the geometric dimensions of the cross section increase (Fig. 1). This is due to the action of internal force factors when the beam operates in the vertical plane, which is the most economical version of the bridge for the perception of bending loads. Thus, when developing a mathematical model of a bridge-type crane, we take as a basis the structural scheme of the bridge, symmetrical about the vertical axis (Fig. 2).

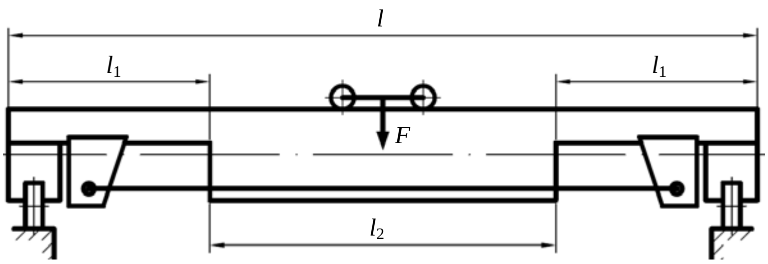

Fig. 2. Structural diagram of the crane bridge

In the design scheme, under l_1 the lengths of the symmetrical sections of the beam are indicated, with the moments of inertia of the cross sections – I_1. Similarly, l_2 – the length of the middle section of the beam with the moment of inertia of the cross section – I_2, and $I_2 > I_1$.

When drawing up the design scheme, we assume that all elements of the beam are solid bodies, the beam operates in the elastic stage and is supported by ideal hinges. The most unfavorable case for the bridge and of interest to us will be when the workload *F* is above the support. The bending of the beam in this case will have a maximum value. In such a combination of loads, the design scheme of a prestressed bridge can be represented by the corresponding scheme shown in Fig. 3, where *l* is the span length of the beam; *e* is the eccentricity of the application of the axial load *S*; *f* – bending (deflection) of the beam in the middle of the span; EI_i is the bending stiffness of the crane bridge in the vertical plane for the corresponding ***i***-th section.

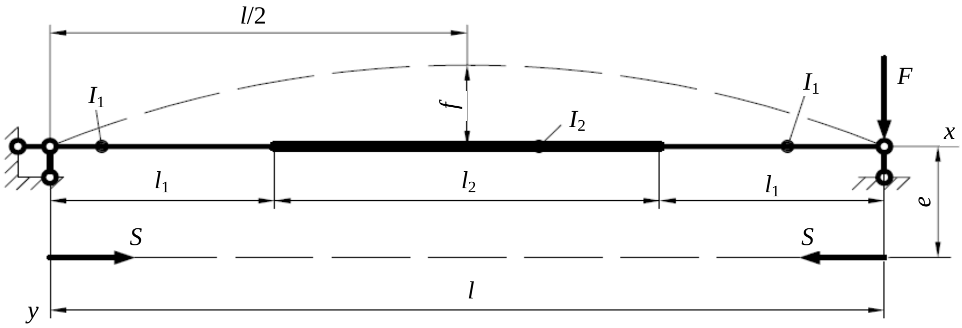

Fig. 3. Calculation scheme of the span beam of the crane bridge

Since the calculation scheme of the beam is symmetrical about the vertical axis, we will consider one, for example, the left, half of the bridge. Taking into account the conditions at the embedded end of the beam, and also applying the notation:

$$\frac{S}{EI_1} = k_1^2 \text{ and } \frac{S}{EI_2} = k_2^2,$$

we give the differential equations of the deflection arrows, as well as their solutions for sections with moments of inertia of the cross section I_1 and I_2, respectively:

$$\frac{d^2 y_1}{dx^2} = k_2^2 \, (f - y_1 + e),$$

$$\frac{d^2 y_2}{dx^2} = k_1^2 \, (f - y_2 + e).$$

Then

$$y_1 = f + c_1 \cos k_2 x + c_2 \sin k_2 x + e,$$

$$y_2 = f \, (1 - \cos k_1 x) + e,$$

where x – current coordinate of the location for determining an arbitrary deflection (bend) y.

The transition point from one section of the beam, the moment of inertia of the section of which is equal to I_1, to another section, with the moment of inertia of the section I_2, is determined by the coordinate:

$$x = l - (l_2 + l_1) = L.$$

We proceed from the fact that the deflection in the middle of the bridge is equal to f, and also from the fact that at the transition point, at $x = L$, both sections of the elastic span curve have the same deflection and the same tangent, we obtain the following equations:

$$f = f + c_1 \cos k_2 l + c_2 \sin k_2 l + e,$$

$$f + c_1 \cos k_2 L + c_2 \sin k_2 L + e = f \, (1 - \cos k_1 L) + e \, ,$$

where $c_1 = -c_2 \, tg k_2 l - e \sec k_2 l, \quad c_2 = \dfrac{f \cos k_1 L \cos k_2 l - e \cos k_2 L}{\sin k_2 l_1}.$

After some transformations, we obtain a formula for determining the deflection in the middle of the span:

$$f = \frac{k_2 \, e}{k_1 \sin k_1 L \, \left(1 - \dfrac{k_2 \cos k_2 l \ \operatorname{ctg} k_1 L}{k_1 \sin k_2 l_1} \ (\cos k_2 L - \operatorname{tg} k_2 l \sin k_2 L)\right)} \times \\ \times \left(\frac{\sin k_2 L}{\cos k_2 l} - - \frac{\cos k_2 L \ (\sin k_2 L \ \operatorname{tg} k_2 l + \cos k_2 L)}{\sin k_2 l_1} \right). \tag{1}$$

It can be seen from the resulting equation (1) that the ratio of the lengths l_1, l_2 and the geometric characteristics of the cross sections I_{x1}, I_{x2}, I_{y1} and I_{y2} corresponding to these sections significantly affect the magnitude of the deformation of the bridge. In engineering structures, as a rule, the moments of inertia of the sections of bridges in vertical planes are much larger than the moments of the same bridges in horizontal planes, $I_{x,i} >> I_{y,i}$. This does not exclude the loss of stability of the compressed beam in one of the two main working planes.

In this connection, there is a need to obtain an equation for determining the magnitude of the longitudinal force when it approaches its critical value. Its value for our case can be determined from the above equations:

$$\frac{k_1}{k_2} = \frac{c_2 \ (\operatorname{tg} k_2 l - e \sec k_2 l) \sin k_2 L + \cos k_2 L}{\sin k_1 L}. \tag{2}$$

After simple transformations of equation (2), we obtain an expression for determining the critical value of the longitudinal load:

$$S_{cr} = r \, \frac{E \, I_2}{l^2},$$

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where r is the specifying design coefficient for the critical longitudinal force, which depends on the ratio of the moments of inertia of the cross sections along the span of the main beam (I_1/I_2), as well as the lengths of the supporting sections of the beam l_1 to the length of the beam itself by the span l –(l_1/l).

Research results

Using expression (2), the values of the refinement coefficient r were calculated, some of which are given in Table 1. The lengths l_1 of the beam sections with the moment of inertia I_1 ($I_1 < I_2$) were taken based on the recommendations for designing crane bridges to the supporting sections, and are given for the following range:

$$l_1 \approx (0.1...0.2)\,l\,.$$

Thus, using expression (2), or using the data of the refinement coefficient r given in Table 1, the critical value of the longitudinal force S_{cr} was determined for beams with different values of the moments of inertia of the cross sections I_1, I_2, and lengths l_1, l_2, for these sections.

Table 1

Values of the specifying calculation coefficient r

I_1/I_2 \ l_1/l	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55
0.10	9.30	8.99	8.58	7.61	6.55	5.85	4.52	4.41	4.01	3.98
0.15	9.46	9.35	9.15	8.51	7.51	6.39	5.62	5.23	4.61	4.10
0.2	9.59	9.47	9.35	8.76	8.11	7.41	6.71	6.04	5.21	4.21
0.25	9.68	9.56	9.39	8.97	8.57	7.93	7.10	6.38	6.02	5.01
0.30	9.77	9.61	9.45	9.09	8.64	8.22	7.60	6.98	6.43	5.73
0.35	9.76	9.69	9.52	9.25	8.91	8.39	8.27	7.69	7.01	6.23
0.40	9.79	9.70	9.67	9.39	9.08	8.94	8.51	8.21	7.59	6.71
0.45	9.81	9.76	9.71	9.42	9.11	8.99	8.66	8.32	7.89	7.00

According to the expressions obtained, mathematical studies of the deformed state of a pre-stressed beam were carried out, the geometric characteristics of the cross section of which correspond to an I-beam with profile number No. 24M. The eccentricity with which the longitudinal forces S act on the beam was taken h = 200 mm.

Some of the results obtained are shown in Figures 4 and 5 and in Table 2. The study took into account the influence of the longitudinal force on the deformation of the bridge in the vertical plane. In this connection, in Table 2, the effect of loads on the beam is estimated by the ratio of compressive S_i and transverse F_i(F_i=0.5 m; 0.63 m;1.0 m) forces – S_i/F_i=1.0; 1.5; 1.75; 2.

Table 2

Conditional deflections of the span beam

l, m	S_i/F_i	Load capacity, t								
		0.5			0.63			1.0		
		S/S_{cr}	$y/[y]$	$y_r/[y]$	S/S_{cr}	$y/[y]$	$y_r/[y]$	S/S_{cr}	$y/[y]$	$y_r/[y]$
10.5	1.0	0.12	–0.10	–0.12	0.15	–0.10	–0.12	0.24	–0.11	–0.13
	1.5	0.18	–0.17	–0.20	0.23	–0.17	–0.21	0.36	–0.18	–0.22
	1.75	0.21	–0.20	–0.23	0.27	–0.20	–0.24	0.43	–0.21	–0.24
	2.0	0.24	–0.23	–0.24	0.31	–0.24	–0.28	0.49	–0.25	–0.28
13.5	1.0	0.20	–0.16	–0.20	0.25	–0.17	–0.21	0.40	–0.17	–0.22
	1.5	0.30	–0.22	–0.28	0.38	–0.24	–0.30	0.60	–0.25	–0.31
	1.75	0.35	–0.27	–0.35	0.44	–0.28	–0.36	0.70	–0.29	–0.37
	2.0	0.40	–0.31	–0.36	0.50	–0.32	–0.37	0.80	–0.33	–0.39
16.5	1.0	0.31	–0.20	–0.23	0.40	–0.21	–0.24	0.62	–0.21	–0.25
	1.5	0.47	–0.30	–0.33	0.59	–0.31	–0.35	0.94	–0.31	–0.36
	1.75	0.55	–0.34	–0.40	0.68	–0.36	–0.43	1.10	–0.36	–0.45
	2.0	0.63	–0.40	–0.45	0.79	–0.41	–0.47	1.25	–0.42	–0.49

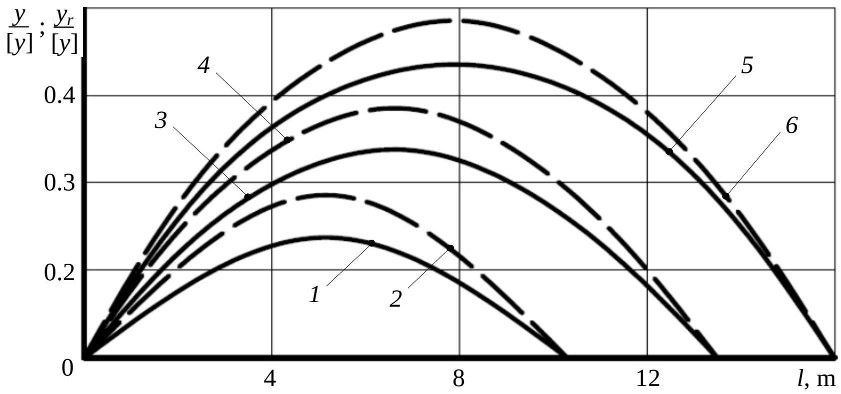

Fig. 4. Curved deflections of crane bridges: 1, 3, 5 – for beams with a constant cross-sectional moment of inertia for spans of 10.5 m, 13.5 m, and 16.5 m, respectively; 2, 4, 6 – for beams with variable cross-sectional moment of inertia for spans of 10.5 m, 13.5 m, and 16.5 m, respectively

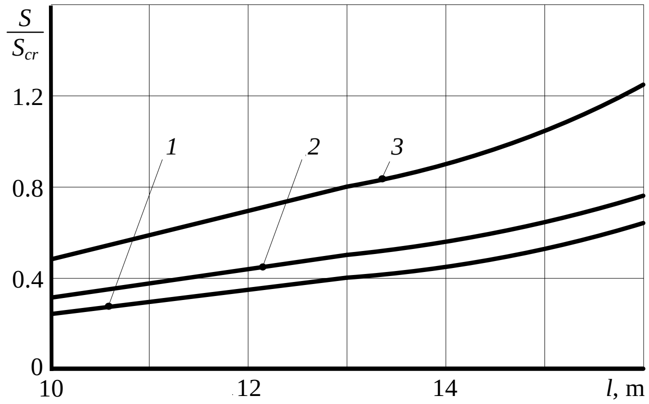

Fig. 5. Values for determining the critical longitudinal load of beams: 1, 2, 4 – for beams with transverse forces, respectively, $F = 0.5$ t, $F = 0.63$ t and $F = 1.0$ t

Bridge deformations are presented in the form of conditional cambers:

$$\frac{y}{[y]};\ \frac{y_r}{[y]},$$

where y_r is understood as the calculated deflection of the bridge, taking into account the coefficient r, which takes into account the relationship:

$$\frac{I_1}{I_2} \text{ and } \frac{l_1}{l},$$

namely, the moments of inertia of the sections and the dimensions of their lengths corresponding to these sections of the beam. Permissible values of conditional deflections were taken for operation mode groups 4K-5K – $[y/l] = 2 \cdot 10^{-3}$.

The analysis of the obtained results has established that when designing span structures with support sections, the deformations of the loaded bridge increase by 5...10 %, and in some cases – up to 18 %.

This can be seen from Figure 4, which shows the arrows of deflections of beams with load capacity $F = 1$ t with spans: $l = 10.5$ m – graphs 1, 2; $l = 13.5$ m – graphs 3, 4 and $l = 16.5$ m – graphs 5, 6. Dotted lines show deflection curves for bridges with variable cross sections. Thus, the beam deflection on graph 2 is 12 % larger than the same beam with constant geometric characteristics of the section along the entire length – graph 1. Similarly, the beam deflection on graph 4 is 18 % than the beam deflection shown on graph 3, and on graph 6 16 % more than in Fig. 5.

Thus, if the actual operation of the superstructure provides for its operation with the considered combination of loads, then this requires special attention when designing this structure. Here it is rec-

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ommended to use the mathematical model of the flying bridge developed by the authors of this work, the design scheme of which is shown in Fig. 3.

Fig. 5 shows the ratios of the calculated values of the axial force S to the critical value of the longitudinal load S_{cr} for beams with load capacities $F = 0.5\ \text{t}$, $F = 0.63\ \text{t}$ and $F = 1.0\ \text{t}$ – graphs, respectively, 1...3. From Table 2 and Figure 5 it can be seen that for this combination of loads, buckling of the beam is possible. Therefore, for example, the bifurcation of the main beam can occur in a horizontal plane, where its moment of inertia of the cross section I_y is minimal (Fig. 3, graph 3). This possibility is observed in cases when, in order to reduce the weight of the beam and ensure its bearing capacity, the calculated value of the eccentric axial load is increased. This also requires special attention when designing prestressed crane bridges with long spans.

Conclusions

Based on the analysis of the stress-strain behavior of the crane bridge, the authors recommend that in traditional calculation schemes, the beam should not be considered as a prismatic rod with a constant cross-section, since the specified schemes do not correspond to the real conditions of its design.

In this regard, a new mathematical model of the prestressed span structure is proposed, which takes into account the maximum approximation of the calculation scheme to the real structural form of the prestressed span structure.

The results obtained in this work can be used in the future to improve existing structures and engineering methods of calculation during design, as well as in the conditions of real operation.

Література

1. Chebrovsky A., Savva Yu. Review of the state of prestressed metal beams and the results of the study of operating crane beams when operating on a moving load. *PNU Bulletin*. 2013. 4 (31). 383–402.
2. Flexural behavior of ungrouted post-tensioned concrete masonry beams with unbonded bars / J.M. Garcia, R.L. Bonett, A.E. Schultz, J. Carrillo, C. Ledezma. *Constr. Build. Mater.* 2019. 203. 210–221.
3. Martovytsky L., Glushko V. Design of metal structures. Krugozir, Ukraine, 2016. 418.
4. Flexural Behavior of a 30-Meter Full-Scale Simply Supported Prestressed Concrete Box Girder / W. Jianqun, T. Shenghua, H Zheng, C. Zhou, M. Zhu. *Appl. Sci*. 2020. 10(9). 30–76.
5. Bonopera M., Chang K., Lee, Zheng-Kuan. State-of-the-Art Review on Determining Prestress Losses in Prestressed Concrete Girders. *Appl. Sci*. 2020. 10. 72–57.
6. Tkachev A., Tkachev O. The influence of the nature of the action of the external load on the stiffness of the main beams of bridge cranes. *Lifting and transport equipment.* 2020. № 1 (62). 51–60.
7. Zhegulsky V., Mironov I., Lukashuk O. Design and calculation of crane metal structures. Ural Publishing House University. 2019.
8. Tkachov A., Tkachov O., Sydorenko I. Improvement of the deformed state of flight beams of bridge cranes. *Faculty of Architecture, Civil Engineering and Applied Arts.* 2020. 2. 4. 118–125.
9. Naidenko E., Bondar O., Boiko A., Fomin O., Turmanidze R. Control Optimization of the Swing Mechanism. In: Tonkonogyi V., Ivanov V., Trojanowska J., Oborskyi G., Pavlenko I. (eds) Advanced Manufacturing Processes III. InterPartner 2021. Lecture Notes in Mechanical Engineering. Springer, Cham, 2022.
10. Iodchik A. Deflections of a steel beam, pre-stressed by bending of the I-beam. *Bulletin of VSSTU*. 2013. 5(44). 45–52.
11. Zou J., Huang Y., Feng W., Chen Y., Huang Y. Experimental study on flexural behavior of concrete Tbeams strengthened with externally prestressed tendons / J. Zou, Y. Huang, W. Feng, Y. Chen, Y.Huang *Mathematical Biosciences and Engineering.* 2019. Vol. 16, Issue 6. 6962–6974.
12. Tkachev A., Tkachev O., Fomin O., Bondar O., Naidenko E. Influence of Horizontal Inertial Loads on the Operation of Overhead Crane Girders. Advances in Design, Simulation and Manufacturing V Proceedings of the 5th International Conference on Design, Simulation, Manufacturing: *The Innovation Exchange, DSVIE-2022*, June 7-10, Poznan, Poland, Volume 2: Mechanical and Chemical Engineering, Odessa, Ukraine, pp. 47–54, 2022.
13. Lou T.J., Lopes S.M. R., Lopes A.V. Numerical modeling of externally prestressed steel 436 – concrete composite beams. *J. Constr. Steel Res*. 2016. 121. 229–236.
14. Qu X., Xu G., Fan X., Bi X. Intelligent optimization methods for the design of an overhead traveling crane. *Chinese Journal of Mechanical Engineering.* 2015. 28(1). 187–196.
15. Liu J., Wang T. Deflection Calculation on Reinforced Concrete Beams Strengthened by Externally Prestressing Transverse Tensioning Method. *Advanced Materials Research.* 2013. 71, 430–436.
16. Nie J., Tao M., Cai C., Li S. Deformation Analysis of Prestressed Continuous Steel-Concrete Composite Beams**.** *Journal of Structural Engineering.* 2009. 135 (11). 94–102.

17. Overhead Crane Camber Deformation Assessment and Energy Analysis / T. Yifei, L. Lijin, S. Guomin, L. Dongbo, L. Xiangdong. *Proceedings of the Institute of Mechanical Engineers*. 2014. Part B (6). 52–55.
18. Tkachev A., Tkachev A.l., Predrag Dasic, Prokipovych I., Kostina M. Static Stiffness of the Crane Bridges under Moving Lood Distributoin. InterPartner-2021, *3rd Grabchenko`s International Conference on Advanced Manufacturing Process*, September 7–10, Odessa, Ukraine, p. 42, 2021.

References

1. Chebrovsky, A., & Savva, Yu. (2013). Review of the state of prestressed metal beams and the results of the study of operating crane beams when operating on a moving load. *PNU Bulletin*, *4* (31), 383–402.
2. Garcia, J.M., Bonett, R.L., Schultz, A.E., Carrillo, J., & Ledezma C. (2019). Flexural behavior of ungrouted post-tensioned concrete masonry beams with unbonded bars. *Constr. Build. Mater.*, *203*, 210–221.
3. Martovytsky, L., & Glushko, V. (2016). *Design of metal structures*. Krugozir, Ukraine.
4. Jianqun, W., Shenghua, T., Zheng, H., Zhou, C., & Zhu, M. (2020). Flexural Behavior of a 30-Meter Full-Scale Simply Supported Prestressed Concrete Box Girder. *Appl. Sci.*, *10*(9), 30–76.
5. Bonopera, M., Chang, K., Lee, Zheng-Kuan. (2020). State-of-the-Art Review on Determining Prestress Losses in Prestressed Concrete Girders. *Appl. Sci.*, *10*, 72–57.
6. Tkachev, A., & Tkachev, O. (2020). The influence of the nature of the action of the external load on the stiffness of the main beams of bridge cranes. *Lifting and transport equipment*, *1 (62*), 51–60.
7. Zhegulsky, V., Mironov, I., & Lukashuk, O. (2019). *Design and calculation of crane metal structures.* Ural Publishing House University.
8. Tkachov, A., Tkachov, O., & Sydorenko, I. (2020). Improvement of the deformed state of flight beams of bridge cranes. *Faculty of Architecture, Civil Engineering and Applied Arts., 2,* 4, 118–125.
9. Naidenko, E., Bondar, O., Boiko, A., Fomin, O., & Turmanidze, R. (2022). Control Optimization of the Swing Mechanism. In: Tonkonogyi V., Ivanov V., Trojanowska J., Oborskyi G., Pavlenko I. (eds) *Advanced Manufacturing Processes III. InterPartner 2021.* Lecture Notes in Mechanical Engineering. Springer, Cham.
10. Iodchik, A. (2013). Deflections of a steel beam, pre-stressed by bending of the I-beam. *Bulletin of VSSTU*, 5(44), 45–52.
11. Zou, J., Huang, Y., Feng, W., Chen, Y., & Huang, Y. (2019). Experimental study on flexural behavior of concrete Tbeams strengthened with externally prestressed tendons. *Mathematical Biosciences and Engineering, 16*, 6, 6962–6974.
12. Tkachev, A., Tkachev, O., Fomin, O., Bondar,O., & Naidenko, E. (2022). Influence of Horizontal Inertial Loads on the Operation of Overhead Crane Girders. Advances in Design, Simulation and Manufacturing *V Proceedings of the 5th International Conference on Design, Simulation, Manufacturing:* The Innovation Exchange, DSVIE-2022, June 7-10, Poznan, Poland, Volume 2: Mechanical and Chemical Engineering, Odessa, Ukraine, pp. 47–54.
13. Lou, T.J., Lopes, S.M. R., & Lopes, A.V. (2016). Numerical modeling of externally prestressed steel 436 – concrete composite beams. *J. Constr. Steel Res*., *121*, 229–236.
14. Qu X., Xu G., Fan X., Bi X. Intelligent optimization methods for the design of an overhead traveling crane. *Chinese Journal of Mechanical Engineering.* 2015. 28(1). 187–196.
15. Liu, J., & Wang, T. (2013). Deflection Calculation on Reinforced Concrete Beams Strengthened by Externally Prestressing Transverse Tensioning Method. *Advanced Materials Research, 71*, 430–436.
16. Nie, J., Tao, M., Cai, C., & Li, S. (2009). Deformation Analysis of Prestressed Continuous Steel-Concrete Composite Beams**.** *Journal of Structural Engineering, 135* (11), 94–102.
17. Yifei, T., Lijin, L., Guomin, S., Dongbo, L., & Xiangdong, L. (2014). Overhead Crane Camber Deformation Assessment and Energy Analysis. *Proceedings of the Institute of Mechanical Engineers, B (6),* 52–55.
18. Tkachev, A., Tkachev, A.l., Predrag, Dasic, Prokipovych, I., & Kostina, M. (2021). Static Stiffness of the Crane Bridges under Moving Lood Distributoin. InterPartner-2021, *3rd Grabchenko`s International Conference on Advanced Manufacturing Process*, September 7–10, Odessa, Ukraine, p. 42.

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