

UDC 681.5.015

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DISCRETE MATHEMATICAL STRUCTURES IN THE ANALYSIS AND SYNTHESIS OF PASSIVE AND ACTIVE VIBRATION ISOLATING DEVICES

Л. Бовнегра, К. Кіркопуло, А. Павличко, В. Літвінов, І. Сидоренко. Дискретні математичні структури при аналізі та синтезі пасивних та активних віброізоляційних пристроїв. У роботі розглянуті дискретні математичні структури, що застосовуються для аналізу та синтезу пасивних та активних віброізолюючих пристроїв. Показано, що підходи дискретної математики у вигляді кінематичних та їх подальшого розвитку у вигляді модифікованих кінематичних графів, дозволяють ефективно імітувати функціональну взаємодію між елементами таких пристроїв, описуючи їхню взаємодію відповідними кінцевими математичними структурами. Такий підхід до вирішення задач синтезу віброізолюючих пристроїв дозволяє ефективно проводити повний перебір можливих варіантів і є основою для переходу до проектування їх промислових зразків. У роботі наведено аналіз існуючих пасивних та активних віброізолюючих на основі якого, як приклад, проведено синтез принципово нового пасивного віброізолюючого пристрою з елементами активних систем. Основою синтезу є модифікований кінематичний граф, який уявляє собою результат перетину безлічі елементів модифікованих кінематичних графів відповідно пасивного та активного віброізолюючих пристроїв. Встановлено, що такий модифікований кінематичний граф має два цикли з одним полюсом і дугою, що визначає пружний зв'язок, які входять в обидва цикли. Показано, що отриманий модифікований кінематичний граф синтезованого пристрою швидко трансформується у відповідну структурну схему і може бути конструктивно реалізований у вигляді прототипу даного пристрою. Обґрунтовано, що синтезований у представленій роботі пристрій за своїми структурними ознаками займає проміжне місце в ієрархії віброізолюючих пристроїв між пасивними та активними.

Ключові слова: математичні структури, дискретні структури, кінематичний граф, модифікований кінематичний граф, синтез структури

L. Bovnegra, K. Kirkopulo, A. Pavlyshko, V. Litvinov, I. Sydorenko. Discrete mathematical structures in the analysis and synthesis of passive and active vibration isolating devices. The article deals with discrete mathematical structures used for the analysis and synthesis of passive and active vibration isolation devices. It is shown that approaches of discrete mathematics in the form of kinematic graphs and their further development in the form of modified kinematic graphs make it possible to effectively model the functional interaction between the elements of such devices, describing their interaction with the corresponding finite mathematical structures. Such an approach to solving the problems of synthesis of vibration isolating devices makes it possible to effectively carry out a complete enumeration of possible options and can be the basis for the transition to the design of their industrial samples. The paper presents an analysis of existing passive and active vibration isolating devices on the basis of which, as an example, a synthesis of a fundamentally new passive vibration isolating device with elements of active systems was carried out. The basis of the synthesis is a modified kinematic graph, which is the result of the intersection of the sets of elements of the modified kinematic graphs, respectively, of passive and active vibration isolation devices. It has been established that such a modified kinematic graph has two cycles with one pole and an arc that defines an elastic connection included in both cycles. It is shown that the resulting modified kinematic graph of the synthesized device is very quickly transformed into the corresponding block diagram and can be constructively implemented as a prototype of this device. It is substantiated that the device synthesized in the presented work, according to its structural features, occupies an intermediate place in the hierarchy of vibration isolation devices between passive and active ones.

Keywords: mathematical structures, discrete structures, kinematic graph, modified kinematic graph, structure synthesis

Introduction

Discrete mathematics is an important tool for solving problems of analysis and synthesis of technical systems. This is due to the fact that the problems of analysis and synthesis of most technical systems, which are described by advising discrete mathematical structures, are associated with the construction of specific algorithms in which efficiency, including in terms of computational complexity, is of great importance.

Synthesis of the design of a new vibration isolating device, however, like any other technical device, is the main stage on the way to its design implementation. At this stage, the presence of certain constituent elements in the device is theoretically substantiated, a generalized description of their functional interaction is carried out and requirements for each of them are established, and their influence on the properties of the synthesized device as a whole is determined. Since the same design of a vibra-

DOI: 10.15276/opu.1.67.2023.10

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tion isolating device can receive a different design transformation for specific conditions of its use, which determine its passive or active form, it is advisable to synthesize the design based on a structural analysis of some design prototype, and in some cases several structural prototypes. A constructive prototype is most often understood as a virtual model of a vibration isolation device, which has certain simplifications that do not affect the functional properties of the device. Existing 3D computer design tools, such as AutoCad, Inventor, Te-Flax, etc., allow creating not only computer models of mechanical systems, but also simulating the functional interaction between their elements, performing their kinematic and force calculations [1]. In this case, structural prototypes, as a rule, are final structures. Such structures are associated with the direction of discrete mathematics in the form of finite mathematics, the subject of which is finite graphs, finite groups, finite automata. Supplementing a 3D computer model of mechanical systems with the corresponding finite mathematical structures makes it possible to efficiently carry out a complete enumeration of possible synthesis options [2].

This approach, which determines the simulation study of technical structures, is the basis for the transition to the design of their industrial designs. The development of this approach is a solution to a rather important scientific and applied problem.

Literature analysis

When performing a structural analysis of technical systems in order to identify elements of the same purpose and possessing, as a consequence, the same properties, it is possible to identify elements whose functional purpose is determined only by the specifics of the considered structure. In this case, an additional study of these elements in terms of their functional appropriateness and possible replacement or combination with existing elements is required.

One of effective tools to solve the problems of analysis of structures is the apparatus of discrete mathematics in the form of graph theory, which allows you to model the relationship between elements of various natures, thereby determining the structural properties of the system under consideration [3, 4]. Models based on finite graphs are widely used as structural models of systems since such a model allows their idealized representation in the form of appropriate schemes with concentrated components and poles. In such schemes connection of components among themselves is realized in nodes, which are formed solely by combining the poles. Depending on the number of poles possessed by the considered concentrated component, there are two-pole and multi-pole components, which are called bipolar and multi-polar respectively. Typical representatives of physical systems which allow representation by finite graphs with concentrated components are mechanical structures. Included in their structure elastic elements, dampers, mechanical transmissions are displayed by bipolar poles, and engines – by multipolar poles. Thanks to its clearness and simplicity, this device has recently won wide recognition and widespread use [5].

For mathematical description of composition and structure of physical system two types of relations are usually used: the pole equations describing individual properties of each component without reference to possible connections with other components; the equations of connections reflecting character of connection of various components in the scheme without reference to their individual properties [6].

The pole equation of the bipolar is a functional dependence between two physical quantities characterizing its state (for example, force and movement of the mechanical bipolar). Function describing nonlinear bipolar can be defined by analytical expression, graph or table. Linear bipolar is characterized by parameter which is either constant value (stationary bipolar) or function of time (non-stationary bipolar).

A multipole is described by a system of equations linking the physical quantities at its poles. Often multipole components are represented by a circuit model consisting of bipolar components, each of which is described by a corresponding functional relationship, unlike conventional bipolar, such relationships may contain quantities associated with other components of the circuit model. Eventually, a physical system with concentrated components can always be represented by a circuit consisting of bipolar. In this case, the fundamental physical laws expressing the conditions of equilibrium and continuity (for mechanical systems – D'alambert principle) usually act as the equations of connections. In each specific case, these equations are derived from consideration of the scheme structure, and they should contain the same values as the component equations, which characterize the states of bipolar systems. Thus compatibility of initial equations which transformation allows to receive mathematical model of system in the required form is provided [7, 8].

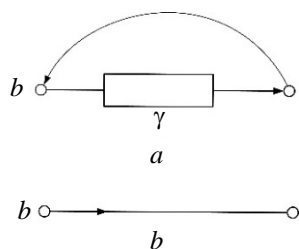


Fig. 1. Bipolar component: model (a); pole graph (b)

A circuit with bipolar components, regardless of its specific physical nature, can be represented by a pole graph. There is one-to-one correspondence between the circuit consisting of bipolar components and its graph: the nodes of the circuit correspond to vertices and the edges of the graph correspond to bipolar components. Orientation of an edge is connected with the direction of counting of physical quantities characterizing the bipolar state. The pole graph is a universal topological model of physical systems with concentrated components. The way to such model lies through idealization of the system (structural diagram) and its abstraction (polar graph). The main value of topological models is that their properties and methods of use can be studied and developed inde-

pendently from physical nature of systems [9]. The specificity of a particular problem is evident at the initial stage when constructing the graph and at the final stage of interpretation of the obtained results. For any two-pole (Fig. 1, a) its pole graph is an arc with two end vertices (Fig. 1, b). In the general case the bipolar equation $f(\gamma, \xi) = 0$ contains two variables γ and ξ . One of them, for example γ , characterizes the state of bipolar relative to the cross section and is oppositely directed to each of its poles. Such variables are called transverse variables (e.g. for mechanical systems this is a force or moment). Another variable ξ characterizes the state of the bipolar relative to its poles (for example, linear (angular) velocity or displacement). Such variables are called longitudinal and their directions are associated with the direction of the path from one pole to the other. Often transverse variables are called follow-up variables, and longitudinal variables are called parallel variables [10].

If the bipolar equation can be explicitly represented in relation to the transverse variable $\gamma = f_y(\xi)$, its corresponding arc is called y – arc, and the value of γ can be considered as a response to ξ . If a bipolar equation can be represented in the form $\xi = f_z(\gamma)$, then its corresponding arc is called z – arc, and the value ξ can be seen as a response to ξ . Bipolar that allow description with respect to both variables are called mutually determined, and their corresponding arcs are called w – arcs. Since of two variables γ and ξ one characterizes the impact and the other the reaction, their positive directions are considered to be mutually opposite. Usually, the directions of arcs are identified with positive directions of readings of transverse variables, and positive directions of readings of longitudinal variables are taken as inverse orientations of arcs.

The polar graph of the system is constructed so as to provide the simplest relations between its structure and the coupling equations [11, 12]. Usually the connection equations are formed for transverse and longitudinal variables in the following form:

- the algebraic sum of transverse variables for any vertex of the graph equals zero:

$$\sum \gamma(t) = 0, \tag{1}$$

- the algebraic sum of longitudinal variables for any vertex of the graph is zero:

$$\sum \xi(t) = 0. \tag{2}$$

When variables are summed algebraically, they are considered positive if their directions coincide with the chosen direction relative to the vertex or contour, and negative if the directions of the variables are opposite to the chosen directions.

The ideal passive bipolar of mechanical systems are resistance (dissipation), mass and stiffness, as well as the source of displacement (velocity) and the source of force (Fig. 2).

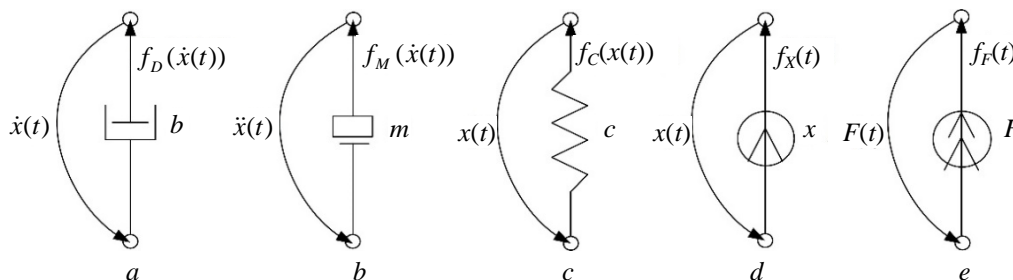


Fig. 2. Models of ideal mechanical (translational) two-terminal networks: dissipation (a); mass (b); rigidity (c); source of movement (d); source of power (e)

The displacement $x(t)$ and its time derivatives are longitudinal variables, and the force $f(t)$ is a transverse variable [12, 13].

Increasing technological requirements to modern equipment requires a significant reduction of negative manifestations associated with vibrations and oscillations. Therefore, it is of interest to apply the presented apparatus of discrete mathematics to the analysis of existing passive and active vibration isolating devices in order to find new, optimal design solutions.

Research methodology

Application of the presented apparatus of discrete mathematics for the analysis of the structure of a fairly common uniaxial passive vibration isolating device is based on a step-by-step transition from its 3D-model, including a fixed base 0 and a movable support 1 and a cylindrical compression spring with between them (Fig. 3, *a*), to the corresponding kinematic (structural) diagram, defining the considered device as a flat mechanism with the degree of movement $W=1$ (Fig. 3, *b*).

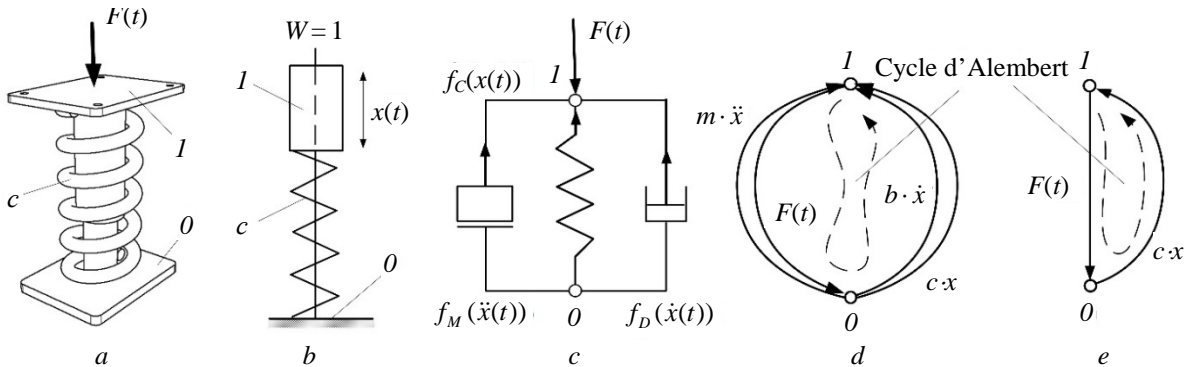


Fig. 3. Formation of the graph of a uniaxial passive vibration isolating device: 3D model (*a*); kinematic (structural) diagram (*b*); scheme based on ideal mechanical two-poles (*c*); bipolar graphs (*d, e*)

Such an approach makes it possible to describe the device in question quickly enough with the corresponding models of ideal mechanical bipolar (Fig. 3, *c*), on the basis of which the direct functional interaction of the elastic connection with fixed 0 and movable 1 parts can be described by elements of discrete mathematics in the form of a pole graph.

The pole graph corresponding to the model of ideal mechanical bipolar, being a logical continuation of the analysis allows us to determine the relationship between the links – poles 1 and 2 in the form of arcs, which form a cycle, which is called a d’Alembert cycle, since the interrelations forming it obey the well-known principle $\sum F(t) = 0$ – the algebraic sum of forces for any vertex of the pole graph is equal to zero (Fig. 3, *d*). If we abstract from inertial $m(\ddot{x})$ and dissipative $b(\dot{x})$ forces, the arcs of the graph define a direct relationship between the external disturbance from the object subject to vibration protection and the internal restoring elastic force generated by the elastic element of the device in the form $F(t) = cx$ (Fig. 3, *e*).

In contrast to passive vibration isolation devices, the structures of active vibration isolation systems have a complex organization. In addition to the basic elements 0 and 1 separated by a reduced elastic coupling, since the device generally uses several elastic elements, it contains control and management elements (Fig. 4, *a*).

The functional interaction between the controls is determined by the signal generated by the control element – sensor R . The signal is transmitted to the control system consisting of the signal amplifier 5 and the differentiating device 6 . In turn, the control system determines the magnitude and direction of the corrective movement of the system link 2 and sends a corresponding command to the electric motor 4 for the corresponding orientation of the link 3 . The presence in the structure of the active vibration isolation system of electronic elements and the motor implies that for their normal functioning it is necessary to have a sufficiently powerful energy source 7 . The transition from the structural diagram of the active vibration isolation system (Fig. 4, *b*) to its diagram based on ideal bipolar poles (Fig. 4, *c*) gives the corresponding pole graph (Fig. 4, *d*). The presence of the arc of the graph corresponding to the energy source characterizes the obtained pole graph of the considered active system as one-connected. The analysis of the graph shows that the arcs determining interrelations between its poles form two cycles: d’Alembert cycle between poles $0, 1, 2$ and control cycle formed by arcs between poles $0, 2, 3, 4, 5, 6$.

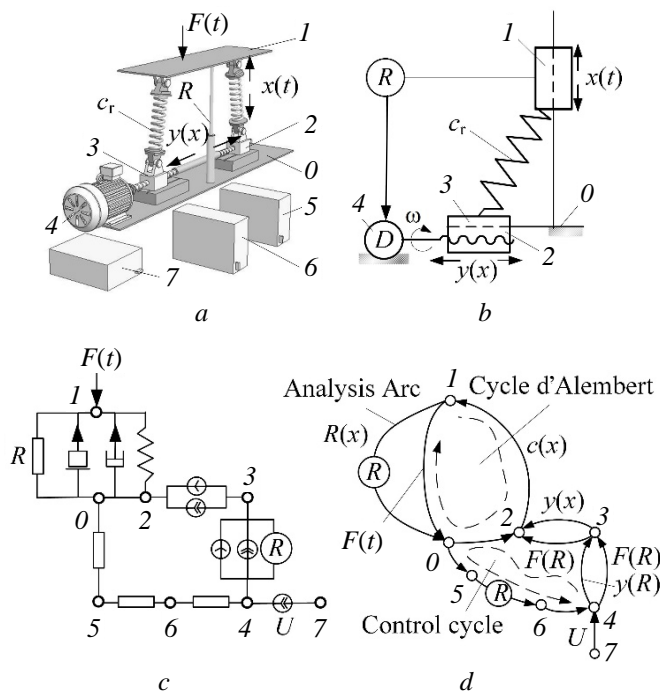


Fig. 4. The sequence of the formation of the graph of electro – mechanical active vibration isolation system: 3D model (a); block diagram (b); scheme based ideal two-terminal networks (c); bipolar graph (d)

It should be noted that in the case when link 2 is stationary, the relationship between poles 0 and 2 is absent, and the d’Alamber cycle of the active vibration isolation system becomes identical to the d’Alamber cycle of the passive vibration isolation device. The interrelationships in the control cycle have a mixed character and obey both d’Alamber’s principle (interrelationships between mechanical elements – poles 0, 2, 3) and Kirchhoff’s laws (interrelationships between electrical elements – poles 4, 5, 6). It is established that the functional purpose of link 2 is both the realization of the place of attachment of the elastic element and the realization of control of the elastic characteristics of the device in the form of some corrective motion. The organization of the above type of control of elastic characteristics is most often carried out on the condition that the elastic elements of the system are all-metal. It is for them, in contrast to non-metallic and combined elastic elements, that the directions of the force and the elastic deformation caused by it coincide and always lie in the same plane.

The analysis of the received pole graph of the considered electro-mechanical active vibration-isolating system allowed to reveal and functional identity of two interrelations determined by the arc of the graph $R(x)$ and the arc of the graph $c_r(x)$ between poles 0 and 2. The identity lies in the fact that the change in both the sensor resistance and the value of strain of the elastic connection determine the change in the same parameter – the external force impact. On this basis, the hypothesis is put forward that the change in the magnitude of the elastic coupling strain can be considered not only as a controlled parameter, but can also be used as a corrective motion after a certain kinematic transformation. The hypothesis put forward is quite logical based on the type of the controlled signal – displacement $x(t)$ and the expected result – the corrective displacement $y(t)$, which determines the change in the elastic characteristics of the device.

Results

In order to substantiate a formalized design algorithm for a passive vibration isolation device with elements of active systems to control its elastic characteristics, several basic principles for the synthesis of such a device are formulated. As the first principle, it is accepted that a passive vibration isolator with elements of active systems can be synthesized only on the basis of all-metal elastic elements, for which, in contrast to non-metal and combined elastic elements, the directions of force and elastic deformation caused by it coincide, and the directions of elastic deformation and correction motion lie in one plane. The second principle, based on the provisions of the first one, limits the spatial realization of the synthesized structure and defines it as a flat mechanism with elastic links whose mo-

bility is $W=1$. The third principle defines the presence of some value of elastic bond deformation, which can be used without transformation as a controllable parameter that reflects the intensity of external vibration and shock effects on the synthesized device. The implementation of the elastic control function by a feedback loop based only on the mechanical elements constituting a closed kinematic chain designed to convert the controlled motion into a corrective one is the fourth principle. The fifth and final principle is that the corrective displacement realized in the closed mechanical feedback loop results in an additional deformation of the elastic coupling elements or a change in their orientation with respect to the external load line.

The above principles can be the basis for a formalized algorithm for analysis and synthesis of a passive vibration isolator with elements of active vibration protection systems. As a base in the proposed algorithm one can use the improved technique of analysis of kinematic schemes of plane mechanisms by means of kinematic graphs, proposed by R.V. Ambartsumyants [14]. In case of applying kinematic graphs, the degree of mobility of a planar mechanism with rigid links, which corresponds to the previously mentioned second principle of the formalized algorithm, can be determined by the expression:

$$W = 3(p - 1) - 2q_5 - q_4, \tag{3}$$

where p is the number of poles of the graph corresponding to the number of rigid links; q_5 is the number of arcs of the graph corresponding to kinematic pairs of class 5; q_4 is the number of arcs of the graph corresponding to kinematic pairs of class 4.

In this paper it is suggested that expression (3) can be used for the analysis of structures of plane mechanisms with elastic links, which corresponds to the first and second presented principles, or of plane mechanisms with links with links of non-mechanical nature. However, application for these purposes of expression (3) is possible with some modification which will take into account not only kinematic pairs characterizing contact of links but also elastic, dissipative links and links of mixed nature: electro-mechanical, electric, pneumatic, etc. that determine functional interrelations between links.

To approve this proposal, consider the previously presented structural diagram of a single-axis passive vibration isolator (Fig. 3, a). When modeling it with kinematic graph, a graph is obtained, which contains two poles ($p=2$) and one arc ($q_5=1$) (Fig. 5, a).

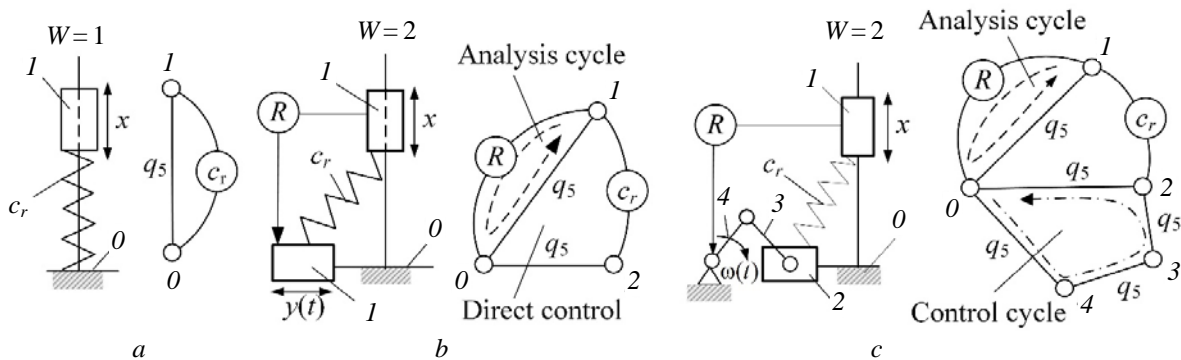


Fig. 5. Modeling of vibration isolating devices by modified kinematic graphs: passive vibration isolating device (a); active vibration isolation system with direct control (b); active anti-vibration system with automatic control (c)

Then, according to expression (3), the degree of mobility is $W=3(2-1)-2\cdot 1=1$. However, in a real passive vibration isolator, its elements do not move until a certain external force is applied. Therefore, if there is no external action, any vibration isolator can be regarded as a conditionally stationary system ($W=0$). In this case, the stationary state of such a system, in addition to the kinematic state, is also determined by the elastic coupling. In this connection, it was proposed to modify the previously presented kinematic graph by adding an arc q_{cr} defining the elastic coupling. Moreover, the properties of this arc determining mobility restrictions are assumed to be similar to the properties of a kinematic pair of class 4 since the elastic coupling admits a similar number of mobility when solving a planar kinematic problem. Taking this into account, the degree of mobility of a uniaxial passive vibration isolator according to the modified kinematic graph is:

$$W = 3(p - 1) - 2q_5 - q_4 - q_{cr} = 3(2 - 1) - 2\cdot 1 - 0 - 1 = 0. \tag{4}$$

Consider the graph of an active vibration-isolating system in the version where the elastic control system is implemented as a linear motor with an electronic control system (Fig. 5, *b*). The mechanical part of the system consists of fixed part *0* and movable part *1*. The elastic coupling is located between the movable part *1* and the linear motor 2. The kinematic graph of such system consists of 3 poles ($p=3$) and two arcs $q_5=2$. According to expression (3) the degree of mobility of the system in question is equal to $W=3(3-1)-2\cdot 2=2$.

However, the presence of two links, namely the elastic link c_r and the electronic control system in the form of the link of non-mechanical nature R , which are defined by the arc q_{cr} between poles of graph 1 and 2 and the arc q_R between poles of graph 0 and 1, allows to consider the active vibration isolating system as conditionally motionless system. Analyzing the derived modified kinematic graph we can distinguish the analysis cycle of external perturbation bounded by the arcs between the poles 0 and 1 and also the direct kinematic control of elastic characteristics in the form of the arc between the poles 0 and 2. The analysis cycle and the control arc have one pole 0 in common, corresponding to the stationary part of the device. It should be noted that the arc defining the elastic relation is not part of the control loop and is the outer arc of the graph. A similar result is obtained when analyzing the structure of an active vibration isolation system with a more complex organization of the elastic control system. The mechanical part of the system consists of stationary 0 and movable 1 parts. The elastic link is located between the movable part 1 and the link 2. Links 2 and 3 form a group of corrective links which are moved by rotation of electric motor 4 (Fig. 5, *c*). Modified kinematic graph of the system in question consists of 4 poles ($p=4$) and five basic arcs $q=5$. According to expression (3), the degree of mobility. $W=3(5-1)-2\cdot 5=2$. As in the previous case, the elastic connection cr and the electronic control system in the form of non-mechanical connection R define the system in question as conditionally motionless. In the obtained modified kinematic graph one can distinguish not only the analysis cycle but also the control cycle, which have one common pole 0 corresponding to the stationary part of the device.

Summarizing the obtained results, the condition for the modified kinematic graph defining the vibration isolator in which the control of its elastic characteristics is implemented is formulated. This condition is the presence of two cycles with one common pole and an arc included in both cycles defining the elastic relation in the non-single-connected modified kinematic graph.

The fulfillment of this condition is the basis for synthesis of a new vibration isolating device with extended functionality, which will be illustrated by the following example. To simplify the synthesis procedure, let us assume that the synthesized device having some elastic coupling in its structure will be constructed using some number of links and only kinematic pairs of class 5. In this case the corresponding modified kinematic graph will include one arc $q_{cr}=1$ and there will be no arcs q_4 in it. Then the zero value of expression (4) which defines conditional immobility of the device corresponds to the certain number of poles and arcs of the modified kinematic graph. For the considered example the number of poles and arcs of such graph $p=4$ and $q_5=4$ correspondingly. Consequently, the graph of the synthesized device besides the poles corresponding to the necessary fixed 0 and movable parts 1 of the device between which there is an elastic connection, must contain two more poles – the corrective links 2 and 3. As it was mentioned before, all poles of the graph are connected by arcs corresponding to kinematic pairs of class 5.

The required solution in the form of organizing two cycles with a common pole 0 and a common arc corresponding to the elastic coupling is quite obvious and has only one option (Fig. 6, *a*).

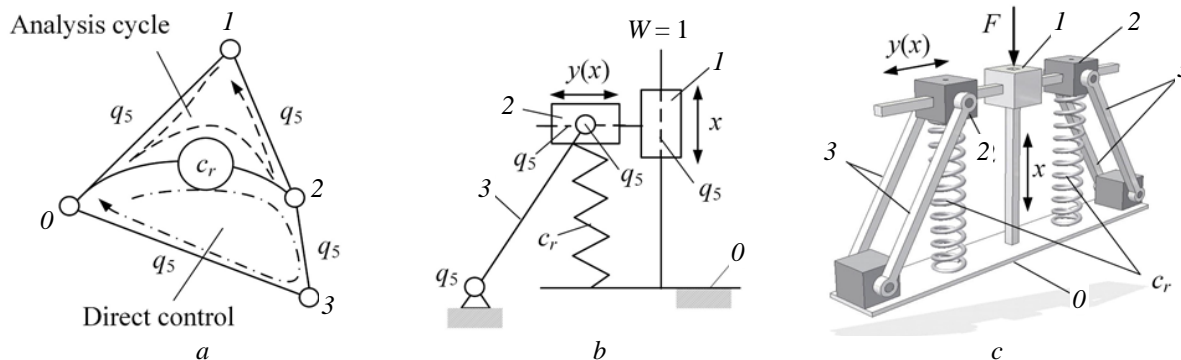


Fig. 6. Synthesis of a passive vibration isolating device with extended functionality: modified kinematic graph (*a*); block diagram of the device (*b*); device prototype (*c*)

The resulting graph allows easy transition to a structural diagram (Fig. 6, *b*) and a structural prototype (Fig. 6, *c*). Thus, as an example, an autonomous hybrid structure of a passive vibration isolator of a new type was synthesized. The synthesized structure, being passive, is self-adapting. Depending on the magnitude of the external action that determines the magnitude of the displacement $x(t)$, the structure implements the possibility of transforming this displacement into the corresponding corrective displacement $y(t)$ that determines the change in the elastic characteristics of the device.

Conclusions

On the basis of this research, the following conclusion is drawn:

1. Discrete mathematics is an important tool for solving problems of analysis and synthesis of technical systems, because its apparatus, in the form of graph theory, allows modelling the relationship between elements of diverse nature, thereby determining the structural properties of the technical system in question.
2. Modelling of real mechanical objects by schemes of discrete mathematics in the form of bipolar components, which are represented by a pole graph, allows revealing certain dependences at interaction between elements of both passive and active technical systems.
3. It has been established that in the non-single-connected modified kinematic graph which defines the vibration isolating device with controllable elastic characteristics there are necessarily two cycles with one pole and an arc which defines elastic connection and which are included into both cycles.
4. The modified kinematic graph with the presence of two cycles with one pole and an arc defining the elastic coupling, entering into both cycles, allows obtaining a structural scheme of a passive vibration isolator with controlled elastic characteristics.
5. Considering the evolution of vibration isolation devices in terms of functional interaction, number and type of control elements of elastic characteristics, it is found that the device synthesized in the presented work takes an intermediate place between passive vibration isolation devices and active vibration isolation systems by its features.

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Received February 05, 2023

Accepted March 27, 2023