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The Method of Trend Analysis of Parameters Time Series of Gas-turbine Engine State

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Abstract. This research substantiates an approach to interval estimation of time series trend component. The well-known methods of spectral and trend analysis are used for multidimensional data arrays. The interval estimation of trend component is proposed for the time series whose autocorrelation matrix possesses a prevailing eigenvalue. The properties of time series autocorrelation matrix are identified.

INTRODUCTION

The problem of improving diagnostic systems for power and energy plants based on gas-turbine engines of aircraft and general-purpose industrial design consists in enhancing statistical inferences concerning their technical state. This problem solving is achieved by developing the methods of trend analysis of the time series generated by gas-turbine engines registered parameters (state variables and output variables).

PROBLEM STATEMENT

The subject of this research are methods of trend analysis allowing to establish regularities of development of the trend component and its confidence intervals [2,4,6]. Known methods of trend monitoring [4,6] allow us just to see that a trend is absent at a predetermined significance level, since this is how the basic hypothesis is formulated [9]. Experience of trend monitoring methods application indicates an unacceptably high level of errors of both the first and second kinds. Methods of the trend analysis enabling direct isolation of a time series trend component provide an opportunity to improve the reliability of statistical conclusions about the technical condition of diagnosed objects. However, there remains the question of the trend selected component significance level and its valuation confidence interval.

The purpose of this study is to substantiate the approach to interval estimation of the trend component of a time series formed by the totality of the object registration parameters deviations from its diagnostic model (DM).

The central hypothesis of this research is the assumption that in the process of operation of complex energy facilities their characteristics gradual natural degradation occurs, which inevitably leads to the presence of long-term trend parameters in the recorded data. Therefore, only the reference state difference from DM shows changes in the technical state of the object.The subordinate hypothesis consists in the fact that the reliability of statistical inferences about the technical condition of the diagnosed object may be improved by the trend component interval estimate at the specified significance level.

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Following the well-known methods of trend analysis (SSA, "Caterpillar") [3], it is assumed that full information about the time series change is contained in the trajectory matrix X_t with a dimension of $n \times k$:

$$X_t = \begin{bmatrix} x_k & x_{k+1} & \dots & x_n \\ x_{k-1} & x_k & \dots & x_{n-1} \\ \dots & \dots & \dots & \dots \\ x_1 & x_2 & \dots & x_{n-k} \end{bmatrix}. \quad (1)$$

The following model of the trend and noise components is used as a statistical model of data generation [6, 7]:

$$\vec{x}_k = \left[x_k \ \ x_{k+1} \ \ x_{k+2} \dots x_{k+n-1} \right] = \vec{x}_{tr} + \vec{x}_{noise}. \quad (2)$$

The problem of the matrix (1) analysis consists in division of its rows (2) at a given significance level. The said problem is solved by successive implementation of the following stages:

1. The problem is solved for the matrix eigenvalues (1)

$$X_t X_t^T \vec{u}_i = \lambda_i \vec{u}_i, \quad (3)$$

where columns $\vec{u}_i$ form the matrix U from orthogonal vectors of the matrix $X_t X_t^T$.

2. The matrix of principal components of the time series is determined [1,5]:

$$F = U^T X_t, \quad (4)$$

where its rows are arranged according to decrease of eigenvalues of matrix $X_t X_t^T$.

3. The matrix (1) rows are expanded in terms of theprincipal components (4)

$$\vec{x}_s = \sum_{i=1}^{k} b_{si} \vec{f}_i, \quad (5)$$

where $s = \overline{1,k}$, b_{si} – interference ratios determined by solving the overdetermined $(n > k)$ system of linear algebraic equations:

$$F^T \vec{b}_s = \vec{x}_s. \quad (6)$$

4. Solution (6) is found with the use of pseudoinverse matrix [5]:

$$\vec{b}_s^T = \left(F F^T \right)^{-1} F \vec{x}_s^T. \quad (7)$$

Since $\left(F F^T \right) = diag\{\lambda_i\}$, $i = \overline{1,k}$, then it follows from (7) that

$$b_{sj} = \vec{x}_s \vec{f}_j^T \Big/ \left(\vec{f}_j \vec{f}_j^T \right) = \lambda_j^{-1} \vec{x}_s \vec{f}_j^T, \quad (8)$$

where $\vec{f}_j$ are the rows of the principal components matrix. If the eigenvalue $\lambda_1 = \lambda_{\max}$ corresponds to the trend component, then according to (8) we receive

$$\vec{x}_{tr,s} = b_{s1}\vec{f}_1. \tag{9}$$

After selection of the trend component (9), statistical properties of the remainder $\vec{x}_{noise}$ are defined. If the residual time series meets the belonging criteria of the sample from the general totality of normally distributed random variables at a predetermined significance level, then the row division (2) is correct. If, at the same time, the root-mean-square deviation of the residual sample does not exceed the measurement errors, then the row division corresponds to the physical features of the processes.

According to the experience of practical application [6,7,8] of this approach of a time series trend component selection, if the series includes a statistically significant trend, then the latter is bound to the maximum eigenvalue of the matrix of autocorrelations $X_t X_t^T$, which is significantly greater than the other eigenvalues. In this regard, as a hypothesis, it may be assumed that this condition (a significant difference of the first eigenvalue from all others) can be deemed the actual condition of the presence of a statistically significant trend in the sample.

To confirm the proposed hypotheses and determine the conditions of its application, let us first consider the following

Lemma. *If the elements r_{ij} of matrix R are such that*

$$r_{ii} = 1, i = \overline{1,k},$$
$$r_{ij} = r, i \neq j,$$

then its eigenvalues are as follows:

$$\lambda_1 = 1 + (k-1)r,$$
$$\lambda_i = 1 - r,$$

and the elements of the first eigenvector $\vec{u}_1$, corresponding to the first eigenvalue are equal to each other: $u_{11} = u_{12} = u_{13} ... = u_{1k}$.

Consequence. *Should the condition of normalization of $\vec{u}_1\vec{u}_1^T = 1$ satisfied, the elements of the first eigenvector $\vec{u}_1$, that corresponds to the first eigenvalue are equal to* $u_{11} = u_{12} = u_{13} ... = u_{1k} = 1/\sqrt{k}$.

Proof. For the purpose of proof, let us consider equation $\det(\lambda E - R) = 0$ and establish that the following equations are correct for the Lemma conditions

$$SpR = \sum_{i=1}^{k} \lambda_i, \qquad \det R = \prod_{i=1}^{k} \lambda_i.$$

It is evident that the first condition is satisfied:

$$SpR = \sum_{i=1}^{k} \lambda_i = \lambda_1 + (k-1)\lambda_i = 1 + (k-1)r + (r-1)(1-r) = k.$$

The second condition means that

$$\det R = \prod_{i=1}^{k} \lambda_i = [1 + (k-1)r](1-r)^{k-1}. \tag{10}$$

If we subtract the first row of R matrix from its second row, the latter from the third row, etc., the following matrix will be the result

$$\begin{bmatrix} r-1 & 1-r & 0 & 0\ldots & 0 \\ 0 & r-1 & 1-r & 0\ldots & 0 \\ \ldots & & & & \\ r & r & r & r\ldots & 1 \end{bmatrix}$$

At this, the matrix determinant is not changed. Let us add all columns of the obtained matrix to the first one:

$$\begin{bmatrix} 0 & 1-r & 0 & 0\ldots & 0 \\ 0 & r-1 & 1-r & 0\ldots & 0 \\ \ldots & & & & \\ 1+(k-1)r & r & r & r\ldots & 1 \end{bmatrix}$$

Upon transformation of the obtained matrix determinant by the first column, the final result will look as follows:

$$\det R = [1+(k-1)r](1-r)^{k-1},$$

which is precisely (10).

Since the sum of elements of any row of the matrix R is equal to the first eigenvalue, the elements of the first eigenvector $\vec{u}_1$, corresponding to the first eigenvalue are equal to each other. The Lemma is proved.

Theorem. *Let us consider a given time series and its trajectory matrix (1). If the trajectory matrix satisfies the hypothesis of its rows equicorrelation, the first component (9) of the time series centred by the first principal component is a moving average of this series.*

Such a hypothesis is disproved at the given significance level of the distribution χ^2 by crucial statistics of correlation of features (rows of the trajectory matrix).

If, in addition, the noise component satisfies the conditions of sampling from the general totality of independent normally distributed random variables, the confidence interval of the trend component is determined by the interval estimate of the average.

Proof. It is needed. Let the matrix of autocorrelations $X_t X_t^T$ belong to a class of matrices R. Then

$$\vec{f}_1 = \vec{u}_1 X_t, \qquad b_1 = \lambda_1^{-1}\vec{x}_s \vec{f}_1^T.$$

As $u_{11} = u_{12} = u_{13} = \ldots = u_{1k} = 1/\sqrt{k}$, then $\vec{f}_1 = \sqrt{k}\cdot mean(X_t)$. From (9) we get
$\vec{x}_{tr,s} = b_{s1}\vec{f}_1 = k\lambda_1^{-1}[(mean(X_t))^T \vec{x}_s]mean(X_t)$.

It is easily seen that $k[(mean(X_t))^T \vec{x}_s] = \lambda_1$, according to the Lemma conditions. Consequently, $\vec{x}_{tr,s} = mean(X_t)$.

Sufficiency follows from the singleness of solution of the linear algebraic equations for the first eigenvector corresponding to the first eigenvalue

$$(\lambda_1 E - X_t X_t^T)\vec{u}_1 = 0.$$

If the normalized matrix of autocorrelations $X_t X_t^T$ does not belong to the class of matrices R, but there is evidence to suggest statistical validity of such supposition, then the known [1] statistics of features correlation is to be used.

The correlation hypothesis (normalized matrix of autocorrelations $X_t X_t^T$ belongs to the class of matrices R) is refuted when the following condition is fulfilled:

$$\zeta = (A - BC)(n-1)/(1-r)^2 > \chi^2(\alpha, N), \quad (11)$$

where α is the level of significance, $N = (k+1)(k-1)/2$ is the number of freedom degrees,

$$A = \sum_{\substack{i,j=1 \\ i \neq j}}^{k} \left(r_{ij} - r\right)^2, \; B = \left(\sum_{i=1}^{k} \left(r_i - r\right)^2\right), \; C = r\left(2-r\right)\left(k-1\right)^2 / \left[k - \left(k-2\right)\left(1-r\right)^2\right],$$

$$r = \left(\sum_{\substack{i,j=1 \\ i \neq j}}^{k} r_{ij}\right) / \left(k\left(k-1\right)\right), \; r_i = \left(\sum_{\substack{j=1 \\ i \neq j}}^{k} \left(r_j\right)\right) / \left(k-1\right),$$

and r_{ij} – the elements of the normalized autocorrelation matrix $X_t X_t^T$.

As the experience of practical application shows, statistics (11) corresponds to the real time series, if the average row correlation coefficient is close to 1 (as is the case in the example considered in [1]) or to 0 (the row is not correlated, *i.e.*, a trend is absent). In the case where the average row correlation coefficient has an intermediate value, the Student's test of equality of the average with respect to the rows of the normalized autocorrelations matrix may be used.

To assess the conditions of the proposed approach applicability, solved were the applied problems of interval estimation of the trends of the GTE state parameters when the said engine is a part of aircraft power plants and gas compressor units.

The problem of assessing the technical condition of a three-shaft gas-turbine engine as part of a gas compressor unit in its normal operation with a duration of ~18 months (operating time 5.650 hours [6]) was set and solved. The source of input data is a database of the following parameters recording: speed of turbines of high and low pressure N_{HP}, N_{LP}, pressure of the high pressure compressor P_{HPC}, temperature behind the low-pressure turbine T_{LPT}*, and the pressure and temperature at the engine inlet P_{IN} and T_{IN}*.

The results of the above studies are presented in Figures 1, 2, 3, and 4. The interval assessment was made at a confidence level of 0.95 with a window of 20 terms of series. Figures 1 and 2 show the results of interval estimation of the trends of deviations from the diagnostic model.

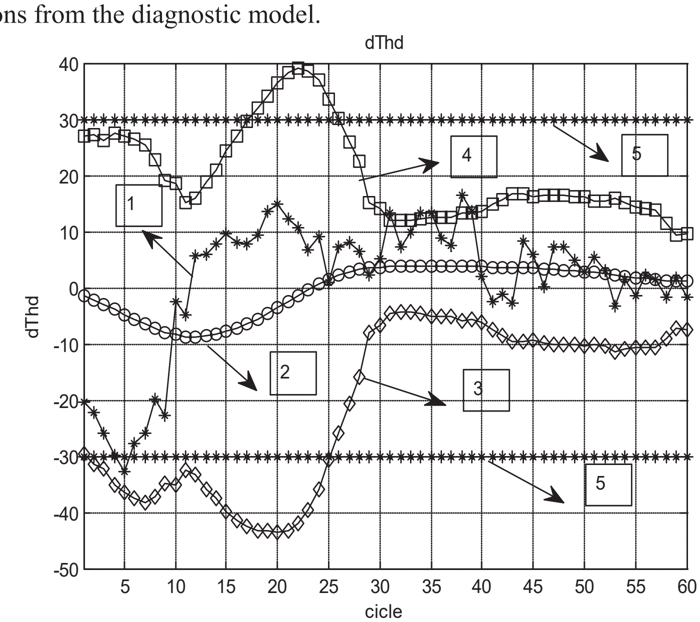

FIGURE 1. Results of processing the temperature time series: 1 – sample of deviations; 2 – sample trend; 3, 4 – confidence intervals; 5 – set limits

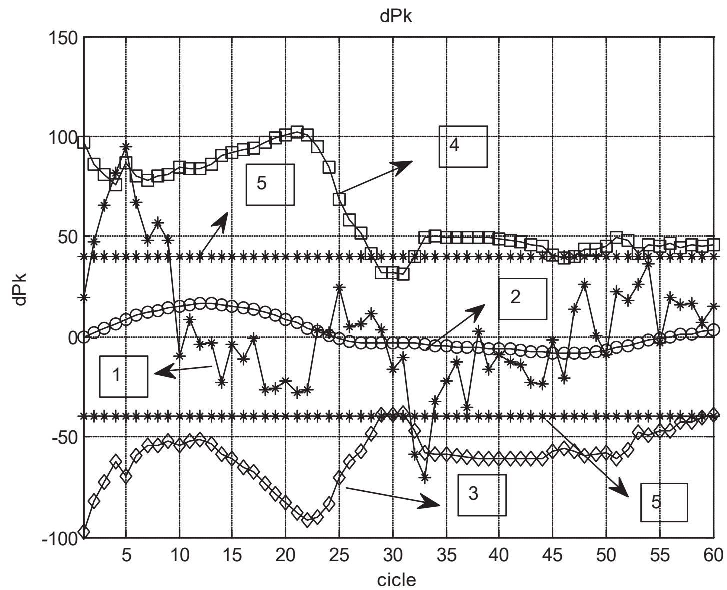

FIGURE 2. Results of processing the pressure time series: 1 – sample of deviations; 2 – sample trend; 3, 4 – confidence intervals; 5 – set limits

As it follows from the above illustrations, at a given confidence level one can argue that the sample estimate of the trend is within established boundaries. Figures 3 and 4 present the results of interval estimation of the trends of the deviations from the diagnostic model. The achieved results allow to formulate the following statistical conclusion: at a given confidence level the hypothesis that the sample estimate of the trend will be out of range should be rejected.

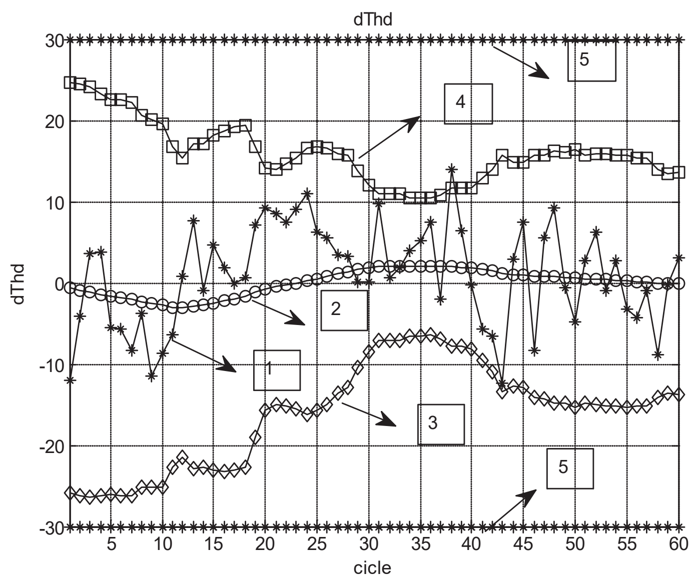

FIGURE 3. Results of processing the temperature time series: 1 – sample of deviations; 2 – sample trend; 3, 4 – confidence intervals; 5 – set limits

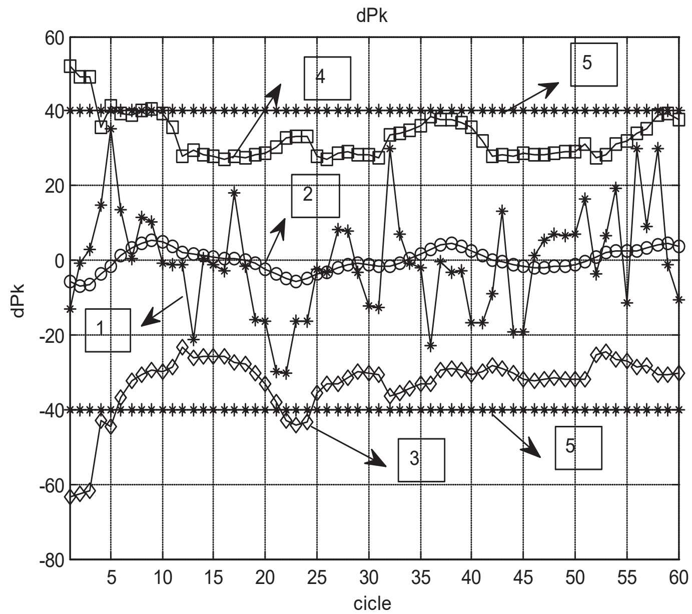

FIGURE 4. Results of processing the pressure time series according to the diagnostic model: 1 – sample of deviations; 2 – sample trend; 3, 4 – confidence intervals; 5 – set limits

The databases of recording the technical condition of the sustainer propulsion system (SPS) of the aircraft Il-76 with a two-shaft engine over a long-term operation [6] were analysed. In accordance with the approach described in [6], analysed were series of registration parameters deviations from polynomial regression models of variables interdependencies in steady modes (static characteristics). The results of applying the proposed approach to SPS is illustrated in Figure 5, which shows the sample, trend and its interval estimation for a time series of deviations from the polynomial static model of the pressure increase behind the compressor.

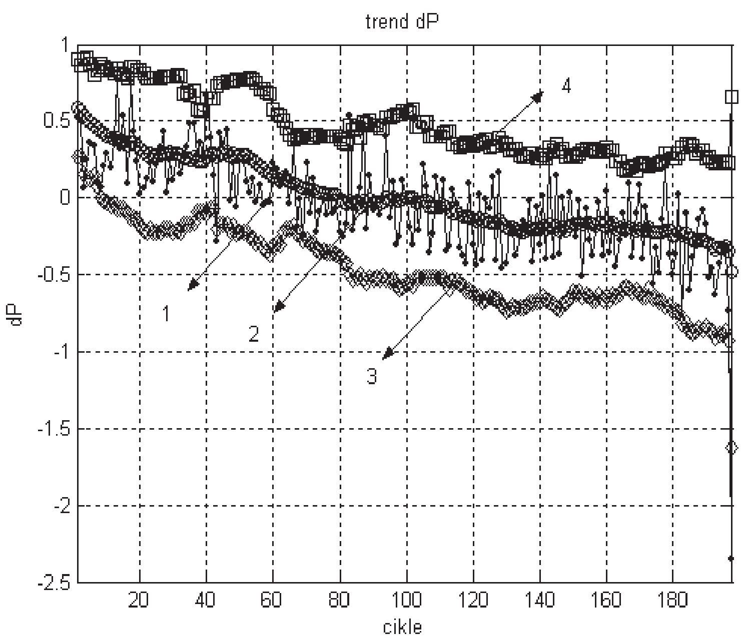

FIGURE 5. Pressure parameters: 1 – sample of deviation from DM, 2 – trend, 3 – lower confidence interval, 4 – upper confidence interval

The trajectory matrices of a number of engine parameters, which were recorded for 218 flight cycles (over a year) were analysed. The trend analysis window was 20 flight cycles.

The above Theorem is confirmed for all analysed time series. Figure 6 illustrates the predominance of the first eigenvalue in the distribution of the time series autocorrelations matrix eigenvalues of the pressure parameters behind the compressor, and Figure 7 presents the distribution of the first eigenvector elements.

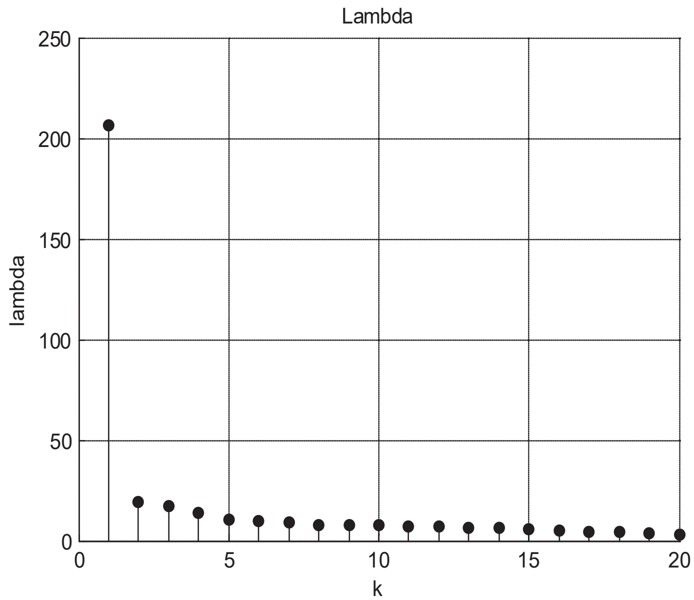

FIGURE 6. Distribution of eigenvalues

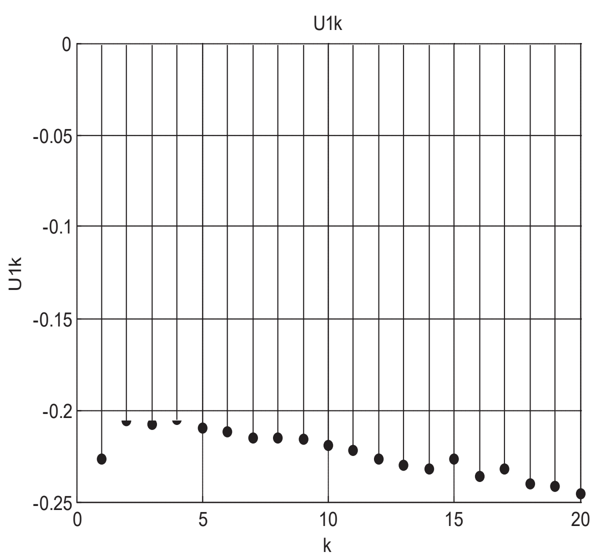

FIGURE 7. Distribution of elements of the first eigenvector

Student's test of equicorrelation of the trajectory matrix rows is confirmed at a significance level of 0.05 for the analysed time series. The root-mean- square deviation of the trend obtained in (9) and from the moving average amounted to 0.45%. The confidence intervals of the trends were determined for a probability of 0.9999. The analysed parameters are characterised by the extension of confidence intervals in the final phase of SPS operation that may be a significant diagnostic sign.

Thus, estimation of the trend component by the method of moving average has no statistically significant difference as compared with the assessment carried out by known methods. However, the computational complexity of such approach is significantly less and enables setting the trend component confidence intervals.

CONCLUSIONS

The combination of the method of trend components selection and that of probabilistic evaluation of its confidence intervals enables increasing the reliability of diagnostic conclusions on the technical condition of complex power facilities. The methods were combined by forming multidimensional arrays out of the studied object technical condition recorded data and their analysis by the method of principal components. For the time series whose autocorrelation matrix has a prevailing eigenvalue, the interval estimation of a trend component was found.

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