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Information support to solve direct dynamic problem for the previously disturbed electromechanical systems

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ABSTRACT

The article is devoted to the creation of methodological foundations for solving a direct problem of dynamics for linear dynamic systems, the motion of which is described by ordinary differential equations with nonzero initial conditions. Consideration of the motions of linear dynamic systems allows to simplify the mathematical apparatus used and to solve motion determination problems by using a known approach based on transfer functions. However, due to the fact that the classical definition of transfer functions does not involve taking into account non-zero initial conditions, which are caused by the presence of initial deviations of the coordinates of the control object from their desired values, in our work we use the Laplace-Carson transformation to find the corresponding images and write the equations of motion in operator form. This approach, in contrast to the generally accepted one, led to the introduction of information about the initial conditions of motion in the right-hand side of the corresponding operator differential equations and necessitated the generalization of the vector of control signals by including in it components that take into account the initial conditions of motion of the system under consideration. Such transformations made it possible to generalize the concept of a matrix transfer function as a matrix linear dynamic operator, which consists of two components that define disturbed free and controlled forced movements. The use of such an operator makes it possible to study the dynamics of the considered linear system both separately for each of the components of the generalized vector of controlling influences, and in the complex, thus solving the direct problem of the dynamics of linear systems.

As an example, we show the use of the proposed approach for motion analysis of a DC motor with nonlinear fan friction based on its piecewise linearized model.

Keywords: Information support, dynamical system; direct dynamic problem; transfer function; initial states; matrix methods; linear differential operator; Laplace-Carson transformation

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INTRODUCTION

It is difficult to imagine the current stage of human development without using control theory developments. These developments are widespread in various branches of industry [1], transportation [2], [5], technology [3], science [4], communications [6], and much more.

Modern developments in control theory are based on some control methods which allow us to study control system motions [7, 8], [9], stability of these motions [10, 11], and design controllers to form the desired motions [12, 13], [14]. The problem of controllers' design is compounded by the presence of external and internal disturbances and/or uncertainties.

That is why a lot of methods and approaches to designing closed-loop control systems are developed [15, 16], [17, 18], [19, 20], [21]. One of them is based on the intellectual control paradigm and allows us to design controllers by intellectual control methods such as neuro and fuzzy control [22, 23], [24].

The main problem with these approaches using is quite a big error which is caused by subjective factors and the main feature of these approaches is not necessary to know control plant parameters and structure.

The next group of methods is based on using a plant model while a closed-loop system is being designed [25, 26]. Various methods of plant modeling can be used: state space equations [27], transfer functions [28, 29], and so on. This group of control design methods allows us to reduce control error and increase system performance.

Quite big number of methods to represent system description, study its motions and stability are based on the transfer functions apparatus which allows us in clear form represent systems differential equations and show its structure.

LITERATURE REVIEW

Search in the IEEEXplore library for phrase "transfer function" gives more than 4000 results, and more than 700 of them are for keyphrase "transfer function with initial conditions".

Analysis of some of these publications shows that authors use three approaches to study systems with initial conditions:

1. The initial condition problem is solved in the implementation stage of system model, when the continuous time model is discretized and initial conditions setting up for corresponding numerical integrator [30]. In their paper authors does not study influence of initial conditions into system dynamic. It is clear that such a solution gives only partial solution of the considered problem because of it gives complex simulation results, but does not allow to find components of this solution. We think that this solution can be obtained if only initial condition left as the signal source. But it can require quite huge time to study effects of all system disturbances. Moreover, such study requires changing model program. In general, this solution of initial condition problem for transfer function means transforming transfer function into state space equations domain and considering non-zero system state in this domain. Generally speaking, it is not necessary to consider transfer function in this case.

2. The most similar approach to the proposed by us is found in [31] where authors study usage of Laplace transformation with non-zero initial conditions to define some analog of fractional order transfer function. The main drawback of this paper is using of Laplace transformation which provides additional integral dependence for step function. This fact can confuse control theory professionals who use Laplace-Carson transformation to define transfer function and exclude additional integrators. Moreover, in their paper authors consider only a few cases and perform weak enough generalization of their results which very hard to use to analyze complex multichannel dynamical systems.

The given brief analysis shows the research relevance and expediency.

PROBLEM STATEMENT

The use of transfer functions gives us the necessary visualization of the designed control system by representing it with a block diagram. Also, many control methods are known to operate with these diagrams and optimize and transform them. That is why the use of transfer functions is convenient enough while the control system is studied and designed.

The main problem with transfer functions' usage is that these functions are defined as some integrodifferential operator with zero initial conditions. It is clear that this fact reduces the area of using these functions and does not allow to design a control system for plants with a non-zero initial state as well as does not allow to study system disturbed motions which are used as the basis to solve various control optimization problems.

We offer to avoid this drawback by considering previously-disturbed dynamical system motion and using it to generalize the transfer function definition to have possibility to take into account the system initial state.

PURPOSE AND OBJECTIVES OF THE STUDY

The purpose of the work is to improve method of study system responses for continuous-time linear, linearized, and piecewise-linear dynamical systems which have already operated before studying is being started due to the external or internal disturbances.

To achieve the goal, the following tasks are established**.**

1. Development of a method to define transfer function for the previously disturbed dynamical system.

2. Generalization of transfer function concept as well as concept of input signals.

3. To show the use of proposed approach by considering an illustrative example and make conclusion about possibility and peculiarities of proposed approach implementation.

Our paper organized as follows: at first, we define the generalized transfer function which contrary to conventional way takes into account system initial conditions, and then this transfer function is used to model the generalized linear plant which motion starts from non-zero state. We prove the correctness and benefits of our method in section Results and Discussion where DC motor drive is modeled and simulated. We use the transformed equations to solve the direct dynamic problem and define the generalized transfer function to study drive motions under the known external signals and internal initial conditions.

1. METHOD

Let us consider the generalized single-input single output controllable plant which dynamic is described with ordinary differential equations in normal form

$$\begin{aligned}&\frac{d}{dt}y_j=\sum_{i=1}^{n}a_{ij}y_i;\\&\frac{d}{dt}y_n=\sum_{i=1}^{n}a_{in}y_i+b_nu_n,\quad i=1,\ldots,n-1,\end{aligned}\tag{1}$$

here y_i are plant state variables, a_{ij}; b_n are plant factors which are defined as some functions of its parameters and u_n is a control effort, n is the plant dimension.

The usage of differential equations to define plant dynamics rases the problem of taking into account initial conditions for plant and observer. We call plant motions which start with non-zero initial conditions, as initially-disturbed motions. Plant is called by us as initially-disturbed one in this case. It is clear that these initial conditions should be considered while the plant dynamic is being studied as well as its motion is being planned.

That is why we complete (1) with their initial conditions y_{i0}, $\hat{y}_{i0}$ and rewrite the studied dynamical system as follows

$$\begin{aligned}&\frac{d}{dt}y_j=\sum_{i=1}^{n}a_{ij}y_i;\quad \frac{d}{dt}y_n=\sum_{i=1}^{n}a_{in}y_i+b_nu_n,\\&y_i(0)=y_{i0},\quad i=1,\ldots,n-1;\end{aligned}\tag{2}$$

We call (2) as the full controlled initially-disturbed plant equations in the normal form. These equations are written down by using known physical laws and dependencies and they allow us to study the real physical processes in the considered plant by its observable variables.

Let us apply Laplace-Carson transformation to plant and observer state variables

$$\begin{aligned}&y_i(t)\to Y_i(s);\quad u_n(t)\to U_n(s);\\&\frac{d}{dt}y_i(t)\to sY_i(s)-sy_{i0},\end{aligned}\tag{3}$$

here s is a Laplace operator.

One can use transformations (3) to rewrite (2) in the operator form

$$\begin{aligned}&sY_i(s)-sy_{i0}=\sum_{i=1}^{n}a_{ij}Y_i(s),\quad i=1,\ldots,n-1;\\&sY_n(s)-sy_{n0}=\sum_{i=1}^{n}a_{in}Y_i(s)+b_nU_n(s).\end{aligned}\tag{4}$$

It is clear that contrary to known control methods and approaches, which are based on usage of plant motions equations in the operator form, equations (4) allows us to take into account initial states of plant and observer. The use of these initial states allows us to study, plan, and control of plant motions in the correct way. At the same time, the use of classical approaches to design control signals for initially-disturbed plant can cause reducing stability level for the plant closed-loop control system.

Let us use (4) to model the plant motion. Here we assume that plant parameters and input signal are known and plant state variables can be defined as solution of (4).

Let us write down these equations in matrix form as follows

$$s\mathbf{Y}=\mathbf{a}\mathbf{Y}+\mathbf{b}U+s\mathbf{y_0},\tag{5}$$

where **Y** is a n-th sized vector of plant state variables; $\mathbf{y_0}$ is a n-th sized vector of plant initial states;
a is nxn-th sized matrix of plant factors which defined its free motion; **b** is n-th sized vector of plant controlled-motion factors. We use capitalized letters here to address Laplace-Carson transformation for plant variables and the Laplace operator is skipped to improve formulas reading.

One can use (5) to define vector of plant state variables as a result of operator equation solution

$$\begin{aligned}\mathbf{Y}&=(s\mathbf{E}-\mathbf{a})^{-1}\mathbf{b}U+(s\mathbf{E}-\mathbf{a})^{-1}s\mathbf{y_0}=\\&=(s\mathbf{E}-\mathbf{a})^{-1}(\mathbf{b}U+s\mathbf{y_0}),\end{aligned}\tag{6}$$

here power –1 means inverse matrix; **E** is an identity matrix.

Analysis of (6) allows us to make an obvious conclusion: plant motions depend on input signal as well as initial conditions. Furthermore, the similarity of the first and second summands in (6) allows us to consider initial conditions as an additional input signal and define generalized plant input signal as follows

$$\mathbf{V}_2 = \left(\mathbf{b}U \quad s\mathbf{y_0}\right)^T . \tag{7}$$

The generalized control signal allows us to rewrite matrix plant motion equation in such a way

$$\mathbf{Y} = \left(s\mathbf{E} - \mathbf{a}\right)^{-1} \mathbf{E}_2 \mathbf{V}_2, \quad \mathbf{E}_2 = \left(\mathbf{E} \quad \mathbf{E}\right). \tag{8}$$

One can use (8) to define following matrix transfer function

$$\mathbf{W}(s) = \frac{\mathbf{Y}(s)}{\mathbf{V}_2(s)} = \left(s\mathbf{E} - \mathbf{a}\right)^{-1} \mathbf{E}_2 . \tag{9}$$

We call this transfer function as the full generalized plant transfer function and consider it as nx2n-th dimensional differential operator. One can use this operator to define plant motions which depends on control inputs and initial states. It is necessary to say that the extension of control inputs vector $\mathbf{V}_2$ (7) can cause wrong thinks about possibility to control plant by changing components of vector $s\mathbf{y}_0$ during the plant operating time. To prevent this possible misunderstanding, we claim that the components of initial states vector sy_0 are defined only as scaled Heaviside step functions.

We show the use of the proposed approach by considering a solution of direct dynamic problem for a nonlinear dynamical system with single-variable nonlinearity.

EXPERIMENT SETUP

1. DC ELECTRIC DRIVE MODELING

Let us consider the differential equation of DC electric drive

$$\begin{aligned} &\frac{d}{dt}\omega = -\frac{h}{J}\omega^2 + \frac{c}{J}I_a - \frac{1}{J}T_c; \\ &\frac{d}{dt}I_a = -\frac{c}{T_a R_a}\omega - \frac{1}{T_a}I_a + \frac{1}{R_a T_a}U_a , \end{aligned} \tag{10}$$

where ω is a DC motor speed, I_a is a DC motor current, h is a friction factor, c is a constructive factor, J is a DC drive inertia, T_c is a DC drive load torque, R_a is an armature resistance, U_a is a DC voltage, T_a is an electromagnetic constant

$$T_a = \frac{L_a}{R_a} , \tag{11}$$

where L_a is an armature inductance.

In our paper we study the electric drive based on DC motor DPR-72 with following nominal parameters (Table 1).

Table 1. **Drive parameters**

No.	Parameter	Value
1	Armature resistance, R_a	2.9 Ohm
2	Armature inductance, L_a	0.0027 H
3	EMF factor, c	0.052 Wb
4	Idle speed, ω_0	520 s^{-1}
5	Nominal current, I_{anom}	1.3 A
6	Nominal voltage, U_{an}	27 V
7	Moment of inertia, J	$1.86 \cdot 10^{-5}$ kgm^2
8	Friction factor, h	$2 \cdot 10^{-7}$ Nms^2
9	Nominal torque, T_{dnom}	0.04 Nm

Source: **compiled by the authors**

It is clear that nonlinear function in the first equation of (10) makes cardinal influence on electric drive dynamic and stability. We offer to take into account the above-mentioned noninearity and represent it with piecewise linear function

$$f_{pwl}(\omega) = \begin{cases} k_1\omega + \omega_{f1} & \text{if} \quad \omega < \omega_1; \\ \vdots & \\ k_n\omega + \omega_{fn} & \text{if} \quad \omega_{n-1} < \omega < \omega_n , \end{cases} \tag{12}$$

here k_i and ω_{fi} are piecewise linear approximation factors and ω_i is fracture points. The simplest two-branches piecewise linear function which can be used to approximate the square nonlinearity is shown in Fig.1.

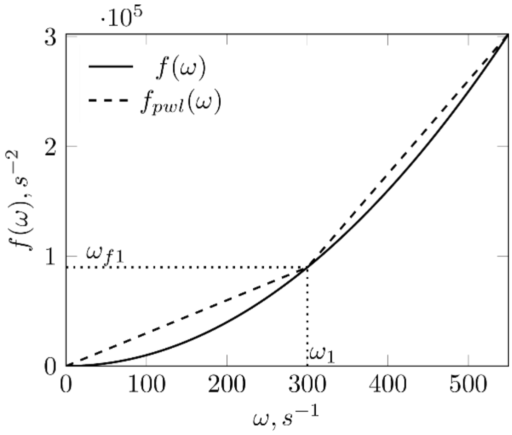


Fig. 1. **Piecewise linear approximation of drive nonlinearity**

Source: **compiled by the authors**

All studies in the paper are performed for such function so we rewrite it as follows

$$f_{pwl}(\omega)=\begin{cases} k_1\omega & \text{if} \quad \omega<\omega_1; \\ k_2\omega+\omega_{f1} & \text{if} \quad \omega\geq\omega_1. \end{cases} \quad (13)$$

It is clear that if one replaces continuous nonlinear function with piecewise linear one, he can use linear control methods to perform system analysis.

Let us rewrite (10) for each from intervals, where the considered piecewise linear function is defined, by taking into account (11) and (12) as well as following factors

$$\begin{aligned} &a_{11i}=-\frac{k_i h}{J};\ a_{12}=\frac{c}{J}; m_{10i}=-\frac{h\omega_{fi}}{J};\ m_1=-\frac{1}{J}; \\ &a_{21}=-\frac{c}{T_a R_a};\quad a_{22}=-\frac{1}{T_a};\quad m_2=\frac{1}{R_a T_a} \end{aligned} \quad (14)$$

in such a way

$$\begin{aligned} &\frac{d}{dt}\omega=a_{11i}\omega+a_{12}I_a+m_1T_c+m_{10i}; \\ &\frac{d}{dt}I_a=a_{21}\omega+a_{22}I_a+m_2U_a. \end{aligned} \quad (15)$$

The first equation in (15) has a free term m_{10} which is defined by features of the used piecewise linear approximation. That is why we offer to consider (15) as the generalization for (2).

It is clearly understood that (15) define electric drive dynamic and static in all intervals of the considered piecewise linear function. In this case, the studied electromechanical system can be considered as variable structure dynamical system which dynamic is defined as follows

$$\begin{aligned} &\frac{d}{dt}\omega=a_{12}I_a+m_1T_c+\begin{cases} a_{111}\omega & \text{if} \quad \omega<\omega_1; \\ a_{112}\omega+m_{102} & \text{if} \quad \omega\geq\omega_1; \end{cases} \\ &\frac{d}{dt}I_a=a_{21}\omega+a_{22}I_a+m_2U_a. \end{aligned} \quad (16)$$

Moreover, in case of the studying a high-speed system motions it is necessary to take into account what these motions start from some values which are defined by switching system structure.

We take into account Laplace-Carson transformation for linear differential equations with non-zero initial states (3) and rewrite (15) into operator form

$$\begin{aligned} &s\omega=a_{11i}\omega+a_{12}I_a+m_1T_c+s\omega(0)+m_{10i}; \\ &sI_a=a_{21}\omega+a_{22}I_a+m_2U_a+sI_a(0). \end{aligned} \quad (17)$$

We claim that (17) is the generalization of a well-known motion equation for DC electric drive. Contrary conventional drive model, our equations take into account drive operating mode and can be used to study changing of drive coordinates which is caused by changing armature voltage or external torque. Furthermore, (17) gives us the possibility to consider a nonlinear friction.

Let us rewrite (17) into matrix form

$$s\mathbf{Y}=\mathbf{AY}+\mathbf{MU}+s\mathbf{Y_0}+\mathbf{M}_0, \quad (18)$$

here

$$\begin{aligned} &\mathbf{Y}=(\omega \quad I_a)^T;\quad \mathbf{U}=(T_c \quad U_a)^T; \\ &\mathbf{Y_0}=(\omega(0) \quad I_a(0))^T. \\ &\mathbf{M}=\begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix};\quad \mathbf{A}=\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix};\quad \mathbf{M_0}=\begin{pmatrix} m_{10i} \\ 0 \end{pmatrix} \end{aligned} \quad (19)$$

One can use (18) with (41) to study the considered system dynamic as well as its steady state, define its frequency responses, design controller and more.

2. SOLUTION OF DIRECT DYNAMIC PROBLEM

Let us solve direct dynamic problem for the system (18). It is understood that this solution in the matrix form can be written down as follows

$$\mathbf{Y}=(s\mathbf{E}-\mathbf{A})^{-1}(\mathbf{MU}+s\mathbf{Y_0}+\mathbf{M_0}). \quad (20)$$

Matrix expression (42) allows us to define full state space vector **Y** which components depend on input signals vector **U**, initial state vector $\mathbf{Y_0}$, and system parameters matrices **A** and **M**.

$$\begin{aligned} \omega=&\frac{a_{12}m_2U_a+m_1sT_c-m_1a_{22}T_c+}{a_{11i}a_{22}-a_{12}a_{21}-(a_{11i}+a_{22})s+s^2}+ \\ &+\frac{m_{10i}s-a_{22}m_{10i}}{a_{11i}a_{22}-a_{12}a_{21}-(a_{11i}+a_{22})s+s^2} \\ &+\frac{+s^2\omega(0)-a_{22}s\omega(0)+a_{12}sI_a(0)}{a_{11i}a_{22}-a_{12}a_{21}-(a_{11i}+a_{22})s+s^2}; \\ I_a=&\frac{a_{21}m_1T_c+m_2sU_a-a_{11i}m_2U_a}{a_{11i}a_{22}-a_{12}a_{21}-(a_{11i}+a_{22})s+s^2}+ \\ &+\frac{a_{21}m_{10i}+a_{21}s\omega(0)+s^2I_a(0)-a_{11i}sI_a(0)}{a_{11i}a_{22}-a_{12}a_{21}-(a_{11i}+a_{22})s+s^2}. \end{aligned} \quad (21)$$

Analysis of (21) allows us to study effect of each input signal and/or system initial state.

To perform such studies, we offer to rewrite (21) and define each drive state variable as linear combinations of its inputs and initial states which are weighted with some differential operators

$$\begin{aligned}
\omega &= \frac{m_1(s-a_{22})}{a_{11i}a_{22}-a_{12}a_{21}-(a_{11i}+a_{22})s+s^2}T_c + \\
&+\frac{a_{12}m_2}{a_{11i}a_{22}-a_{12}a_{21}-(a_{11i}+a_{22})s+s^2}U_a + \\
&+\frac{(s-a_{22})s}{a_{11i}a_{22}-a_{12}a_{21}-(a_{11i}+a_{22})s+s^2}\omega(0) + \\
&+\frac{a_{12}s}{a_{11i}a_{22}-a_{12}a_{21}-(a_{11i}+a_{22})s+s^2}I_a(0) + \\
&+\frac{m_{10i}s-a_{22}m_{10i}}{a_{11i}a_{22}-a_{12}a_{21}-(a_{11i}+a_{22})s+s^2}; \\
I_a &= \frac{a_{21}m_1}{a_{11i}a_{22}-a_{12}a_{21}-(a_{11i}+a_{22})s+s^2}T_c + \\
&+\frac{m_2(s-a_{11i})}{a_{11i}a_{22}-a_{12}a_{21}-(a_{11i}+a_{22})s+s^2}U_a + \\
&+\frac{a_{21}s}{a_{11i}a_{22}-a_{12}a_{21}-(a_{11i}+a_{22})s+s^2}\omega(0) + \\
&+\frac{(s-a_{11i})s}{a_{11i}a_{22}-a_{12}a_{21}-(a_{11i}+a_{22})s+s^2}I_a(0) \\
&+\frac{a_{21}m_{10i}}{a_{11i}a_{22}-a_{12}a_{21}-(a_{11i}+a_{22})s+s^2};
\end{aligned} \quad (22)$$

The usage of (22) gives us the possibility to study electric drive motions which are caused by different voltages, external torques, and initial conditions. Contrary to the well-known conventional approach the use of general solution (22) of direct dynamic problem allows us to take into account possible changing of plant structure.

If one defines differential operators in (22) as components of matrix transfer function

$$\begin{aligned}
W_{11} &= \frac{m_1(s-a_{22})}{a_{11i}a_{22}-a_{12}a_{21}-(a_{11i}+a_{22})s+s^2}; \\
W_{12} &= \frac{a_{12}m_2}{a_{11i}a_{22}-a_{12}a_{21}-(a_{11i}+a_{22})s+s^2}; \\
W_{13} &= \frac{(s-a_{22})s}{a_{11i}a_{22}-a_{12}a_{21}-(a_{11i}+a_{22})s+s^2}; \\
W_{14} &= \frac{a_{12}s}{a_{11i}a_{22}-a_{12}a_{21}-(a_{11i}+a_{22})s+s^2}; \\
W_{15} &= \frac{m_{10i}s-a_{22}m_{10i}}{a_{11i}a_{22}-a_{12}a_{21}-(a_{11i}+a_{22})s+s^2}; \\
W_{21} &= \frac{a_{21}m_1}{a_{11i}a_{22}-a_{12}a_{21}-(a_{11i}+a_{22})s+s^2}; \\
W_{22} &= \frac{m_2(s-a_{11i})}{a_{11i}a_{22}-a_{12}a_{21}-(a_{11i}+a_{22})s+s^2}; \\
W_{23} &= \frac{a_{21}s}{a_{11i}a_{22}-a_{12}a_{21}-(a_{11i}+a_{22})s+s^2}; \\
W_{24} &= \frac{(s-a_{11})s}{a_{11i}a_{22}-a_{12}a_{21}-(a_{11i}+a_{22})s+s^2}; \\
W_{25} &= \frac{a_{21}m_{10i}}{a_{11i}a_{22}-a_{12}a_{21}-(a_{11i}+a_{22})s+s^2}
\end{aligned} \quad (23)$$

and takes into consideration the generalized input vector

$$\mathbf{V} = \begin{pmatrix} T_c & U_a & \omega(0) & I_a(0) & 1 \end{pmatrix}^T \quad (24)$$

as well as the generalized state vector

$$\mathbf{Y} = \begin{pmatrix} \omega & I_a \end{pmatrix}^T, \quad (25)$$

he can rewrite (22) in terms of the generalized matrix transfer function

$$\mathbf{Y} = \mathbf{W}(s)\mathbf{V}, \quad (26)$$

here

$$\mathbf{W}(s) = \begin{pmatrix} W_{11} & W_{12} & W_{13} & W_{14} & W_{15} \\ W_{21} & W_{22} & W_{23} & W_{24} & W_{25} \end{pmatrix}. \quad (27)$$

The last component in (24) equals to 1 and allows us to take into account changing of system structure.

It is clearly understood that the transfer function (27) allows us to simplify the describing of system dynamic. In the most general case one can consider this component as some switching function which defines system structure by using current values of its state variables.

In such a way one can consider (26) as the solution of direct dynamic problem for (20) in the most general case.

Analysis of (22) and (23) allows us to claim that transfer functions W_{12} and W_{22} are conventional transfer function for the speed and current as functions of armature voltage. Transfer functions W_{11} and W_{21} are not commonly used but one can obtained them if consider drive torque as input signal. Other transfer functions are not defined in classical control theory of DC electric drives.

3. NUMERICAL SOLUTION OF DIRECT DYNAMIC PROBLEM FOR LINEARIZED DC SERIES ELECTRIC DRIVE

Let us specify these transfer functions by taking into account the system variable structure and possible operation modes.

3.1. The case of drive starts from zero stationary state

In this case all initial conditions $\omega(0)$ and $I_a(0)$ should be taken as zero as well as approximation factor m_{100}. Such an assumptions allows us to rewrite (22) in the following way

$$\omega = \frac{m_1(s-a_{22})}{a_{111}a_{22}-a_{12}a_{21}-(a_{111}+a_{22})s+s^2}T_c + \frac{a_{12}m_2}{a_{111}a_{22}-a_{12}a_{21}-(a_{111}+a_{22})s+s^2}U_a;$$
$$I_a = \frac{a_{21}m_1}{a_{111}a_{22}-a_{12}a_{21}-(a_{111}+a_{22})s+s^2}T_c + \frac{m_2(s-a_{111})}{a_{111}a_{22}-a_{12}a_{21}-(a_{111}+a_{22})s+s^2}U_a. \quad (28)$$

Expressions (28) are trivial and allow us to define drive motions by using only external inputs.

We use Python 3.6.15 with Scipy 1.3.3 library to simulate drive dynamic by using shown in Fig.2 code.

Usage of such an approach to simulate system dynamic is trivial and clear: lines 8-9 define drive parameters, rows 10-12 define model factors, row 13 defines external signals, lines 16-17 define numerator and denominator for the considered transfer functions, lines 19-20 create these transfer functions as LTI dynamical objects, rows 25-26 defines units step responses for each from above-defined transfer functions, and line 27 defines system output for the given input signals. Other lines allow us to study dynamic of another state variable. Given code gives us the possibility to

```
6   from scipy import signal
7
8   Ra=2.9; Ta=8e-3; c=0.052
9   J=1.86e-5; h=2e-7; k1=300
10  a111=-k1*h/J; a12=c/J
11  m101=0; m1=-1/J
12  a21=-c/Ra/Ta; a22=-1/Ta; m2=1/Ra/Ta
13  Ua=15; Tc=0.04
14
15
16  W11_num=[m1,-m1*a22];   W12_num=[a12*m2]
17  W1i_den=[1,-(a111+a22),a111*a22-a12*a21]
18
19  W11=signal.lti(W11_num, W1i_den)
20  W12=signal.lti(W12_num, W1i_den)
21
22  W21_num=[a21*m1];  W22_num=[m2,-m2*a111]
23  W2i_den=[1,-(a111+a22),a111*a22-a12*a21]
24
25  t,w_Ua=signal.step(W12,N=1e3)
26  t,w_Tc=signal.step(W11,N=1e3)
27  w=w_Ua*Ua+w_Tc*Tc
28
29  W21=signal.lti(W21_num, W2i_den)
30  W22=signal.lti(W22_num, W2i_den)
31
32  t,Ia_Ua=signal.step(W22,N=1e3)
33  t,Ia_Tc=signal.step(W21,N=1e3)
34  Ia=Ia_Ua*Ua+Ia_Tc*Tc
```

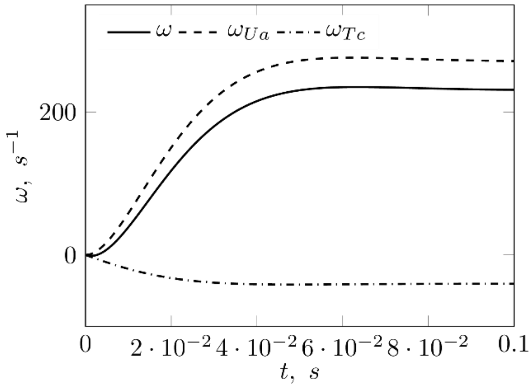
Fig.2 **Simulation code**

Source: **compiled by the authors**

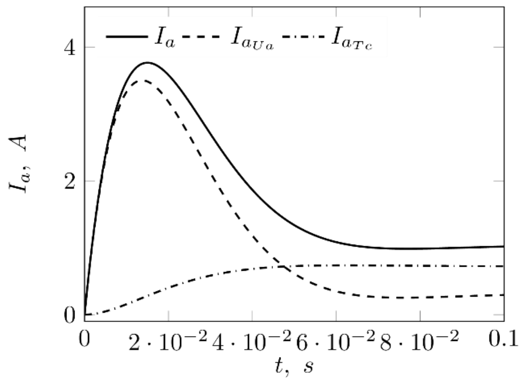
define simulation results as 1D-arrays, which can be easy visualized by using Python libraries or saved into file to perform analysis in other software products. We use Python here due to its open-source license and huge spreading in various platforms and operational systems from single-chip computers like Raspberry Pi to big cloud services. Nevertheless, one can use other mathematical software to perform system studies. For example, opensource-licensed SageMath, Octave, FreeMat, and proprietary-licensed MathSoft MatLab and Waterloo Maple have the similar methods to perform systems analysis and the use of these methods are quite similar to the above-considered.

We show simulation results of the designed in Fig. 2 model in Fig. 3.

Analysis of obtained simulation results as well as above-given formulas shows trivial results for drive speed and current as well as their components which are caused by different input signals. The shown in Fig. 3 results does not differ from known ones. This fact proves the correctness of defined factors and terms in (28).



a – Drive speed



b– Drive current

Fig.3. **Simulation results of drive starting**

Source: **compiled by the authors**

3.2. The case of drive deceleration from some speed to another non-zero speed

Now we continue to show the proposed approach usage and we turn our attention into the case when drive has been already operates with some speed and current under some armature voltage U_a and external torque T_c. We study the transients in the drive which are caused by reducing drive voltage.

Classical approach based on the transfer function use does not allow to perform such study due to impossibility to set some initial values for the transfer function. The most known approaches to study such systems are based on the decomposition of transfer function into elementary blocks and setting initial conditions for each integrator, using a MATLAB Simulink Extra block which allows setting up initial values for derivatives in transfer function, and the use of DC drive model which is designed as solution of (15). It is necessary to say that all above-mentioned approaches allow only simulate studied dynamical system and it is very hard to use them for solving other tasks such as closed-loop system design. Moreover, the first and second approaches require to define initial values for derivatives of output variable, which can be quite complex problem for big system.

That is why we turn our attention into (26) which matrices for the considered case can be rewritten as follows

$$\mathbf{V} = \begin{pmatrix} T_c & U_a & \omega(0) & I_a(0) \end{pmatrix}^T;$$
$$\mathbf{W}(s) = \begin{pmatrix} W_{11} & W_{12} & W_{13} & W_{14} \\ W_{21} & W_{22} & W_{23} & W_{24} \end{pmatrix}. \quad (29)$$

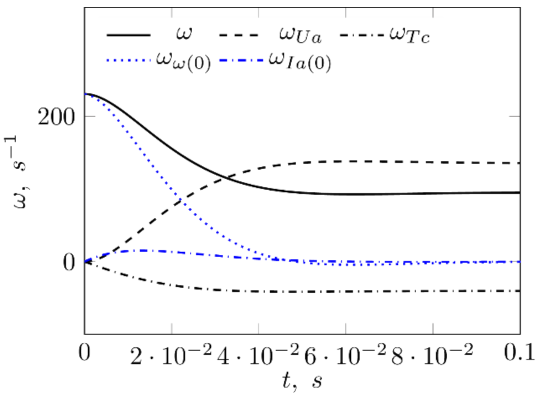
Usage of the above-mentioned software requires to define linear time-invariant subsystems, which we define by using transfer functions W_{11}-W_{14}, W_{21}-W_{24} from (23).

We define initial conditions components of **V**

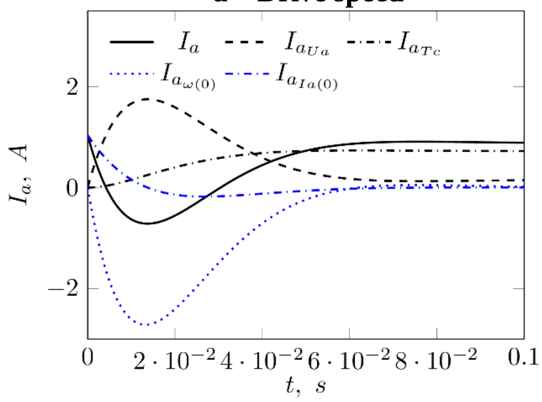
$$I_a(0) = \frac{a_{21} m_1 T_c - a_{111} m_2 Ua}{a_{111} a_{22} - a_{12} a_{21}};$$
$$\omega(0) = \frac{a_{12} m_2 U_a - a_{22} m_1 T_c}{a_{111} a_{22} - a_{12} a_{21}}. \quad (30)$$

as solution of (17) by assuming that drive operates in steady state.

Simulation results for the considered case are shown in Fig.4. Here we study the twice reducing of armature voltage.



a – Drive speed



b – Drive current

Fig.4. **Simulation results of drive decelerating**

Source: **compiled by the authors**

The given in Fig. 4 simulation results show drive motion under voltage changing and affecting of various external and internal disturbances. These results are also quite trivial but the comparison of solid black and dotted blue curves in Fig.4a allows us to generalize the definition of the disturbed motion which should be defined by taking into account both initial and final values of drive state variables. Initial variation of disturbed motion coordinates should be defined as difference between the initial and final values of the considered state variable instead of only its initial values which are considered now. Thus, the disturbed motions should be defined in the various ways for cases of physically reproducible and irreproducible motions.

3. The case of drive switching structure study

In this case we use all components of matrix transfer function (27). Moreover, since the studied electric drive is considered as variable-structure dynamical system we study it in two stages.

In the first stage we consider drive motion from the given by (30) initial conditions. We think that drive starts its motion under armature voltages equals to 27V from initial conditions which are correspond steady state with armature voltage equals to 15V. This stage finishes while drive speed reaches ω_1. It becomes when current I_a reaches values I_{a1}.

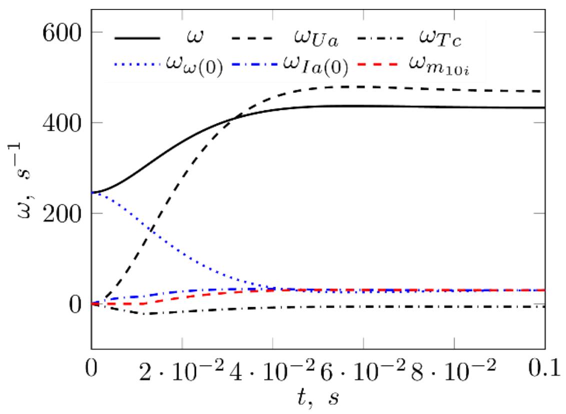
We use these values as the initial conditions in the second stage. In this stage model terms change their values and we consider the drive operation in the second branch of piecewise linear function (13).

One can find simulation results for study of variable structure system in Fig. 5.

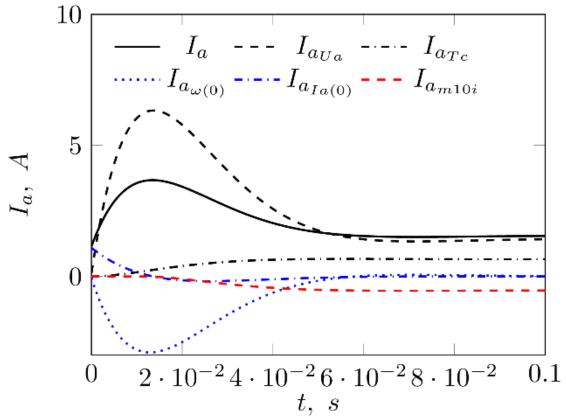
Analysis of the given in Fig. 5 transients proves the system variable structure and shows that the drive speed reaches value 300 s^{-1} near 0.01s. In this time system factors change their values and some of transients become nonexponential one and take high-order discontinuities.

DISCUSSION OF THE RESULTS

The above-given formulas and simulation results show that the proposed approach is highly formalized one because of the using of matrix and operator calculus to design system model. This approach allows to model and simulate liner, linearized, or piecewise-linear dynamical systems which motions can be described with linear ordinary differential equations. One can study system dynamic by solving these equations in both analytical and numerical ways.



a – **Drive speed**



b – **Drive current**

Fig.5. **Simulation results of drive accelerating**

Source: **compiled by the authors**

The advantages of the developed approach to solve direct dynamic problem are clear algorithm of model design with strong mathematical backgrounds.

One can formulate this algorithm as follows:

1. The advantages of the developed approach to solve direct dynamic problem are clear algorithm of model design with strong mathematical backgrounds. One can formulate this algorithm as follows.

2. The studied system dynamic is described with linear ordinary differential equations by using known physical laws or energy approaches.

3. System equations are transformed into the Laplace-Carson operator form by using the generalized transformation and taking into system initial conditions.

4. The generalized system input vector is defined by taking into account system inputs, initial conditions and switching functions.

5. Components of the generalized matrix transfer function are defined by using (20) or similar equations.

6. Taking into account (26) or similar makes it possible to define system dynamic in the operator

form. The use of matrix calculus allows us to define all system state variables at the same time.

Defined in such a way transfer functions allows us to generalize the channels which affect on system motion. We offer to append to list of well-known from classical control theory feedforward channel and channel of external disturbances the such components as initial conditions channel and variable structure channel.

Contrary to conventional approach to study system dynamic by using mathematical models in MatLab Simulink, for example, our approach allows to study effect of all components of the generalized input vector on system dynamic simultaneously. Contrary to the conventional approach, this can cause consuming additional calculation resources to perform such study but at the same time it reduces time for system dynamic simulation. Moreover, for the systems with fixed structure calculation by using (28) and similar can easy parallelized if step response for each channel is calculated as different process.

In the case of study variable-structure dynamical system one should split system motion into several stages and define for each stage initial conditions and transfer function components.

Also, contrary to the known approaches, the use of transfer function-based approach gives us the possibility to check system stability and define the domains of system stable operating.

It is clear that the system dynamic can be studied not only by applying unit step function as it is shown in the previous section but any input signals in feedforward and external disturbances channels can be used to define system response.

Thus, the sphere of effective application of the proposed approach is study of system dynamic in the parallel way for linear dynamical systems and series-parallel for variable-structure systems. Moreover, the defined in such a way transfer functions can be used to solve inverse dynamic problem and define input vector components by known transfer functions and system output vector.

CONCLUSIONS

The generalization of transfer function as the matrix linear differential operator makes it possible to take into accounts not only system control signal and external disturbances but also allows to consider system initial state. Such system dynamic describing is a very formal and it allows solving direct dynamic problem in strong mathematical way. Since the proposed approach is based on matrix methods one can use it to operate with wide range single-channel and multi-channel dynamical system, study theirs motions and consider full system motions or motions of each subsystem in which the system can be split. One can find that the proposed approach can be used to model and simulate both continuous and variable structure dynamical systems.

We see the future development of our method in the spreading it into the class of discrete-time dynamical systems as well as use it to solve the inverse dynamic problem and design controller which allows to reach the desired motion trajectories for both continuous-time and discrete-time dynamical systems.

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Інформаційне забезпечення до вирішення задач динаміки попередньо-збурених електромеханічних систем

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АНОТАЦІЯ

Стаття присвячена створенню методологічних засад для розв'язання прямої задачі динаміки для лінійних динамічних систем, рух яких описується звичайними диференціальними рівняннями з ненульовими початковими умовами. Розгляд рухів лінійних динамічних систем дозволяє спростити використовуваний математичний апарат та розв'язувати задачі визначення руху шляхом використання відомого підходу, який базується на передавальних функціях. Однак, через те, що класичне визначення передавальних функцій не передбачає прийняття до уваги ненульових початкових умов, які викликані наявністю початкових відхилень координат об'єкта керування від їх бажаних значень, в нашій роботі ми використовуємо перетворення Лапласа Карсона для знаходження відповідних зображень та запису рівнянь руху у операторній формі. Такий підхід, на відміну від загально-прийнятого, призвів до введення у праві частині відповідних операторних диференціальних рівнянь інформації про початкові умови руху та обумовив необхідність узагальнення вектору сигналів керування шляхом включення в нього компонент, що враховують початкові умови руху розглядаємої системи. Такі перетворення дозволили узагальнити поняття матричної передавальної функції як матричного лінійного динамічного оператора, який складається з двох компонент, що визначають збурений вільний та керований вимушений рухи. Використання такого оператора дозволяє досліджувати динаміку лінійної системи як окремо по кожному з компонент узагальненого вектору керуючих впливів, так і в комплексі, вирішуючи тим самим пряму задачу динаміки лінійних систем.

В якості прикладу ми показуємо використання запропонованого підходу для аналізу рухів двигуна постійного струму з нелінійним вентиляторним тертям на основі його кусково-лінеаризованої моделі.

Ключові слова: інформаційне забезпечення; динамічна система; пряма задача динаміки; передавальна функція; початкові стани; матричні методи; лінійний диференціальний оператор; перетворення Лапласа- Карсона

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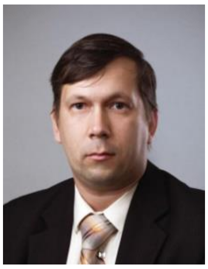

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