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ELEMENTARY CALCULATION OF CLIP CONNECTIONS WITH INCOMPLETE SWEEP OF SHAFT

І.П. Шацький, Л.Я. Роп'як. **Елементарний розрахунок клемових з'єднань з неповним обхопленням вала.** Розглянуто перспективні конструкції роз'ємних клемових (гвинтово-фрикційних) з'єднань з неповним охопленням вала, які застосовуються в машинах і механізмах нафтогазової галузі. Сформульовано контактні задачі взаємодії півступиць з валом для симетричних і несиметричних з'єднань. У свою чергу для несиметричних відносно стягувального болта конструкцій вивчаються два варіанти взаємодії: з забороною та із свободою бокового переміщення. На підставі апріорних припущень щодо законів розподілу контактного тиску, прийнятих в традиційних навчальних курсах «Деталі машин», розроблено інженерну методику розрахунку клемових з'єднань. При цьому різним характерам спряження деталей (із зазором, узгоджене, з натягом) відповідають зосереджений, косинусний і сталий (лінійний) розподіли контактних напружень. Встановлено аналітичні залежності граничних моментів та сил зрушування від кутів охоплення, сили затягування болта і від трибологічних властивостей спряжуваних деталей складального вузла.
Ключові слова:клемове з'єднання, неповне охоплення вала, розрахунок.

I.P. Shatsky, L.Ya. Ropyak. **Elementary calculation of clip connections with incomplete sweep of shaft.** The article describes promising structures of clip (screw and friction) connections with incomplete sweep of shaft used in machines and mechanisms for the oil and gas industry. The contact problems of interaction between semi-hubs and shaft for the symmetric and asymmetric connections are formulated. For structures that are asymmetric relatively the joint bolt two types of interaction are investigated: with and without lateral displacement. Based on a priori assumption about the distribution laws of contact pressure accepted in traditional courses of "Machine Details" an engineering method for calculating of clip connections is developed. Herewith different types of details coupling (with a gap, matched, with tension) correspond to concentrated, cosine and sustainable (linear) distributions of contact stresses. There are determined an analytical dependences of boundary points and breakloose force on spanning angles, bolt tightening force and tribological properties of joined parts of subassembly.
Keywords: clip connection, incomplete sweep of shaft, calculation.

Introduction. Clip (screw and friction) connections belong to the class of nominally fixed friction joints, in which required normal pressure is created by tightening bolts. There are designs with split and one-piece hub. Traditionally units with complete sweep of shaft are commonly used. But sometimes in structural considerations applicable are coupling in which semi-clips cling the part of cylindrical surface of shaft. Such a situation arises from the need to regulate the angle of parts fixing. An example of such a unit is combination of blades with hub in the axial fans of pressurization of gas pumping units.

Literature review. The methods of typical calculations of most commonly used clip connections with complete sweep of shaft are worked out in details today [1…3]. Instead, the quality issues of gathering of clip connections with partial sweep of shaft are in fact unexplored. Especially it concerns the units, that are asymmetric relatively the joint bolt.

Aim of the Research. In the article the theoretical aspects of modeling the interaction of elements of detachable clip connections with incomplete sweep of shaft are considered. Aim of the research is to develop methods of engineering evaluation of bearing capacity of structures for different types of details coupling. The declared simplicity of calculations is that they are held not on the basis of rigorous solutions of contact problems, but on the basis of intuitive a priori assumptions about the

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contact pressure distribution law as it is done in traditional courses of “Machine Details” while calculation the detachable clips with complete sweep of shaft.

Main Body. Let’s consider split clip connections (Fig. 1). They consist of semi-hubs sweeping the shaft with diameter $D=2R$ in areas with angles of inclination 2β and $\alpha+\beta$ respectively in symmetric and asymmetric versions. Suppose angles α and β are acute, so every single arc of contact is no longer than semicircle: $0\le 2\beta\le\pi$, $0\le\alpha+\beta\le\pi$. Connection is tightened with through-bolt so that the main vector of bolt tightening forces is P and passes through the axis of the shaft. Let’s find the dependency of friction torque form tightening force.

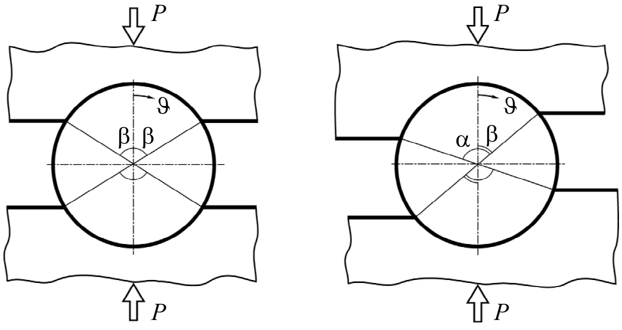

Fig. 1. Scheme of clip connection with split hub that partially sweep the shaft: symmetric (a) and asymmetric (b) versions

Symmetric and asymmetric connections including the different character of parts coupling (with a gap, adjusted and with tension) let’s consider separately. Let's start with the simplest situations.

Calculation of symmetric connection (Fig. 1, *a*). Running polar angle ϑ is measured from the vertical axis of symmetry clockwise. So for the upper semi-hub the dense clinging of shaft is possible on area $\vartheta\in(-\beta,\beta)$.

Let $q(\vartheta)$ is contact pressure that in this case is π-periodic even function. The main vector of contact stresses applied to, for example, upper semi-hub has to be balanced by tightening force is

$$\sum Y_i = R\int_{-\beta}^{\beta} q(\vartheta)\cos\vartheta\, d\vartheta - P = 0 .$$

Hence

$$P = 2R\int_0^{\beta} q(\vartheta)\cos\vartheta\, d\vartheta . \tag{1}$$

Let’s find the moment of friction forces M acting on the shaft while attempting to turn it at two semi-hubs:

$$M = 2R\int_{-\beta}^{\beta} fq(\vartheta)R d\vartheta = fD\cdot 2R\int_0^{\beta} q(\vartheta) d\vartheta , \tag{2}$$

where f is dry friction coefficient.

Comparing (1) and (2), we find

$$M = fPD\frac{\int_0^{\beta} q(\vartheta)\, d\vartheta}{\int_0^{\beta} q(\vartheta)\cos\vartheta\, d\vartheta} . \tag{3}$$

Postulating in (3) contact pressure distribution laws for a particular character of coupling, we obtain the dependences of moment from the force P and angle β.

When planting clip with large clearance ($R_{cl} > R$), when contact of semi-hub with the shaft passes through the generating line $\vartheta = 0$ and after tightening of bolts — a narrow strip, the contact pressure was approximated by concentrated force per unit length

$$q(\vartheta) = p\delta(\vartheta) / R,$$

where $\delta(\vartheta)$ is Dirac function;

$$p = P / L,$$

L — strip contact length (deep into Fig. 1).

Then the numerator and denominator in (3) are the same constants, and

$$M = fPD. \tag{4}$$

When adjusted (agreed) planting in conditions of equality of radii of the shaft circles and semi-hub the contact pressure distribution is taken close to the cosine: $q(\vartheta) = q_{\max}\cos\vartheta$, $\vartheta \in (-\beta, \beta)$. By calculating the integrals in (3)

$$M = fPD\frac{4\sin\beta}{2\beta + \sin 2\beta}. \tag{5}$$

The presence of tension ($R_{cl} < R$) in untensioned condition in fact means the presence of gap at small ϑ and the point of contact at $\vartheta \approx \pm\beta$. When tightening the bolt the contact pressure localizes at the edges around of angles $\vartheta = \pm\beta$, but after tightening mentioned gap is eliminated and further growth of P leads probably to equalization of contact pressure. So in this case let be $q(\vartheta) = q = \text{const}$ and after calculating integrals in (3)

$$M = fPD\frac{\beta}{\sin\beta}. \tag{6}$$

Calculation of asymmetric connection (Fig. 1, *b*). Let $\vartheta \in (-\alpha, \beta)$ be the arc of probable contact and $q(\vartheta)$ be the contact pressure that in this case is π-periodic function.

Calculating the vertical component of vector of contact stresses and the main moment of friction forces instead of (1) and (2) we obtain respectively

$$P = R\int_{-\alpha}^{\beta} q(\vartheta)\cos\vartheta d\vartheta, \tag{7}$$

$$M = fDR\int_{-\alpha}^{\beta} q(\vartheta)d\vartheta. \tag{8}$$

From (7) and (8) it is easy to find out the dependence between the friction torque and the total strength of bolt tightening

$$M = fPD\frac{\int_{-\alpha}^{\beta} q(\vartheta)d\vartheta}{\int_{-\alpha}^{\beta} q(\vartheta)\cos\vartheta d\vartheta}. \tag{9}$$

The feature of asymmetric connection is that depending on the way the semi-hubs are mounted by bolt there are two possible situations:

— fixed connection without mutual horizontal displacement of semi-hubs;

— free connection with the possibility of mutual lateral displacement of parts.

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In first case the relative horizontal displacement equals zero and horizontal component of the contact stresses vector is unknown. And vice versa in second case the relative horizontal displacement is unknown and the projection of the main contact stresses vector on horizontal axis is zeroed:

$$\sum X_i = R\int_{-\alpha}^{\beta} q(\vartheta)\sin\vartheta\, d\vartheta = 0\,. \tag{10}$$

The last condition is used to establish the degree of asymmetry of contact pressure distribution.

Depending on the character of semi-hub coupling with the shaft let's obtain the different expressions for the friction torque in asymmetric clip connection.

When planting the clip with a gap regardless of presence or absence of lateral displacement the contact pressure is concentrated in $\vartheta = 0$. Then from (9) also can be obtained (4).

In case of agreed contact for the task without lateral displacement we postulate the law $q(\vartheta) = q_{\max}\cos\vartheta$. Calculating integrals in (9)

$$M = fPD\frac{4(\sin\alpha + \sin\beta)}{2(\alpha+\beta) + \sin 2\alpha + \sin 2\beta}\,. \tag{11}$$

In case of presence of lateral displacement we accept cosine distribution of contact pressure with displaced maximum

$$q(\theta) = q_{\max}\cos(\vartheta - \vartheta_0)\,, \tag{12}$$

where angle ϑ_0 can be calculated from (10).

Calculating quadratures in (9) and (10) after simple transformations

$$M = fPD\cdot 4\frac{(\sin\alpha + \sin\beta)\cos\vartheta_0 - (\cos\beta - \cos\alpha)\sin\vartheta_0}{(2(\alpha+\beta) + \sin 2\alpha + \sin 2\beta)\cos\vartheta_0 - (\cos 2\beta - \cos 2\alpha)\sin\vartheta_0}\,,$$

$$\mathrm{tg}\,\vartheta_0 = \frac{\cos 2\beta - \cos 2\alpha}{2(\alpha+\beta) - \sin 2\alpha - \sin 2\beta}\,.$$

It is easy to ensure that $\vartheta_0 > 0$ if $\alpha > \beta$.

Excluding ϑ_0, finally we get

$$M = fPD\frac{2(\sin\alpha + \sin\beta) - (\alpha + \beta - \sin(\alpha+\beta))}{(\alpha+\beta)^2 - (\alpha+\beta)(\sin 2\alpha + \sin 2\beta) - \sin^2(\alpha+\beta)}\,. \tag{13}$$

In order to guarantee the correctness of obtained result the inequality performing should be provided in (12)

$$q(\vartheta) \ge 0\,, \quad \vartheta \in (-\alpha, \beta)\,.$$

This inequality is performed if $\cos(-\alpha - \vartheta_0) \ge 0$. Hence

$$\alpha \ge \frac{\pi}{2} - \vartheta_0$$

or

$$\mathrm{tg}\,\alpha \ge \mathrm{ctg}\,\vartheta_0 = \frac{2(\alpha+\beta) - \sin 2\alpha - \sin 2\beta}{\cos 2\beta - \cos 2\alpha} \tag{14}$$

are the conditions under which it is possible to use of formula (13) when $\alpha > \beta$. In other words β does not have to be less than α, then the area of contact covers the entire range $(-\alpha, \beta)$. In violated inequality (14) in some areas $(-\alpha, -\alpha_1)$ the contact is broken, then instead α in (13) it should be taken $\alpha = \alpha_1(\beta)$, where α_1 is the root of the transcendental equation

$$\operatorname{tg}\alpha_1 = \frac{2(\alpha_1+\beta)-\sin 2\alpha_1-\sin 2\beta}{\cos 2\beta-\cos 2\alpha_1}.$$

Let's finally consider the asymmetrical coupling with tension. In the absence of lateral displacements we accept $q(\vartheta)=\mathrm{const}$, $\vartheta\in(-\alpha,\beta)$. Then from (9)

$$M = fPD\frac{\alpha+\beta}{\sin\alpha+\sin\beta}. \quad (15)$$

To connect with an additional degree of freedom in the horizontal direction let's assume that the contact pressure is distributed linearly

$$q(\vartheta)=A+B\vartheta,\ \vartheta\in(-\alpha,\beta). \quad (16)$$

Then

$$M = fPD\frac{A(\alpha+\beta)+B(\beta^2-\alpha^2)/2}{A(\sin\alpha+\sin\beta)-B(\alpha\sin\alpha+\cos\beta)},$$

where from (10)

$$\frac{A}{B}=\frac{\sin\alpha+\sin\beta-\alpha\cos\alpha-\beta\cos\beta}{\cos\beta-\cos\alpha}>0.$$

Excluding A/B, finally we get

$$M = fPD\frac{\alpha+\beta}{2}\frac{2(\sin\alpha+\sin\beta)-(\alpha+\beta)(\cos\alpha+\cos\beta)}{2-2\cos(\alpha+\beta)-(\alpha+\beta)\sin(\alpha+\beta)}. \quad (17)$$

Summary table

Type of connection	Character of coupling	Contact pressure distribution	Friction torque
Symmetric	with a gap	concentrated force	$M = fPD$
	adjusted	cosine	$M = fPD\frac{4\sin\beta}{2\beta+\sin 2\beta}$
	with tension	uniform	$M = fPD\frac{\beta}{\sin\beta}$
Asymmetric without lateral displacement of semi-hubs	with a gap	concentrated force	$M = fPD$
	adjusted	cosine	$M = fPD\frac{4(\sin\alpha+\sin\beta)}{2(\alpha+\beta)+\sin 2\alpha+\sin 2\beta}$
	with tension	uniform	$M = fPD\frac{\alpha+\beta}{\sin\alpha+\sin\beta}$
Asymmetric with lateral displacement of semi-hubs	with a gap	concentrated force	$M = fPD$
	adjusted	cosine with displaced maximum	$M = fPD\frac{2(\sin\alpha+\sin\beta)-(\alpha+\beta-\sin(\alpha+\beta))}{(\alpha+\beta)^2-(\alpha+\beta)(\sin 2\alpha+\sin 2\beta)-\sin^2(\alpha+\beta)}$
	with tension	linear	$M = fPD\frac{\alpha+\beta}{2}\frac{2(\sin\alpha+\sin\beta)-(\alpha+\beta)(\cos\alpha+\cos\beta)}{2-2\cos(\alpha+\beta)-(\alpha+\beta)\sin(\alpha+\beta)}$

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Checking in (16) inequality $q > 0$ we find the condition of result (17) applicability

$$A - B\alpha \geq 0,$$

$$\alpha \leq \frac{\sin\alpha + \sin\beta - \alpha\cos\alpha - \beta\cos\beta}{\cos\beta - \cos\alpha}. \tag{18}$$

If α is too high and (18) is violated, then again in (17) it should be taken $\alpha = \alpha_1(\beta)$, where α_1 is the root of the transcendental equation that depends from β

$$\alpha_1 \leq \frac{\sin\alpha_1 + \sin\beta - \alpha\cos\alpha_1 - \beta\cos\beta}{\cos\beta - \cos\alpha_1}.$$

In this case the contact happens only on area $\vartheta \in (-\alpha_1, \beta)$.

Results. The final formulas for the calculation of limit values of moments that can be transmitted by connections with incomplete sweep of shaft are consolidated in Table

In the case of mutual displacement of components additionally by axial force Q (perpendicular to the plane of Figure) in the last column of the table M should be replaced by $\sqrt{M^2 + (QR)^2}$.

Conclusions. On the basis of a priori assumptions about the distribution of contact stresses there are determined an analytical dependences of boundary points and breakloose forces in detachable clip connection on spanning angles, bolt tightening force and tribological properties of details surfaces. The degree of adequacy of the results it is advisable to find out using numerical analysis of contact problems of elasticity theory.

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