



cogwheel heat content. The temperature field at the heating stage is described by a mathematical dependence, which is a solution of a one-dimensional differential equation of heat conductivity. To determine the temperature  $T_H(x, \tau_H)$  at the heating stage, we can use the equation [5]

$$T_H(x, \tau_H) = \frac{2q}{\lambda} \sqrt{a\tau_H} \operatorname{ierfc} \frac{x}{2\sqrt{a\tau_H}} + T_0, \quad (1)$$

in which by denoting  $\frac{x}{2\sqrt{a\tau_H}} = \xi$ , we remind the well-known relations

$$\operatorname{ierfc} \xi = \left[ \frac{1}{\sqrt{\pi}} \exp(-\xi^2) - \xi \operatorname{erfc} \xi \right]; \operatorname{erfc} \xi = 1 - \operatorname{erf} \xi;$$

$$\operatorname{erf} \xi = \frac{2}{\sqrt{\pi}} \int_0^{\xi} \exp(-\varepsilon^2) d\varepsilon.$$

In the formula (1)  $q$  is the heat flux density ( $\text{W}/\text{m}^2$ ),  $a$  stands for the temperature conductivity ( $\text{m}^2/\text{s}$ ),  $\lambda$  for the heat conductivity ( $\text{W}/(\text{m}\cdot\text{K})$ ),  $x$  for the dimensional coordinate along the depth of the surface layer (m),  $\tau_H = 2h_H/V_f$  for the maximum dimensional heating time at the heating stage (s),  $V_f$  for the velocity of the source in the direction of the  $z$  axis (axial feed or velocity of the part, m/s),  $h_H$  for the maximum value of the

$$\begin{aligned} T_C(x, t_C) = & \int_0^{\infty} \left[ \frac{1}{2\sqrt{\pi a t_C}} \left\{ \exp\left(-\frac{(x-x')^2}{4a t_C}\right) + \exp\left(-\frac{(x+x')^2}{4a t_C}\right) \right\} - \right. \\ & \left. - A \exp(a t_C A^2 + A(x+x')) \times \operatorname{erfc}\left(\frac{x+x'}{2\sqrt{a t_C}} + A\sqrt{a \cdot t_C}\right) \right] f(x') dx' + \\ & + aA \int_0^{t_C} \left[ \frac{\exp\left(-\frac{x^2}{4a(t_C - \tau_C)}\right)}{\sqrt{\pi a(t_C - \tau_C)}} - A \exp(aA^2(t_C - \tau_C) + Ax) \times \right. \\ & \left. \times \operatorname{erfc}\left(\frac{x}{2\sqrt{a(t_C - \tau_C)}} + A\sqrt{a(t_C - \tau_C)}\right) \right] \varphi(\tau_C) d\tau_C, \end{aligned} \quad (3)$$

in which

$$f(x') = \frac{2q\sqrt{a\tau_H}}{\lambda} \left[ \frac{1}{\sqrt{\pi}} \exp\left(-\frac{x'^2}{4a\tau_H}\right) - \frac{x'}{2\sqrt{a\tau_H}} \operatorname{erfc}\left(\frac{x'}{2\sqrt{a\tau_H}}\right) \right] + T_0,$$

where  $\tau_H = 2h_H/V_f$  stands for the maximum heating time at the heating stage (s),  $t_C$  for cooling time (s);  $A = \frac{\alpha_h}{\lambda}$  for the reduced heat transfer coefficient,  $\alpha_h$  for heat transfer coefficient ( $\text{W}/(\text{m}^2\cdot\text{K})$ ),  $\varphi(\tau_C)$  for the starting temperature of the lubricoolant ( $^{\circ}\text{C}$ ), which can vary over the cooling time interval  $\tau_C$ , and  $0 \leq \tau_C \leq t_C$ .

provisional value  $h$  ( $0 \leq h \leq h_H$ ) at the heating stage, that is, the actual half-width of the real heat source (m),  $T_0$  is the initial temperature of the machining workpiece (room temperature, constant value).

The density of the heat flux  $q$  is obtained by averaging the instantaneous value of this parameter  $q(r_x)$ , and taking into account for each point of the involute profile with an instant radius vector [6]

$$q(r_x) = e_c \psi \frac{dQ}{dS_c} = \frac{P}{V_f S_{cc}} \psi \frac{V_f t_n(r_x)}{\sqrt{D t_v(r_x)}}, \quad (2)$$

where  $e_c$  and  $Q$  are the specific grinding energy and material removal rate (in  $\text{J}/\text{mm}^3$  and  $\text{mm}^3/\text{s}$ ),  $S_c$  stands for the contact area ( $\text{m}^2$ ),  $P$  for the grinding power (W),  $S_{cc}$  for the cross-section area in the grinding wheel movement direction ( $\text{m}^2$ ),  $\psi$  for the share of heat into the workpiece,  $t_n(r_x)$  and  $t_v(r_x)$  are the normal and vertical depths of cutting at an involute profile separate point (m),  $D$  is the instant diameter of the grinding wheel in the considered cross-section of its profile (m).

To determine the temperature at the cooling stage, which follows immediately after heating, with the initial conditions obtained during the heating stage, the following equation can be used [5]:

## 2.2 Technique for decision making

Thus, with a known type and a method of supplying the lubricoolant, for controlling the temperature at the cooling time interval it is possible to control the  $\alpha_h$  coefficient value (convection coefficient), the magnitude of the output temperature of the lubricoolant  $\varphi(\tau_C)$ , and the grinding modes, that is, to regulate the vertical cutting depth  $t_v$  and the axial feed  $V_f$ . Moreover, the value of  $t_v$  affects the maximum heating temperature as well as the value of  $V_f$  determines the time of heating and cooling, which affects both on the  $T_H$  and the achieved temperature level  $T_C(x, t)$  at the end of the cooling stage in the “heating-cooling” cycle on each working stroke. Thus, under otherwise identical conditions (cutting speed, grinding wheel characteristics, lubricoolant kind, etc.),

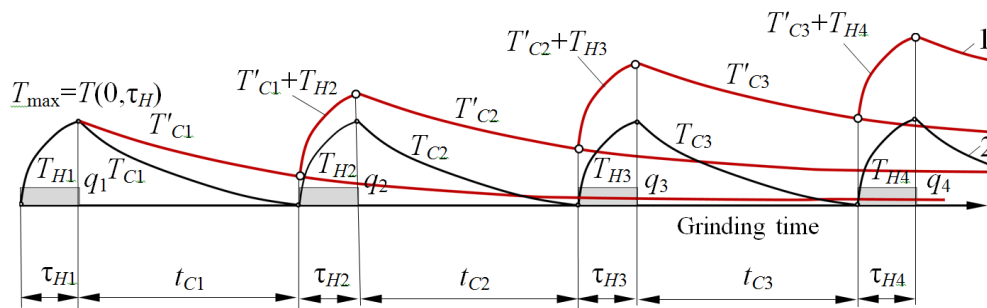


Figure 1– Grinding temperature changing with accumulation heat energy (line 1) and without it (line 2)

With a local increase in the temperature of individual grinded sections of the toothed surface, the temperature field is asymmetric in the symmetric body of the workpiece. This will lead to temperature ununiformed deformations of the heated sections, and, in consequence, to the cogwheel accuracy parameters deviations after the cogwheel cooling.

## 3 Results

### 3.1 Gear grinding with and without working stroke omission

There are two structures of the “heating-cooling” cycle: the up-and-down grinding without working stroke omission (Figure 2 a) and the only up grinding with the omission (Figure 2 b). In the cycle structure without working stroke omission the grinding wheel makes a working stroke with the length of  $l_1 + B + l_2$  ( $B$  is the width of the tooth rim,  $l_1$  and  $l_2$  are the grinding wheel approach and overtravel lengths), i.e. consistently passes the points 1-2-3 (up grinding), which are located in the beginning, middle and end of the length of the tooth rim (Figure 2 a). On the reverse working stroke, the grinding wheel makes reverse displacement (down grinding), i.e. consistently passes the points 3-2-1. In this case, the

the control and optimization parameters may include elements of cutting modes:  $t_v$  and  $V_f$ .

The cooling time  $0 \leq t_C \leq \infty$  is counted from the heating interval end. The task is to determine such mode parameters  $t_v$  and  $V_f$ , under which the heating stage with the surface temperature  $T_{max} = T(0, \tau_H)$  (Figure 1, line 1) will be changed by a cooling stage at which the temperature will change in the required manner (Figure 1, line 2). The control is to choose the grinding modes  $t_v$  and  $V_f$  which will result in the absence of heat accumulation (Figure 1, line 2):  $T_{C1}$ ,  $T_{C2}$ ,  $T_{C3}$  and  $T'_{C1}$ ,  $T'_{C2}$ , and  $T'_{C3}$  are necessary (completely cooled surface) and actual (not completely cooled surface) surface temperature dependence on time in the first, second and third working passes, respectively.

greatest amount of heating gets the point 3, because at this point, the cooling time is the smallest and is equal to

$$t_1 = \frac{2l_2}{V_f} \quad (4)$$

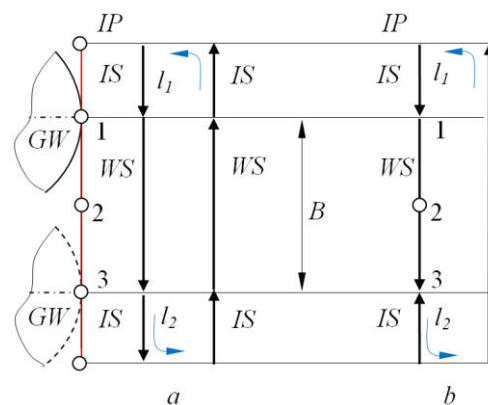


Figure 2 – The grinding cycle structures, in which GW is a grinding wheel; IP, IS and WS are the initial position, single and working strokes respectively

When repeating the “heating-cooling” cycle, the point 1 gets the greatest heating amount, because at this point, the cooling time is the smallest and is equal to

$$t_2 = \frac{2l_1}{V_f} \quad (5)$$

In the structure of the cycle with the working stroke omission, the grinding wheel makes a working stroke  $l_1 + B + l_2$  consistently passing the points 1-2-3 (up grinding), which belong to the beginning, middle and end of the tooth rim length (Figure 2 b). Before the reverse stroke the grinding wheel does not move radially to the cutting depth  $t_v$  that is this reverse stroke is idle and the grinding wheel makes idle stroke with the length of  $l_2 + B + l_1$  (points 3-2-1 without heating) getting to the initial position. When repeating the “heating-cooling” cycle, heating at the up grinding receives the point 1, that is at this point, the cooling time is the smallest and is equal to

$$t_3 = \frac{2l_1 + 2l_2 + 2B}{V_f} \quad (6)$$

### 3.2 Reduction and equalization of temperature

Assuming  $l_1 = l_2 = l$  for the structure with the working stroke omission we can see that cooling time according to formula (6) increases more than twice because the ratio of  $t_3/t_1$  is equal to the  $t_3/t_2$  and is  $2(2l + B)/2l = 2 + B/l$ .

Next, we introduce the following symbols for the surface temperature (i.e.  $x = 0$ ) for heating and cooling:  $T_H(0, \tau_H) = T_H$  and  $T_C(0, t_C) = T_C$ , respectively. For both structures of the cycle the influence of the axial feed  $V_f$  and the vertical depth of cutting  $t_v$  on the heating time  $\tau_H$  and the heating temperature  $T_H$  by the formula (1) and the cooling temperature  $T_C$  by the formula (3)

are established with the following initial data:  $x = 0$  (on surface);  $e_c = 50 \text{ J/mm}^3$ ;  $\psi = 0,8$ ; profile angle  $\alpha = 20^\circ$  or  $\frac{\alpha \pi}{180^\circ}$  rad;  $D = 400 \text{ mm}$ ;  $a = 5.68 \cdot 10^{-6} \text{ m}^2/\text{s}$ ;

$\lambda = 24 \text{ W}/(\text{m} \cdot \text{K})$ ;  $\alpha_h = 10\,000 \text{ W}/(\text{m}^2 \cdot \text{K})$ ;  $A = \frac{\alpha h}{\lambda} = 416,67 \text{ m}^{-1}$ ;  $T_0 = 0 \text{ }^\circ\text{C}$ ;  $\varphi(\tau_C) = 15 \text{ }^\circ\text{C}$ ;  $l_1 = l_2 = 7.86 \text{ mm}$ ;  $B = 24 \text{ mm}$ . The axial feed  $V_f$  varies in the range of 500...7000 mm/min, and the vertical depth of grinding  $t_v$  takes the following fixed values:  $t_v = 0.015 \text{ mm}$  (Tables 1, 3) and  $t_v = 0.074 \text{ mm}$  (Tables 2, 4). With increasing  $V_f$  for  $t_v = \text{const}$  (Tables 1, 2), the heating time  $\tau_H$  decreases and the heating temperature  $T_H$  increases. With the increasing  $t_v$  at the same values for  $V_f$  (pairs of Tables 1, 2, as well as Tables 3, 4), the heating time  $\tau_H$  and heating temperature  $T_H$  increase.

For the cycle with the working stroke omission (Tables 3, 4), the heating time  $\tau_H$  and the heating temperature  $T_H$  did not change compared with the previous structure (without working stroke omission), therefore,  $\tau_H$  and  $T_H$  in Table 3 and Table 4 are not given. There are both cooling time  $t_C$  and cooling temperature  $T_C$  changing at  $t_v = 0.015 \text{ mm}$  (Table 3) and  $t_v = 0.074 \text{ mm}$  (Table 4). The reason for the difference between the parameters  $t_C$  and  $T_C$  in cycles with and without working stroke omission is the only one - the increase in cooling time  $t_C$  in the cycle structure with working stroke omission.

Table 1 – Influence the  $V_f$  on the  $\tau_H$ ,  $T_H$ ,  $t_C$ , and  $T_C$  for the cycle structure without working stroke omission at  $t_v = 0.015 \text{ mm}$

$V_f$ , mm/min	500	1000	1500	2000	2500	3000	3500
$\tau_H$ , s	0.2939	0.1469	0.09798	0.07348	0.05879	0.04899	0.04199
$T_H$ , $^\circ\text{C}$	42.42	59.99	73.47	84.84	94.86	103.91	112.24
$t_C$ , s	1.8868	0.9434	0.6289	0.4717	0.3774	0.3145	0.2695
$T_C$ , $^\circ\text{C}$	11.598	12.002	12.789	13.903	14.906	15.606	16.556
$V_f$ , mm/min	4000	4500	5000	5500	6000	6500	7000
$\tau_H$ , s	0.03674	0.03266	0.02939	0.02672	0.02449	0.02261	0.021
$T_H$ , $^\circ\text{C}$	119.986	127.268	134.143	140.696	146.956	152.957	158.731
$t_C$ , s	0.23585	0.20964	0.18868	0.17153	0.15723	0.14514	0.13477
$T_C$ , $^\circ\text{C}$	17.494	18.416	19.317	20.205	21.072	21.922	22.755

Table 2 – Influence of  $V_f$  on  $\tau_H$ ,  $T_H$ ,  $t_C$ ,  $T_C$  for cycle structure without working stroke omission at  $t_v = 0.074$  mm

$V_f$ , mm/min	500	1000	1500	2000	2500	3000	3500
$\tau_H$ , c	0.65287	0.32644	0.21762	0.16322	0.13057	0.10881	0.09327
$T_H$ , °C	140.428	198.595	243.228	280.856	314.006	343.977	371.537
$t_C$ , c	1.88678	0.94339	0.62893	0.4717	0.37736	0.31446	0.26954
$T_C$ , °C	18.61	26.35	33.541	40.264	46.604	52.618	58.348
$V_f$ , mm/min	4000	4500	5000	5500	6000	6500	7000
$\tau_H$ , c	0.08161	0.07254	0.06529	0.05935	0.05441	0.05022	0.04663
$T_H$ , °C	397.19	421.284	444.072	465.747	486.456	506.32	525.433
$t_C$ , c	0.23585	0.20964	0.18868	0.17153	0.15723	0.14514	0.13477
$T_C$ , °C	63.832	69.097	74.168	79.065	83.805	88.402	92.869

Table 3 – Influence the  $V_f$  on the  $t_C$  and  $T_C$  for the cycle structure with working stroke omission at  $t_v = 0.015$  mm

$V_f$ , mm/min	500	1000	1500	2000	2500	3000	3500
$t_C$ , c	9.53357	4.76678	3.17786	2.38339	1.90671	1.58893	1.36194
$T_C$ , °C	12.604	11.976	11.657	11.494	11.426	11.42	11.458
$V_f$ , mm/min	4000	4500	5000	5500	6000	6500	7000
$t_C$ , c	1.1917	1.05929	0.95336	0.86669	0.79446	0.73335	0.68097
$T_C$ , °C	11.529	11.624	11.738	11.867	12.008	12.158	12.317

Table 4 – The same as Table 3 at  $t_v = 0.074$  mm

$V_f$ , mm/min	500	1000	1500	2000	2500	3000	3500
$t_C$ , c	9.53357	4.76678	3.17786	2.38339	1.90671	1.58893	1.36194
$T_C$ , °C	13.57	14.342	15.457	16.726	18.078	19.477	20.902
$V_f$ , mm/min	4000	4500	5000	5500	6000	6500	7000
$t_C$ , c	1.1917	1.05929	0.95336	0.86669	0.79446	0.73335	0.68097
$T_C$ , °C	22.339	23.78	25.218	26.651	28.074	29.487	30.889

Let's perform a comparison of the cooling temperatures for two grinding cycle structures at  $t_v = 0.015$  mm (Fig. 3, a) and  $t_v = 0,074$  mm (Fig. 3, b). There are three ways to achieve the lowest cooling temperature  $T_C = 11.6^\circ\text{C}$  for  $t_v = 0,015$  mm (Fig. 3, a): 1) when grinding without working stroke omission with axial feed  $V_f = 0.5$  m/min (point A), 2) when grinding with working stroke omission with axial feed  $V_f = 1.8308$  m/min (point B, and 3) the latter at  $V_f = 4226.9$  mm/min (point C).

There are two ways to achieve the lowest cooling temperature  $T_C = 18.6^\circ\text{C}$  for  $t_v = 0.074$  mm (Figure 3 b) when grinding without working stroke omission with axial feed  $V_f = 0.5$  m/min (point D; 2) when grinding with working stroke omission with an axial feed  $V_f = 2.6444$  m/min (point E).

The time to machine by gear grinding both without and with working stroke omission can be determined by the following formulas, respectively

$$T_M = \left( \frac{B + l_1 + l_2}{V_f} \frac{z_{\max}}{t_v} + \frac{T_{IND}}{60} \right) z; \quad (7)$$

$$T_O = \left( \frac{B + l_1 + l_2}{V_f} 2 \frac{z_{\max}}{t_v} + \frac{T_{IND}}{60} \right) z, \quad (8)$$

where  $B$  stands for the width of the tooth rim ( $B = 24$  mm),  $z$  for the number of cogwheel teeth ( $z = 40$ ),  $l_1 = l_2 = 7.86$  mm,  $z_{\max}$  for the grinding stock for machining a cogwheel in the finish (third) stage ( $z_{\max} = 0,1$  mm),  $T_{IND}$  for the indexing time (cogwheel angular turning for one tooth,  $T_{IND} = 4$  s).

To ensure the lowest cooling temperature  $T_C = 11.6^\circ\text{C}$ , the minimum time to machine is equal to 7.632 min (Table 5) which is obtained in the with working stroke omission cycle structure at  $t_v = 0.015$  mm and  $V_f = 4226.9$  mm / min.

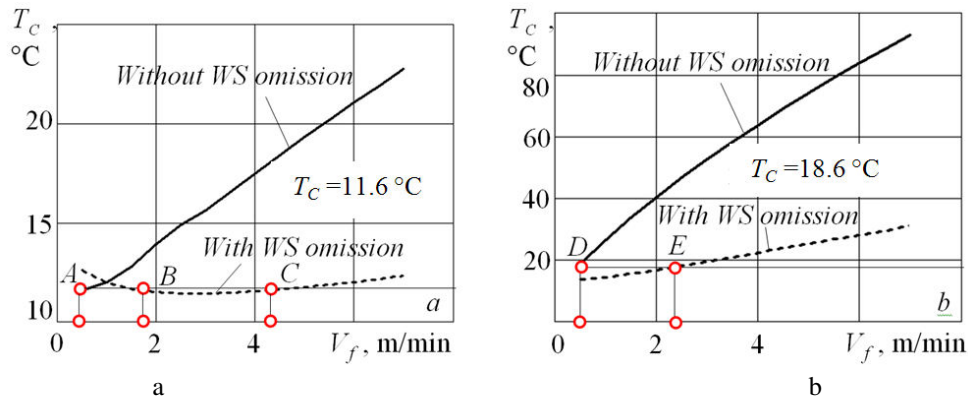


Figure 3 – Cooling temperature  $T_C$  vs axial feed  $V_f$  for different grinding cycle structures at  $t_v = 0.015$  mm (a) and  $t_v = 0.074$  mm (b)

Table 5 – Determining the time to machine in gear grinding

Grinding cycle structure of the	Without working stroke omission	With working stroke omission	
The lowest cooling temperature $T_C = 11.6$ °C at $t_v = 0.015$ mm			
Grinding conditions	$V_f = 500$ mm/min (point A)	$V_f = 1830.8$ mm/min (point B)	$V_f = 4226.9$ mm/min (point C)
Time to machine, min	23.852	14.238	7.632
The lowest cooling temperature $T_C = 18.6$ °C at $t_v = 0.074$ mm			
Grinding conditions	$V_f = 500$ mm/min (point D)	$V_f = 2644.9$ mm/min (point E)	
Time to machine, min	6.961	4.29	

To provide the cooling temperature  $T_C = 18.6$  °C (more than  $11.6$  °C), the minimum time to machine  $T_M = 4.29$  min (Table 5) is obtained in the cycle structure with working stroke omission at  $t_v = 0.074$  mm and  $V_f = 2644.9$  mm/min. From these five variants (points A, B, C, D, and E in Figure 3) we choose the variant with the minimum value  $T_C = 11.6$  °C (points A, B, C), since the theoretical value is  $T_C = 0$ . It can be seen (Table 5) that the minimum time to machine  $T_M = 7.632$  min is obtained in the cycle structure with working stroke omission at  $t_v = 0.015$  mm and  $V_f = 4226.9$  mm/min ( $T_C = 11.6$  °C).

Thus, the study of the gear grinding temperature models (1) and (3) both at the heating (1) and cooling (3) stages allowed to recommend the working stroke omission in the structure of this operation and to assign the appropriate gear grinding modes for the finish (third) gear grinding stage both for  $t_v = 0.015$  mm (points A, B, C in Figure 3) and  $t_v = 0.074$  mm (points D, E).

## 4 Conclusions

As the depth of gear grinding  $t_v$  increases the maximum cooling temperature  $T_C$  increases as well. In the gear grinding cycle structure with working stroke omission cooling time  $t_C$  is greater than that without working stroke omission. As the axial feed  $V_f$  increases the cooling temperature  $T_C$  for the cycle structure with working stroke omission at  $t_v = 0.015$  mm and  $t_v = 0.074$  mm does not practically change, while for the cycle structure without working stroke omission the cooling temperature  $T_C$  increases. The smallest cooling temperature  $T_C$  is reached at  $t_v = 0.015$  mm, so it is expedient to accept the minimum possible vertical depth of grinding  $t_v$  on the finishing (third) gear grinding stage, for example, for the finishing stage we take  $t_v = 0.015$  mm.

Moreover, in this stage, the axial feed  $V_f$  is chosen not only from the condition of maximum productivity (maximum  $V_f$ ), but also from the condition of surface roughness (in the interval of  $V_f$  from 1830.8 to 4226.9 mm/min) and the smallest elastic deformation in the gear grinding system.

## References

1. Kalashnikov, S. N. et al. (1990). *Proizvodstvo zubchatykh koles: Handbook*. Moscow, Mashinostroyeniye [in Russian].
2. Lishchenko, N. V. (2016). Opredeleniye intensivnosti zuboshlifovaniya na osnove analiticheskogo uravneniya evol'venty. *Suchasni tekhnologii v mashinobuduvanni*, Issue 11 [in Russian].
3. Dekapolitov, M. I. (2011). *Povysheniye effektivnosti profil'nogo zuboshlifovaniya tsilindricheskikh koles putem rascheta parametrov staticheskoy naladki: Ph.D. thesis*. Specialties 05.02.07 – Tekhnologiya i oborudovanie mekhanich. i fiziko-tekhnologicheskoy obrabotki; 05.02.08 – Tekhnologiya mashinostroyeniya. Moscow.
4. Nishimura, Yu., Katsuma, T., Ashizawa, Y., Yanase, Y., Masuo, K. (2008). Gear grinding processing developed for high-precision gear manufacturing. *Mitsubishi Heavy Industries, Ltd. Technical Review*, Vol. 45, No. 3, 33–38.
5. Carslaw, H. S., Jaeger, J. C. (1959). *Conduction of heat in solids (2nd ed.)*. London, Oxford University Press.
6. Larshin, V. P. (1999). Tekhnologiya mnogonitochnogo rez'boshlifovaniya pretsizionnykh khodovykh vintov. *Proceedings of Odessa National Polytechnic University*, Odessa, Issue 2(8), 87–91 [in Russian].

## Зниження та вирівнювання температури профільного зубошліфування

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**Аноація.** Розроблено методику визначення режимів зубошліфування на третьому завершальному етапі профільного зубошліфування на верстаті з ЧПК, виходячи із забезпечення вирівнювання температури по периферії і найменшого нагрівання зубчастого колеса. Для цього використано формули для визначення температури зубошліфування на етапі нагрівання та охолодження оброблюваної поверхні. На етапі нагріву температурне поле поширюється по глибині поверхневого шару і змінюється в часі. У момент закінчення етапу нагріву, миттєве температурне поле, яке загасає по глибині поверхневого шару, є початковою умовою для визначення температури на етапі охолодження. Тому температура охолоджувальної поверхні залежить не тільки від регульованого часу охолодження, але також від миттєвого розподілу температури в поверхневому шарі, яке враховується як початкова умова при моделюванні на етапі охолодження. Знайдено оптимальні умови охолодження, в тому числі за рахунок пропусків робочих ходів, які реалізовані зубошліфуванням без установки глибини різання.

**Ключові слова:** профільне зубошліфування, температура зубошліфування, етап нагріву, етап охолодження, структура циклу, пропуск робочого ходу, вирівнювання температури.