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DYNAMIC MODEL OF A PRODUCTION ASSEMBLY LINE AS AN ABSORBING MARKOV CHAIN

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Assembly lines are generally accepted means of mass and serial production, and even used for the manufacture of small-batch products for individual orders. They are flow-oriented production systems. When (re)setting up an assembly line, the important task of balancing it always arises. It originated with the beginning of the use of assembly lines and consists in the rational distribution across jobs of the total workload required for the manufacture of a unit of production. This expedient helps to increase the capacity of the assembly line with some reduction in the requirements for the qualification of the workforce. During the first forty years of the assembly lines, only trial and error methods were used for balancing. Next other balancing methods were developed for different structures and operating conditions of the lines. The article proposes a method for controlling and regulating the incomplete production of a production line, which consists in the periodic redistribution of workers in workplaces, that is, in the redistribution of the incomplete production (in the opposite direction). The purpose of regulation is the approximate alignment of the generation of workplaces and the increase in productivity of the entire line, assuming that the workers do not enter the production line from the outside. The specific problem of the assembly line stability is solved by the methods of the Markov chain theory. A model of the assembly line as an absorbing Markov chain has been constructed. The statement proved in the work formulates the conditions under which the shop management can even out the workstations of the production line with a given accuracy using the reserve labor force. The represent of the production line by the Markov chain allows one to find the only number of workers that must be redistributed between workplaces in order to achieve the goal, and the required number of redistributions per shift.

Keywords: production assembly line, modeling, Markov chain, regulation, stability.

Introduction

In the cybernetic direction of scientific development, the problems of the production organization and control were initially distinguished by three trends [1]: the use of the theory of autoregulation methods, the use of the theory of information systems with feedback methods, biophysical modeling. Later, to study specific aspects of the production systems, including the production lines (PL) of the assembly, other non-named models were proposed, for example, stochastic models based on network representations [2], which can describe the assembly lines functioning in time and when describing to remove unrealistic assumptions about the deterministic duration of technological transitions. During the first forty years of the assembly lines, only trial and error methods were used for balancing. Later, other balancing methods were developed for different structures and operating conditions of the lines. [3,4,5]. Heuristic methods aimed at minimizing costs were considered [6], criteria for physical demand [7] and even individual orders [8] were introduced. The proposed methods range from simple to fairly complex and time consuming, involving, for example, the use of genetic algorithms [9].

Throughout the extensive literature on this topic, assembly PL have not been considered as an absorbing Markov chain, even when methods of operations research were used to analyze production lines. Reviews of balancing methods [eg, 10-12] and analysis of the stability of the balance of submarines [13] do not contain a reference to such a model. Further it is shown how to represent assembly lines by Markov chain. The representation is used to solve a problem related to the stability of assembly lines.

Content statement of the problem

PL assembly consists of linked workplaces (WP). At any moment taken as the initial one, each WP has a certain amount of work in progress, including zero. Due to a change in the efforts of workers and for other reasons, non-completed production moves from one WP to another. If the movement is not controlled, the uneven distribution of work in progress on the WP in separate periods of time can lead to a decrease in the productivity of the line.

Consider one way to control and regulate: in each period of time the master of the workshop adds workers to the “narrow” WP, removing them from the points of the line with zero work in progress. The number of those added and withdrawn are the same in all periods, so that in fact there is a redistribution of existing workers. The goal of the shop management in this case is the approximate alignment of the WP production, which improves the performance of the entire line.

The first task in the study of the issue raised is to find the conditions under which the shop management will be able to achieve their goals. Secondly, to achieve the goals, if possible, it is necessary to find the number of workers that should be withdrawn and transferred in each period, and the required number of periods per shift.

The main (and real) assumption in these related tasks is that the number of workers is limited from above by the allocated staff, workers do not receive from outside to the production line. For a still closer approach to reality, let us assume that the number of workers on a line is not forbidden to decrease for a certain period. This can be represented as a consequence of disability. We will also assume that it is possible to move labor from any WP to any other. To realize this possibility, organization of parallel WP is required.

Mathematical wording

It can be considered that redistributing additional labor between the WP, the master redistributes (in the opposite direction) the work in progress. We represent the amount of unfinished production by vector strings, the *i*-th component of which corresponds to the unfinished production at the *i*-th WP. Then $\vec{e} = \|e_i\|$ sets the incomplete production at various points of the production line at the present time; $\vec{c} = \|c_i\|$ – incomplete production, added (and withdrawn) by the master in each period of time; $\vec{g} = \|g_i\|$ – the goal to which the master aspires. It is clear from the definition that $\vec{e}$ and $\vec{g}$ are non-negative, and $\vec{c}$ contains positive and negative components. If positive, this means an increase, and if negative, a decrease in uncompleted production (an increase in labor force) at the *i*-th WP.

Let p_{ij} denote the share of the *j*-th WP that has switched to the *i*-th WP during the period. Since $0 \le p_{ij} \le 1$, we can consider p_{ij} as probabilities (most often, within the framework of the stated conditions, the values $p_{ij} = 0$ and $p_{ij} \cong 0.5$ will occur). This allows you to apply the theory of Markov chains. The “states” of the chain are WP; the possibility of transfer of labor between any pair of WP means that these states are communicating.

If the additional labor force is constant, then $\sum_i p_{ij} = 1$ and the matrix $P = \| p_{ij} \|$ is the matrix of transition probabilities of the ergodic chain. In the accepted assumptions, labor can decrease, therefore we denote $P_п = \| p_{ij} \|$ and add a fictitious WP that collects all the disappearing labor. Then we obtain an absorbing chain with one absorbing state for which the ergodic hypothesis [3] does not hold. Nevertheless, we first consider what the distribution of work in progress will be over *n* periods for the ergodic case under the assumption that the matrix P_n does not change during many transitions.

Since the initial amounts are given by the vector $\vec{e}$, then after one period on the *i*-th WP there will be $\sum_j e_j \cdot p_{ij}$ of work in progress. This means that the vector $\vec{e}P$ specifies the distribution of work in progress in one period; similarly, $\vec{e}P^2$ sets it in two periods, and $\vec{e}P^s$ – in *s* periods. Thus, the labor force invested in the production line at the beginning changes the distribution of the object of labor to $\vec{c}\,P^s$ after s periods, the investment in the first period changes the distribution to $\vec{c}\,P^{s-1}$ and so on. Consequently, after s periods the total amount of work in progress at the production line will be

$$\vec{m} = \vec{e}P^s + \sum_{k=0}^{s} \vec{c}P^k. \quad (1)$$

The first requirement for the total amount of work in progress is that the vector (1) must be non-negative for any s. With this restriction, it is necessary to provide $\vec{m} \to \vec{g}$.

The calculations for the case of the absorbing chain are formally similar. Since we are only interested in the labor force inside the production line, $P_п$ plays the role of P in expression (1), and by analogy we arrive at the following:

$$\vec{m}_п = \vec{e}P^s_п + \sum_{k=0}^{s} \vec{c}P^k_п. \quad (2)$$

Again it is necessary to require that $\vec{m}_п$ be non-negative for s any and converge to the goal $\vec{g}$.

Stability conditions for production line

When studying Markov absorbing chains, it is convenient to bring the matrix of transition probabilities to the canonical form

$$P = \begin{Vmatrix} I & 0 \\ R & P_п \end{Vmatrix},$$

when absorbing states are written first and form a unit matrix I, the remaining elements belong to the zero matrix 0, and R and $P_п$ – non-negative matrices, including the probabilities of transitions from non-absorbing states. For the final absorbing chain $P^s_п \to 0$, because the process will necessarily fall into the absorbing state. It is also true [4] that the stronger statement is that the infinite series $L = I + P_п + P^2 + \ldots$ always converges to the limit $(I - P_п)^{-1}$, therefore the fundamental matrix

$$L = (I - P_п)^{-1} = I + P_п + P^2 + \ldots$$

plays an important role.

After this, it is obvious that the first member of vector (2) tends to 0, and the second – to $\vec{c}L$. Therefore, to achieve the goal, it is necessary to fulfill the condition with $\vec{c}L = \vec{g}$, which means that if the goal $\vec{g}$ is attainable, then there is a single vector

$$\vec{c} = \vec{g}(I - P_п), \quad (3)$$

leading to a goal. Substituting in the expression (2) instead of $\vec{c}$ its value (3), we get

$$\vec{m}_п = \vec{e}P^s_п + \vec{g}(I - P_п) \sum_{k=0}^{s} P^k_п = \vec{e}P^s_п + \vec{g}(I - P_п^{s+1}) \geq 0$$

or

$$(\vec{g}P_п - \vec{e})P^s_п \leq \vec{g} \qquad (\text{for } s \geq 0). \quad (4)$$

It is tediously in practice to check the entire mass of conditions recorded, so the following statement should be useful, limiting the number of conditions to be checked.

S t a t e m e n t. If, after k periods of redistribution of labor, work in progress chosen as a goal in any WP is not less than the labor intensity given by the vector $(\vec{g}P_п - \vec{e})P^k_п$, then (4) is feasible for all $s \geq k$.

P r o o f. Denote by $\vec{r} = (\vec{g}P_п - \vec{e})P^k_п$. Then $(\vec{g}P_п - \vec{e})P^s_п = \vec{r}P^{s-k}_п$, and the elements of the matrix $P_п$ of any degree do not exceed unity. Therefore, the components of the vector $\vec{r}P^{s-k}_п$ are bounded above by the value $\sum_i |r_i|$. Consequently, if the inequality $\sum_i |r_i| \le \min g_i$ (or $\sum_i |r_i| \le g_i$, if the PL synchronization is perfect) is right, then the expression (4) is valid for $s \ge k$, and since $P^k_п \to 0$, then $r \to 0$. Hence, for sufficiently large k holds $\sum_i |r_i| \le \min g_i$, and the assertion is proved.

Now we have an effective procedure for checking conditions (4) in a reasonable number of steps. We calculate $(\vec{g}P_п - \vec{e})P^s_п$ for all s, starting with $s = 0$. If inequality (4) is violated for any s, then the goal $\vec{g}$ is unachievable. But if there is a vector $(\vec{g}P_п - \vec{e})P^s_п$ satisfying the condition of the statement, then it is never violated and, therefore, $\vec{g}$ is achievable. Often, calculations can be stopped earlier, and namely: if the calculated vector does not exceed the previous one, and expression (4) is true for any k.

We will now solve the second of the assigned tasks – we will find what labor must be transferred to the PL during this period with the attainable goal $\vec{g}$. If $\vec{1}$ is a column vector, all elements of which are equal to one, we can find $\vec{c}\vec{1}$, and this equals $\vec{g}(I - P_п)\vec{1}$. Let $\vec{q} = (I - P_п)\vec{1} \geq 0$. Then the total sum should be $\vec{g}\vec{q}$. This means that the column vector $\vec{q}$ turns the goal into labor, which must be moved along the PL each period. Since the working force disappears from the system in a random absorbing chain, it is not surprising that $\vec{c}\vec{1} > 0$. This means, however, that $c \ge 0$, since $\vec{g}(I - P_п)$ has a negative component.

An interesting feature of solution (3) for PL is that $\vec{c}$ does not depend on the vector of the initial work in progress $\vec{e}$.

Conclusion

It is difficult to expect that a single model could describe all aspects of the functioning of a system. And in the absence of comprehensive, particular models should be applied for different categories of questions regarding the same system. In the light of such an approach to modeling, the proposed representation of production lines for assembly by Markov chains should help clarify the processes occurring in the lines and, ultimately, their design.

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ДИНАМІЧНА МОДЕЛЬ ПОТОКОВОЇ ЛІНІЇ СКЛАДАННЯ ЯК ПОГЛИНАЮЧОГО МАРКОВСЬКОГО ЛАНЦЮГА

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Складальні лінії є загальноприйнятим засобом масового і серійного виробництва і навіть застосовуються для виготовлення продукції невеликого обсягу за індивідуальними замовленнями. Вони представляють собою орієнтовані на потік виробничі системи. При (пере)налаштуванні складальної лінії завжди виникає важлива задача її балансування. Вона з'явилася в переліку задач з початком використання складальних ліній і полягає в раціональному розподілі по робочих місцях повного робочого навантаження, необхідного для виготовлення одиниці продукції. Цей інструмент допомагає підвищити пропускну спроможність складальної лінії при деякому зниженні вимог до кваліфікації робочої сили. Протягом перших сорока років існування складальних ліній для балансування використовувалися тільки методи проб і помилок. Потім були розроблені інші методи балансування для різних структур і умов роботи ліній. У статті запропоновано спосіб контролю і регулювання незавершеного виробництва потокової лінії, що полягає в періодичному перерозподілі робочих по робочих місцях, тобто в перерозподілі незавершеного виробництва (в зворотному напрямку). Мета регулювання – наближене вирівнювання вироблення робочих місць і підвищення за рахунок цього продуктивності всій лінії при допущенні, що ззовні робочі на потокову лінію не надходять. Конкретна задача стійкості лінії складання вирішується методами теорії марковських ланцюгів. Побудована модель потокової лінії складання як поглинаючого марковського ланцюга. Доведене в роботі твердження формулює умови, при яких керівництво цеху може з заданою точністю вирівняти вироблення робочих місць потокової лінії, використовуючи резервну робочу силу. Подання потокової лінії марківськім ланцюгом дозволяє знайти ту єдину кількість робочих, яка повинна перерозподілятися між робочими місцями для досягнення мети, і потрібне число перерозподілів за зміну.

Ключові слова: потокова лінія складання, моделювання, марківський ланцюг, регулювання, стійкість.