Computer Modeling of Nonlinear Flutter of Viscoelastic Based Plate as a Sustainable Mechanical Engineering Approach in Aircraft Structures' Design

Andriy Verlan [1], Bakhtiyar Khudayarov [2], Anatolii Verlan [3], Oleksandr Fomin [4] and Sergii Polozhaenko [5]

[1] *Norwegian University of Science and Technology, NTNU in Gjøvik, postboks 191, NO-2802 Gjøvik, Norway*
[2] *Tashkent institute of irrigation and agricultural mechanization engineers, 39 Kari Niyazov Street, Tashkent 100000, Uzbekistan*
[3] *Institute of modeling problems in power engineering named after G. E. Pukhov, 15, General Naumov str. Kyiv, 03164, Ukraine*
[4,5] *Odessa Polytechnic State University, 1, Shevchenko ave., Odesa, 65044, Ukraine*

Abstract

The article is dedicated to computer modeling of a viscoelastic based plate flutter in design of aeronautical structures in sustainable mechanical engineering. The mathematical model of the flutter problem for viscoelastic plates with viscoelastic base is presented. Using the Bubnov-Galerkin method, discrete models of the flutter problem for viscoelastic plates streamlined by the supersonic gas jet are deduced. A numerical method is developed for solving nonlinear integro-differential equations for the viscoelasticity hereditary theory problem with weakly singular kernels. According to the above numerical method with respect to unknowns, a system of algebraic equations is obtained. To solve the system of algebraic equations, the Gauss method is used. An application program package has been developed to enable modeling and studying of the nonlinear dynamic problems for the hereditary viscoelasticity theory with weakly singular kernels. Based on the proposed model, numerical method and algorithm, nonlinear problems for the viscoelastic plates flutter with a viscoelastic base are investigated. The critical flutter velocity of the viscoelastic plates is determined for solving the stability problem in design of aircraft structures.

Keywords

Sustainable mechanical engineering, Mathematical modeling, Computer modeling, Numerical methods and algorithms, Integro-differential equations, Plate flutter, Viscoelasticity, Viscoelastic base, Aircraft design.

1. Introduction

Currently, composite materials with pronounced viscoelastic properties are widely used in aviation and many other branches to increase the sustainability of mechanical engineering. These branches have obtained light, elegant and efficient thin-walled structures. The importance of stability calculations and strength design of the general cycle for such structures has dramatically increased. In this regard, the heredity viscoelasticity theory is attracting more and more attention of researchers. This is evidenced by numerous research papers in recent years which demonstrate the latest achievements of the viscoelasticity theory. The growing interest in this theory is explained by computer technology

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EMAIL: verlandr@gmail.com (A1); b.khudayarov@tiiame.uz (A2); a.f.verlan@gmail.com (A3); fomin@op.edu.ua (A4); polozhaenko@op.edu.ua (A5);
ORCID: 0000-0002-0439-3214 (A1); 0000-0002-2876-8447 (A2); 0000-0002-8565-2037 (A3); 0000-0002-8816-0652 (A4); 0000-0002-4082-8270 (A5)

development which makes it possible to reliably compare a computational experiment, obtained on the basis of mathematical models, with a full-scale experiment.

The study basis of the composite materials deformation processes is the heredity viscoelasticity theory whose specific application depends on the material parameters, product shape and the changes range of environmental conditions. At the same time, significant difficulties, while creating the appropriate models, arise in connection with regard of the viscoelasticity properties and nonlinear effects. It should be noted that the use of traditional materials in aeronautical structures made it possible to apply mathematical models that can already be called simplified ones. It means, they do NOT fully take into account the viscoelasticity properties and other effects. These effects are most pronounced under conditions of supersonic air or liquid flows, i.e. at high velocities which lead to the flutter effect.

Therefore, the previously deduced scientific results in the field of modeling the processes of aircraft elements' behavior at high velocities can't be directly applied in the considered problems. It emphasizes the problem relevance of obtaining adequate mathematical models for dynamics of aircrafts elements built of materials with explicit substantially viscoelastic and non-linear properties and operating in flutter modes.

The mentioned properties of structure materials and the above factors increase research complexity and lead to the need of developing computational methods for studying the viscoelastic elements sustainability of thin-walled structures. Therefore, the development of effective computational algorithms for solving nonlinear integro-differential equations for the viscoelastic elements' dynamic problems of the thin-walled structures elements with weakly singular heredity kernels is urgent.

2. Study of the viscoelastic based plate flutter nonlinear problem

The flutter of plates and flat shells with regard to elastic and viscoelastic base has been studied by a number of authors [1 - 6]. Pouresmaeeli et al. [6] investigated the natural frequency of orthotropic viscoelastic nanoplates lying on an elastic foundation employing the nonlocal classical plate theory. In [1], an infinite plate was investigated. The plate was lying on an elastic base and streamlined by gas flow. Despite a significant amount of researches, relatively few researches have been done on the nonlinear flutter of viscoelastic plates and panels on elastic base.

In this regard, this research paper presents the theoretical study of the viscoelastic plates non-linear flutter. Based on the Bubnov-Galerkin method with the use of quadrature formulas and the exclusion method of the weakly singular operators, an effective computational algorithm has been developed that enables studying of the problem on the viscoelastic plates nonlinear flutter streamlined by supersonic gas flow.

Let's consider the nonlinear problem of the plate flutter taking into account the viscoelastic bases. Let's assume that the plate with sides a and b and thickness h is hinged along the entire contour and streamlined from one side by supersonic gas flow, as shown in Figure 1.

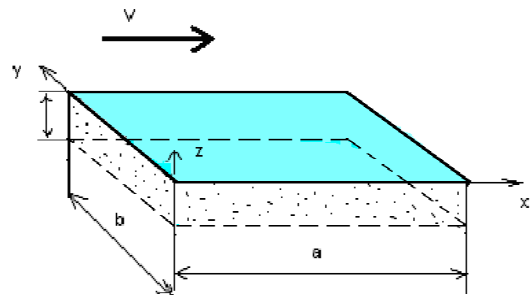

Figure 1: Viscoelastic based plates

Under the assumption made in [1, 7, 8] and taking into account the bases, the vibrations equation of a viscoelastic plate has the following form;

$$\frac{D}{h}\left(1-R^*\right)\nabla^4 w = L\left(w,\Phi\right) - k(1-\Gamma^*)w -$$
$$-\rho\frac{\partial^2 w}{\partial t^2} - \frac{B}{h}\frac{\partial w}{\partial t} - \frac{BV}{h}\frac{\partial w}{\partial x} - \frac{B_1 V^2}{h}\left(\frac{\partial w}{\partial x}\right)^2,$$
$$\frac{1}{E}\nabla^4\Phi = -\left(1-R^*\right)\left\{\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} - \left[\frac{\partial^2 w}{\partial x \partial y}\right]^2\right\}, \quad (1)$$

where $D = \dfrac{Eh^3}{12(1-\mu^2)}$ is flexural rigidity; ρ is material density; h is plate thickness; E is modulus of elasticity; μ is Poison's ratio; w is plate deflection; V is flow velocity; R^* is integral operator with relaxation kernel $R(t)$ with weakly singular property of Abel type; L is differential operator:

$$L\left(w,\Phi\right) = \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 \Phi}{\partial y^2} + \frac{\partial^2 w}{\partial y^2}\frac{\partial^2 \Phi}{\partial x^2} - 2\frac{\partial^2 w}{\partial x \partial y}\frac{\partial^2 \Phi}{\partial x \partial y};$$

Φ is stress function; κ is K-factor (modulus of subgrade reaction); Γ^* is integral operator with relaxation kernel $\Gamma(t)$;

$$\left(\begin{array}{l} \Gamma^* \phi(t) = \int_0^t \Gamma(t-\tau)\phi(\tau)d\tau, \\ \Gamma(t) = A_0 t^{\alpha_0 - 1} \exp(-\beta_0 t) \end{array} \right);$$

$$B = \frac{\aleph p_\infty}{V_\infty}, B_1 = \frac{\aleph(\aleph+1) p_\infty}{4V_\infty^2},$$

where $\aleph$ is polytrophic exponent for gas; p_∞, V_∞ are sound pressure and sound velocity respectively in the unperturbed flow.

Let us search the system (1) solutions in the form of

$$w(x,y,t) = \sum_{n=1}^{N}\sum_{m=1}^{L} w_{nm}(t) \sin\frac{n\pi x}{a} \sin\frac{m\pi y}{b} \qquad (2)$$

$$\Phi(x,y,t) = \sum_{n=1}^{N}\sum_{m=1}^{L} \Phi_{nm}(t) \sin\frac{n\pi x}{a} \sin\frac{m\pi y}{b}$$

After performing the Bubnov-Galerkin procedure, we deduce the system of integro-differential equations (IDE) for $w_{nm}(t)$ and $\Phi_{nm}(t)$. Let's exclude $\Phi_{nm}(t)$ from this system and write down the following nonlinear IDE with respect to the desired function $w_{nm}(t)$:

$$\ddot{w}_{kl} + \lambda^4 \Omega^2 \left[\left(\frac{k}{\lambda}\right)^2 + l^2 \right]^2 (1-R^*) w_{kl} +$$
$$+ \frac{16}{\pi^2} g_k g_l k (1-\Gamma^*) w_{kl} + \frac{12\lambda^4 (1-\mu^2)\Omega^2}{\pi^2}$$
$$\sum_{n,i,j=1}^{N} \sum_{m,r,s=1}^{L} a_{k\ln mirjs} w_{nm} (1-R^*) w_{ir} w_{js} + \qquad (3)$$
$$+ M \dot{w}_{kl} + 2MM^* \sum_{n=1}^{N} \gamma_{kl} w_{nl} +$$
$$+ M_1 M^{*2} \sum_{n,i}^{N} \sum_{m,r}^{L} F_{k\ln mir} w_{nm} w_{ir} = 0,$$

$$w_{kl}(0) = w_{0kl},\ \dot{w}_{kl}(0) = \dot{w}_{0kl},\ k = \overline{1,N};\ l = \overline{1,L};$$

where $\Omega^2 = \dfrac{\pi^4}{12(1-\mu^2)} M_E^2 \left(\dfrac{h}{a}\right)^2$, $M = \aleph M_p^2 \lambda_1$,

$M_1 = \aleph(\aleph+1)\dfrac{M_p^2}{4}$; $M^* = \dfrac{V}{V_\infty}$ – much number;

$M_E = \sqrt{\dfrac{E}{\rho V_\infty^2}}$; $M_p = \sqrt{\dfrac{P_\infty}{\rho V_\infty^2}}$; $\lambda = \dfrac{a}{b}$; $\lambda_1 = \dfrac{a}{h}$;

g_k, γ_{kl}, $F_{k\ln mir}$, $a_{k\ln mir}$ – nondimensional factors.

3. A method for numerical solution of the deduced integro-differential equations for modeling nonlinear flutter of viscoelastic based plate

The systems of the nonlinear IDE (3) are solved numerically using the method proposed in [9-17]. For this purpose, let us write this system in integral form and, using a rational transformation, exclude the weakly singular properties of the integral operator R^*. After having assumed that $t=t_i$, $t_i=i\Delta t$, $i=1,2,\ldots$ (Δt is constant) and after having replaced the integrals with some quadrature formulas for calculating $w_{nm} = w_{nm}(t)$, we deduce the following recurrence relation:

$$w_{ikl} = \frac{1}{1+A_i M} \Bigg\{ w_{0kl} + \left(\dot{w}_{0kl} + M w_{0kl}\right) t_i -$$
$$- \sum_{j=0}^{i-1} A_j \Bigg(M w_{jkl} - (t_i - t_j) \Big\langle -2MM^* \sum_{n=1}^{N} \gamma_{kn} w_{jnl} -$$
$$- \lambda^4 \Omega^2 \left[\left(\frac{k}{\lambda}\right)^2 + l^2 \right]^2 \times \left(w_{jkl} - \frac{A}{\alpha} \sum_{s=0}^{j} B_s e^{-\beta t_s} w_{j-skl} \right) -$$
$$- \frac{12\lambda^4 (1-\mu^2)\ \Omega^2}{\pi^2} \sum_{n,i_1,j_1=1}^{N} \sum_{m,r,s_1=1}^{L} a_{k\ln m i_1 r j_1 s_1} w_{jnm} \times \qquad (4)$$
$$\times \left(w_{ji_1r} w_{jj_1s_1} - \frac{A}{\alpha} \sum_{s=0}^{j} B_s e^{-\beta t_s} w_{j-si_1r} w_{j-sj_1s_1} \right) -$$
$$- \frac{16}{\pi^2} g_k g_l k \left(w_{jkl} - \frac{A_ш}{\alpha_ш} \sum_{s=0}^{j} B_s e^{-\beta t_s} w_{j-skl} \right) -$$
$$- M_1 M^{*2} \sum_{n,i_1=1}^{N} \sum_{m,r=1}^{L} F_{k\ln m i_1 r} w_{jnm} w_{ji_1r} \Big\rangle \Bigg) \Bigg\};$$

$$i = 1,2,\ldots;\ n = \overline{1,N};\ m = \overline{1,L};$$

where A_j, B_s are the numerical factors applied to trapezium quadrature formulas.

Due to the proposed approach, in the algorithm for the numerical solution of the problem in formula (4) the factor at $j = i$ takes zero(0) value, i.e. the last summand of the sum is equal to zero(0). Therefore, the summation is carried out from zero to i -1 ($j = \overline{0, i-1}$). Thus, according to the numerical method with respect to unknowns, we obtain the system of linear algebraic equations.

The calculation results are stated in the table and reflected in the graphs shown in Fig. 2-3 at N=5, L=2. Based on the formula (4), the critical flutter velocity of viscoelastic plates is determined. As a criterion determining the critical velocity V_{cr} we assume that at this velocity the vibratory movement with rapidly increasing amplitudes occurs, which can lead to structure destruction. In the case $V < V_{cr}$, the flow velocity is less than the critical one, the amplitude of the viscoelastic plate vibrations damps [1, 11-13].

For $V=V_{cr}$ the numbers V_1 and V_2 are considered located in the interval (V_0,V_1) in such way that $V_0 < V_1 < V_2 < V_n$. By comparison of the variation low w at $V=V_1$ and $V=V_2$ we can come the following conclusions:

a) if $V<V_1$ the function variation law w is close to the harmonic, it means that V_{kp} can't be located on the interval (V_0,V_1) , i.g. V_{cr} is located on the interval (V_1, V_n);

b) if at $V > V_1$ there is rapid function (w) growth (temporally), then it means that V_{kp} is located on the interval (V_0,V_1).

The processes a) and b), i.e. the exclusion process of the intervals that don't give adverse events, is repeated for (V_0, V_1), or (V_1, V_n), etc. The search ends when the remaining subinterval is reduced to a sufficiently small value.

4. Investigation of the influence of the plate material viscoelastic proper-ties on the critical values of the flutter velocity

As a result of applying the given math model the critical values of the flutter velocity depending on the physical, mechanical and geometric parameters of the plate were obtained as shown in the following Table 1.

Table 1

Dependences of the critical flutter velocity of a viscoelastic based plate on the physical, mechanical and geometric parameters

A	α	β	A_o	α_o	β_o	λ	λ_1	k	V_{cr}
0 0,001 0,01 0,1	0,25	0,05	0,02	0,3	0,01	2,5	250	0,0001	1620 1595 1492 1170
0,1	0,1 0,4 0,7	0,05	0,02	0,3	0,01	2,5	250	0,0001	865 1325 1479
0,1	0,25	0,01 0,1	0,02	0,3	0,01	2,5	250	0,0001	1186 1167
0,1	0,25	0,05	0 0,05 0,3	0,3	0,01	2,5	250	0,0001	1219 1158 1137
0,1	0,25	0,05	0,02	0,1 0,5 0,9	0,01	2,5	250	0,0001	1162 1211 1230
0,1	0,25	0,05	0,02	0,3	0,1 0,5	2,5	250	0,0001	1165 1162
0,1	0,25	0,05	0,02	0,3	0,01	2 2,2 2,7	250	0,0001	718 887 1467
0,1	0,25	0,05	0,02	0,3	0,01	2,5	220 280 300 350	0,0001	1740 846 674 412
0,1	0,25	0,05	0,02	0,3	0,01	2,5	250	0 0,0002 0,0004 0,0006	107 1244 1322 1439

Thus, the influence of the plate viscoelastic properties on the critical values of the flutter velocity was studied.

The calculation results presented in the table show that the solutions of elastic (A = 0) and viscoelastic (A> 0) problems differ significantly from each other.

For example, as parameter A increases from zero to 0.1, the critical flutter velocity decreases by 27.7%.

Further, the influence of the singularity parameter α on the critical flutter velocity was studied. With increasing of α this velocity becomes bigger. For example, the difference between the critical velocity values at α=0,1 and α=0,4 is 53%.

The above table shows that influence of the heredity kernel damping parameter β on the plate flutter velocity is low comparing to influence of viscosity A and singularity α.

This once again confirms that the exponential relaxation kernel is unable for fully description the hereditary properties of the construction material.

The influence of the relative plate thickness parameter λ_1 on the critical flutter velocity V_{cr} is studied.

The calculations were made at λ_1= 220, 280, 300 and 350. The obtained results show that with decrease of the plate thickness (growth of the parameter λ_1) the critical flutter velocity of the viscoelastic declines.

The influence of the plate elongation parameter λ on the critical flutter velocity was investigated.

With increase of λ the critical flutter velocity v become higher, which is explained by the fact that growth of λ (at constant λ_1) leads to the plate size reduction perpendicular to the flow direction and, therefore, the relative rigidity of the system increases.

The tables demonstrate that taking into account the viscoelastic base, the critical flutter velocity increases in comparison to the velocity without taking into account the viscoelastic base. Especially in case of large K-factors (modulus of subgrade reaction), the flutter velocity increases markedly.

The influence of the viscoelastic material properties on the plate vibration amplitudes is shown in Figure 2, where A=0 (1); A=0,005 (2); A=0,1 (3); k=0,0001; α=0,25; β=0,05; λ=2,5; A_0 = 0,1; α_0 = 0,25; β_0 = 0,02; λ_1 = 250; N = 5; L=2.

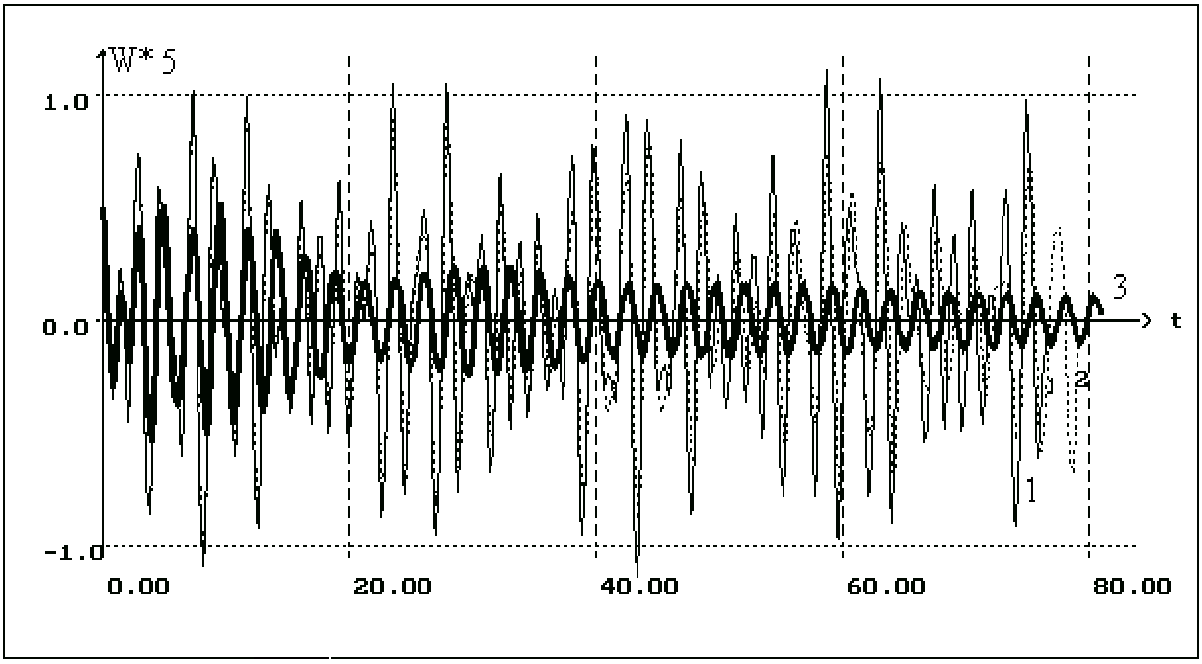

Figure 2: Viscoelastic material properties' influence on the plate vibration amplitudes

As you can see from the Figure 2, with increasing parameter A the oscillations amplitude and frequency decrease.

Figure 3 demonstrates plots of the dimensionless deflection changes depending on the time t for different values of the relative thickness parameter λ, where λ_1=200 (curve 1); λ_1=310 (curve 2); A=0,1; α=0,25; β=0,05; k=0,0001; λ=2,5; A_0=0,11; α_0=0,2; β_0=0,02; *N*=5; *L* =2; V=875 m/s. With increasing of λ_1=*a/h* (thickness reduction) the flutter velocity reduces.

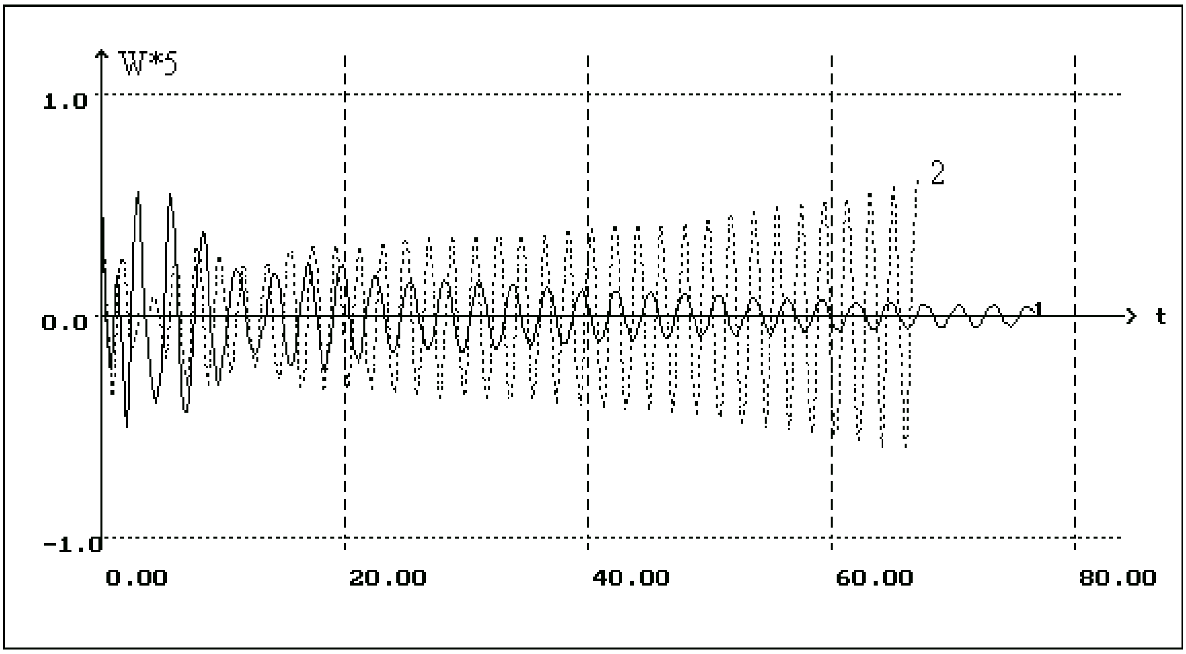

Figure 3: Dimensionless deflection changes depending on time

As you can see from the Figure 2, with increasing parameter *A* the oscillations amplitude and frequency decrease.

Figure 3 demonstrates plots of the dimensionless deflection changes depending on the time *t* for different values of the relative thickness parameter λ, where λ_1=200 (curve 1); λ_1=310 (curve 2); A=0,1; α=0,25; β=0,05; k=0,0001; λ=2,5; A_0=0,11; α_0=0,2; β_0=0,02; *N*=5; *L* =2; V=875 м/с. With increasing of λ_1=*a/h* (thickness reduction) the flutter velocity reduces.

5. Conclusions

Therefore, we can conclude that the singularity parameter α influences not only viscoelastic systems vibrations; it has impact on the critical flutter velocity.

Consequently, regard of such an influence in design of aeronautical structures is of great importance since the smaller the singularity parameter of the structure material is the more intense the dissipative processes in these structures occur.

It should be noted that at a flow velocity lower than V_{cr} the viscoelastic material property decreases the oscillations amplitude and frequency. If the flow velocity is higher than V_{cr} then the material viscoelastic property has a destabilizing effect.

Based on the obtained results, it can be concluded that regard of the plate material viscoelastic properties leads to decrease of the critical flutter velocity V_{cr} where the flutter process begins.

With increasing parameter *A* the oscillations amplitude and frequency decrease. With increasing of parameter λ_1 (thickness reduction) the flutter velocity reduces.

It significantly increases efficiency and stability of the designed and developed aeronautical structures and is a substantial contribution to sustainable manufacturing and mechanical engineering.

6. References

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