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Graph-logical models for (n, f, k) – and consecutive - k-out-of-n – systems

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ABSTRACT

The article is devoted to methods of constructing graph-logical models of fault-tolerant multiprocessor systems. In particular, systems of the type (n, f, k), linear consecutive-k-out-of-n and circular consecutive-k-out-of-n are considered, which are characterized by the failure of the system when a certain number of consecutive processors fail. Graph-logical models can be used to estimate the reliability parameters of fault-tolerant multiprocessor systems by conducting statistical experiments with models of their behavior in the failure flow. The graph-logical models under construction are based on the basic models with a minimum of lost edges. It is determined that to build a graph-logical model of systems of this type, it is sufficient to calculate the maximum possible number of failed processors at which the system remains in operation. A graph-logical model of a basic system that can handle this number of failures is built, without taking into account the sequence of these failures. The next step is to identify all possible consecutive failures that cause the system to fail. Then, the base model is modified in such a way as to reflect the failure of the system when consecutive failures occur. This means weakening the base model on the previously determined vectors. The proposed methods of model construction can be used both for linear and circular consecutive-k-out-of-n systems and for (n, f, k) systems. A minor difference will be in the calculation of some parameters. The paper describes the calculation of such parameters as the maximum allowable number of failures at which the system remains in an operational state, as well as the calculation of the number of all combinations of consecutive failures at which the system fails. Experiments have been conducted to confirm the model's compliance with the system's behavior in the failure flow. Examples are given to demonstrate the process of building graph-logical models for linear consecutive-k-out-of-n, circular consecutive-k-out-of-n and (n, f, k) systems using the proposed methods.

Keywords: Graph-logical models; minimum lost edges-models; non-basic fault-tolerant multiprocessor systems; k-out-of-n systems

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INTRODUCTION

Modern automated control systems (CS) [1, 2] allow reducing human involvement in the control process. Such systems reduce the impact of the human factor and relieve the operator from routine activities, and in some cases, perform tasks with high computational complexity that humans are fundamentally unable to solve in the same time. Typically, CS of complex objects are built on the basis of microprocessor systems that can receive signals from sensors or control devices, process them, and issue an appropriate control signal.

In some industries, such as medicine, military industry, aviation, space industry, banking, and critical infrastructure, failure of the CS can lead to financial and material losses, or even fatal consequences.

Therefore, fault-tolerant multiprocessor systems (FTMS) are used to build control systems. These systems consist of a large number of processors and processors fail. Since fault tolerance is a critically important feature of such systems, much attention is paid to calculating their reliability and safety during FTMS design.

LITERATURE REVIEW AND PROBLEM STATEMENT

Fault-tolerant multiprocessor systems can be classified as basic and non-basic. Basic systems remain operational as long as a certain number of any of its processors are functioning. A non-basic system can behave differently with the same number of failures. Several analytical methods are known for calculating the reliability of a basic system (i.e., k-out-of-n system) [3, 4], [5], but for non-basic systems, the calculations become more complex. Depending on the configuration, and accordingly, the conditions under which the system fails, there are different types of non-basic systems: consecutive-k-out-of-n [6, 7], [8], consecutive-k-within-m-out-of-n [9, 10], [11], consecutive-k-out-of-r-from-n [12, 13], [14], m-consecutive-k-out-of-n [15, 16], [17, 18], (n, f, k) [19, 20], [21], <n, f, k> [20, 21], m-consecutive-k,l-out-of-n [22, 23], [24], kc-out-of-n [17, 18], (r, s)-out-of-(m, n) [25, 26], [27], consecutive-kr-out-of-nr [28], and others. In most of these types of non-basic systems, one of the failure conditions for the entire system is the failure of *k* consecutive processors. Therefore, we will focus on systems where no other conditions are present, specifically on linear and circular consecutive-k-out-of-n systems, as well as on (n, f, k) systems, which are essentially consecutive-k-out-of-n systems but also fail when any *f* processors fail.

In addition to analytical methods for calculating the reliability of FTMS, there are also methods based on statistical experiments. For example, modeling the behavior of the system in a failure flow using graph-logical models (hereinafter referred to as GL-models (graph-logical) [29, 30]. Graph-logical models represent a cyclic undirected graph, where each edge is assigned a Boolean function. The arguments x_i of the edge function represent the states of the processors in the system. The argument x_i takes the value 1 when the processor is functioning, and 0 when the processor has failed. If the function takes the value 0, the corresponding edge is removed from the graph. The loss of connectivity in the graph corresponds to the failure of the entire system.

GL-models can be divided into basic and non-basic. A basic model corresponds to an FTMS containing *n* processors and remains operational when *m* or fewer of them fail (*n*>*m*). Following [31], we will say that vectors can be blocked by weakening – where the model loses connectivity on a vector containing *m* or fewer zeros, or by strengthening – where the non-basic graph-logical model does not lose connectivity on vectors containing more than *m* zeros. Vectors can be blocked in several ways: by changing the edge functions of the graph, altering the structure of the graph, or combining both approaches.

A non-basic graph-logical model can be constructed by modifying a basic model in such a way that its behavior changes compared to the basic model on certain state vectors of the system. This modification of the model results in the blocking of these vectors and alters the model to reflect the behavior of the system in a failure flow.

PURPOSE AND OBJECTIVES OF THE RESEARCH

Despite the existence of known methods for calculating the reliability of systems that fail when *k* consecutive processors fail, the drawback of analytical approaches is that separate formulas or even methods need to be developed for each type of system. Graph-logical models are universal and can be applied to various types of systems. When additional system failure conditions appear, such as system failure when specific processors fail, or when the considered system is part of a larger system where other subsystems fail under another condition, it is quite simple to reflect these conditions in a graph-logical model, while analytical methods may require significant recalculations. Therefore, we will focus on graph-logical models.

Thus, the main goal of this paper is to develop methods for constructing graph-logical models for (n, f, k) systems, linear consecutive-k-out-of-n systems, and circular consecutive-k-out-of-n systems.

MLE-MODELS

A graph-logical model of a basic system that consists of *n* processors and resistant to the failure of any *m* of them is denoted as *K (m, n)*. The methods for constructing a graph-logical model for (n, f, k) systems and both types of consecutive-k-out-of-n systems are based on MLE-models (minimum lost edges) [32]. One of the features of MLE-models is that the graph loses two edges on state vectors of the system that contain *m+1* zeros, and one edge on vectors that contain *m* zeros. The graph does not lose edges on state vectors of the system that contain fewer than *m* zeros. The number of lost edges can be described as

$$\psi(m,l) = \begin{cases} 0, if\ l < m \\ l - m + 1, if\ l \geq m \end{cases}.$$

The number of edges in the graph, and accordingly, the number of edge functions in the MLE-model *K(m, n)*, as demonstrated in [33], can be calculated using the formula:

$$\rho(m,n) = n - m + 1.$$

The main difference between (n, f, k) systems and both types of consecutive-k-out-of-n systems from the basic k-out-of-n systems is the condition for system failure when *k* consecutive processors fail. At the same time, the system will continue to operate even if more than *k* non-consecutive processors fail (but fewer than *f*, in the case of (n, f, k) systems). Essentially, (n, f, k) and consecutive-k-out-of-n systems behave like basic systems, but with the additional failure condition of *k* consecutive processor failures, apart from the failure of more than *m* processors.

Therefore, to construct the graph-logical model of such a system, we first determine the maximum allowable number of failures under which the system remains operational, without considering the condition of *k* consecutive processor failures. This means that we first determine the number *m* and construct the MLE-model of the basic system *K(m, n)*. The next step is to determine the vectors on which the system stops functioning, in other words, all vectors with *k* consecutive zeros. Since it is sufficient to identify only vectors with *k* consecutive zeros, rather than *k* zeros in general, there will be relatively few such vectors. Then, we block all obtained vectors by weakening.

The model can be weakened by modifying two or more edge functions, altering the structure of the graph, or combining both approaches. The resulting GL-model will fully correspond to the behavior of the given system in a failure flow.

(n, f, k) SYSTEMS

An (n, f, k) system consists of *n* linearly arranged processors and fails if and only if at least *f* of its processors fail, or at least *k* of its consecutive processors fail ($k<f$).

To represent an (n, f, k) system as a graph-logical model, we need to find the number *m* – the maximum allowable number of failures in the system under which the system remains operational, without considering the condition of *k* consecutive processor failures. Since it is known that the system fails when any *f* processors fail, it is obvious that the number *m* will be equal to

$$m = f - 1.$$

After determining the number *m*, we construct the basic MLE-model of the system *K (m, n)*.

Next, we identify all vectors with *k* consecutive zeros. The exact number of such vectors can be determined. The starting index of a sequence of *k* zeros can be any index from 1 to *n*. However, to avoid exceeding the vector boundaries, the last possible starting index must allow for the inclusion of *k* processors. Based on this condition, if the sequence starts at index *i*, the last index of the sequence will be

$$i + k - 1 \le n;$$
$$i \le n - k + 1.$$

The number of all possible vectors with *k* consecutive zeros can accordingly be calculated using the formula

$$c(k,m) = n - k + 1 \ (1).$$

After determining all vectors with *k* consecutive zeros, the basic MLE-model *K(m, n)* can be weakened on these vectors.

THEOREM 1

To weaken the given basic MLE-model *K(m, n)* on vectors containing *k* consecutive zeros, it is sufficient to multiply any two edge functions of the GL-model by the function *f′(X)* - the conjunction of all disjunctions of every *k* consecutive arguments of the function.

PROVING

If at least one of every *k* consecutively connected processors remains operational, then the system functions, and accordingly, the function *f′* equals one. At the same time, if *k* consecutive processors fail, *f′* will equal zero. Obviously, this condition for *k* components can be described by a disjunction

$$x_j \vee x_{j+1} \vee \ldots \vee x_{j+k-1},$$

where *j* is any index of an element in the vector from 0 to *n-k+1*. Since the failure of any *k* consecutively connected processors leads to the failure of the entire system, it is necessary to account for the condition that any sequence of *k* zeros in the vector will cause the function *f′* to equal zero. To do this, we construct the conjunction of all possible disjunctions of the function's arguments, starting from the very first element:

$$f' = (x_1 \vee x_2 \vee \ldots \vee x_k) \wedge (x_2 \vee x_3 \vee \ldots \vee x_{k+1}) \wedge \ldots$$
$$\wedge (x_{n-k+1} \vee x_{n-k} \vee \ldots \vee x_n).$$

Or simply describe *f′* as

$$f' = \wedge_{i=1}^{n-k+1} \left(\vee_{j=i}^{i+k-1} x_j \right).$$

In this case, if any vector with *k* consecutive zeros appears, the disjunction that includes all *k* consecutive processors will take the value 0. Accordingly, both modified functions will equal 0, and the graph is guaranteed to lose 2 edges.

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Therefore, the connectivity of the graph is disrupted when a vector with k consecutive zeros appears or when a vector with f zeros appears, thus making the model accurately reflect the behavior of the given system in a failure flow. To simplify calculations, functions with the fewest arguments can be weakened. However, this approach may not be optimal and requires further study. For example, in cases where $k=m$, when k processors fail, the GL-model already loses one edge, so blocking all vectors on two functions by weakening them may be excessive.

Model can also be weakened by changing the structure of the graph. For example, by adding an additional vertex and an edge, whose function will take the value 0 on any vector with k consecutive zeros. Consequently, the modified graph will also lose connectivity on the specified vectors.

Graph can also be modified using the method described in [34]. According to [34], let us denote the condition for system failure when any k consecutive components fail as S. Let $s(X)$ be an expression that depends on the values of the elements of the system's state vector and satisfies this condition. The expression $s(X)$ takes the value 1 if the condition is met and 0 if it is not met. We modify the MLE-model described above by adding two edges with edge functions

$$f_1'(X) = f_2'(X) = \bar{s}(X).$$

These functions will take the value 1 on vectors where the condition is not met, which means that $s(X) = 0$, so the added edges will remain in the graph, and the behavior of the modified model will match the behavior of the basic MLE-model. On vectors where the condition is met, and accordingly $s(X) = 1$, both functions f_1' and f_2' will take the value 0, resulting in the graph losing two edges, thus breaking the graph's connectivity. It is important to note that adding exactly two additional edges is a sufficient condition, but not a necessary one. If needed, the graph can be modified by adding more edges.

The resulting $K'(m, n)$ model will fully correspond to the behavior of a given (n, f, k) system in the failure flow, and it can be easily weakened on other vectors if new conditions appear in the future under which the failure of some processors leads to the failure of the entire system, or vice versa, strengthened on vectors if the system is resistant to failures on which the $K'(m, n)$ model loses its connectivity.

LINEAR CONSECUTIVE-K-OUT-OF-N SYSTEM

Such a system consists of n linearly arranged processors and fails if and only if at least k of its consecutive processors fail. The first and last processors in such a system are not connected to each other, meaning the system is open-ended. To construct the GL-model of such a system, we start by building the basic MLE-model. Unlike (n, f, k) systems, in consecutive-k-out-of-n systems, there is no condition on the number of non-consecutive failures that the system can withstand. As with the previous type of systems, it is important to calculate the maximum allowable number of failures without considering the condition of system failure due to k consecutively connected processor failures. Therefore, it is first appropriate to calculate the maximum allowable number of failures m_l.

THEOREM 2

$$m_l = n - \left\lfloor \frac{n}{k} \right\rfloor.$$

PROVING

Let only every k-th processor remain operational, and let the last k-th processor be at position d, meaning the numbers of the processors that remain operational will be: k, $2k$, $3k$…, dk, where $dk \le n$. Number d can be determined as the largest integer that does not exceed $\frac{n}{k}$, meaning

$$d = \left\lfloor \frac{n}{k} \right\rfloor.$$

Since d is the number of the last sequence of k processors, this number actually determines the minimum allowable number of operational processors in a system at which the system will remain in operation. To determine the maximum allowable number of failed processors, that is, the number m_l, it is sufficient to subtract the obtained number d from the total number of processors n:

$$m_l = n - d = n - \left\lfloor \frac{n}{k} \right\rfloor.$$

Let there be a given linear consecutive-k-out-of-n system, where $n=10$ and $k=3$.

$$m_l = 10 - \left\lfloor \frac{10}{3} \right\rfloor = 7.$$

Table 1 show the state vector of the system with the maximum number of zeros, where the given linear system will remain operational.

Table 1. **The state vector of linear system with the highest number of zeros**

x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
0	0	1	0	0	1	0	0	1	0

Source: **compiled by the authors**

As can be seen, the obtained state vector of the system does not contain *k* consecutive zeros, and therefore, the system will remain operational.

After determining the number m_l, further calculations and the construction of the GL-model for consecutive-k-out-of-n systems are identical to the calculations and construction for (n, f, k) systems.

CIRCULAR CONSECUTIVE-K-OUT-OF-N SYSTEM

Such a system is almost identical to linear consecutive-k-out-of-n systems, except that the first and last processors are directly connected, meaning the system is closed-loop. Accordingly, when calculating the maximum allowable number of failures and the number of all combinations of *k* consecutive failures, it should be taken into account that the failure of the last and first processors will be considered consecutive.

As with the previous two types of systems, we start constructing the GL-model by building the basic MLE-model. Since the given type of system does not have a condition regarding the number of non-consecutive failed components that would cause the entire system to fail, we begin by determining the number m_c.

THEOREM 3

$$m_c = n - \left\lceil \frac{n}{k} \right\rceil.$$

PROVING

Let only every *k*-th processor remain operational, and the last *k*-th processor is at position *d*. Thus, the positions of the processors that remain operational will be: *k*, *2k*, *3k*…, *dk*, where $dk \le n$. Since only every *k*-th processor remains operational, the first $k - 1$ processors will be in a non-operational state. Since the system is circular and the last processor is connected to the first, the last processor must also remain operational. Therefore, the number *d* can be determined as the smallest integer that exceeds $\frac{n}{k}$, meaning

$$d = \left\lceil \frac{n}{k} \right\rceil.$$

Therefore, *d* will be the number that determines the minimum allowable number of operational processors in the system at which the system will remain functional. To determine the maximum allowable number of failed processors, which is the number m_c, it is sufficient to subtract the obtained number *d* from the total number of processors *n*:

$$m_c = n - d = n - \left\lceil \frac{n}{k} \right\rceil.$$

Let there be a given circular consecutive-k-out-of-n system, where $n = 10$ and $k = 3$.

$$m_c = 10 - \left\lceil \frac{10}{3} \right\rceil = 6.$$

Table 2 shows the state vector of the system with the maximum number of zeros, where the given circular system will remain operational.

Table 2. **The state vector of circular system with the highest number of zeros**

x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
0	0	1	0	0	1	0	0	1	1

Source: **compiled by the authors**

Since the state vector of the system does not contain *k* consecutive zeros, the system remains operational.

After determining the number m_c, the MLE-model $K(m_c, n)$ can be constructed.

Obtained GL-model will be weakened on vectors containing *k* consecutive zeros, and it should be taken into account that the system is circular, so the last and first elements of the vector are considered consecutive. Since the sequence with *k* zeros can start from any element of the vector, the number of such vectors will be:

$$c(k, m) = n.$$

EXAMPLES

Example 1. As an example of a linear (n, f, k) system, consider a system consisting of *n* processors, each with *2k* ports for connecting to other processors and nodes (such as sensors, bus controllers, etc.). The system is organized so that connections are established between neighboring processors as well as between processors that are at a distance from 1 to $k-1$. The first and last processors are connected to external nodes. When calculating the reliability of the system, we will not consider the external nodes, as this would add additional conditions for system failure. We will only consider the part that meets the following condition: the system fails in the event of any f processor failures or if the connection between the two terminal nodes of the system is lost.

Let there be a given (n, f, k) system, where $n = 10$, $f = 5$, $k = 3$ (Fig. 1).

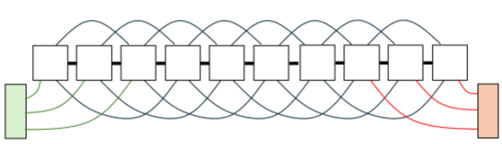

Fig. 1. **Example of a linear system for *n* = 10; *k* = 3**
***Source:* compiled by the authors**

Let's construct the MLE-model of the (n, f, k) system described above. We will determine the maximum allowable number of failures m under which the system remains operational. Accordingly, the graph of the GL-model will not lose connectivity on vectors with m or less zeros:

$$m = f - 1 = 5 - 1 = 4.$$

Now let's build the basic MLE-model *K(4, 10)*. The number of functions will be:

$$\rho(4,10) = 10 - 4 + 1 = 7.$$

Let's construct the graph of the MLE-model *K(4, 10)* (Fig. 2). We will define the edge functions of the GL-model according to [32].

$$f_1 = x_1 \vee x_2 \vee x_3 \vee x_4;$$

$$f_2 = (x_1 \vee x_2 \vee x_3)\left(\left(x_1 \vee x_2\right)\left(x_1x_2 \vee x_3\right) \vee x_4\right) \vee x_5;$$

$$f_3 = (x_1 \vee x_2 \vee x_3)\left(\left(x_1 \vee x_2\right)\left(x_1x_2 \vee x_3\right) \vee x_4x_5\right) \wedge \wedge\left(x_1x_2x_3 \vee x_4 \vee x_5\right) \vee x_6x_7x_8x_9x_{10};$$

$$f_4 = \left(x_1 \vee x_2\right)\left(x_1x_2 \vee x_3\right)\left(x_1x_2x_3 \vee x_4 \vee x_5\right)\left(x_4 \vee x_5\right) \vee \vee\left(x_6 \vee x_7\right)\left(x_6x_7 \vee x_8\right)\left(x_6x_7x_8 \vee x_9 \vee x_{10}\right)\left(x_9 \vee x_{10}\right);$$

$$f_5 = x_1x_2x_3x_4x_5 \vee (x_6 \vee x_7 \vee x_8) \wedge \wedge\left(\left(x_6 \vee x_7\right)\left(x_6x_7 \vee x_8\right) \vee x_9x_{10}\right)\left(x_6x_7x_8 \vee x_9 \vee x_{10}\right);$$

$$f_6 = x_6 \vee (x_7 \vee x_8 \vee x_9)\left(\left(x_7 \vee x_8\right)\left(x_7x_8 \vee x_9\right) \vee x_{10}\right);$$

$$f_7 = x_7 \vee x_8 \vee x_9 \vee x_{10}.$$

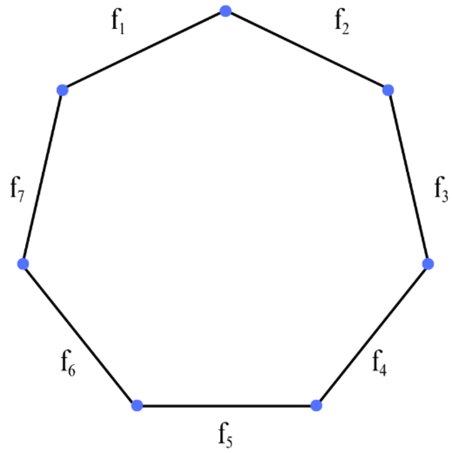

Fig. 2. **The graph of the GL-model *K(4, 10)***
***Source:* compiled by the authors**

In order for the basic GL-model to correspond to the behavior of the given (n, f, k) system in a failure flow, it is sufficient to identify all possible vectors with k consecutive failures and weaken the model on them. We will calculate the number of such vectors using formula (1):

$$c = 10 - 3 + 1 = 8.$$

All eight vectors with three consecutive zeros are listed in Table 3.

Table 3. **All vectors with three consecutive zeros**

No.	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
1	**0**	**0**	**0**	1	1	1	1	1	1	1
2	1	**0**	**0**	**0**	1	1	1	1	1	1
3	1	1	**0**	**0**	**0**	1	1	1	1	1
4	1	1	1	**0**	**0**	**0**	1	1	1	1
5	1	1	1	1	**0**	**0**	**0**	1	1	1
6	1	1	1	1	1	**0**	**0**	**0**	1	1
7	1	1	1	1	1	1	**0**	**0**	**0**	1
8	1	1	1	1	1	1	1	**0**	**0**	**0**

***Source:* compiled by the authors**

Let's weaken any two functions of the basic model on all the above-mentioned vectors. Thus, when a vector with three consecutive zeros appears, the graph will be guaranteed to lose at least two edges. Let these be functions f_1 and f_7. Let's draw the graph of the modified model (Fig. 3).

$$f_1' = (x_1 \vee x_2 \vee x_3 \vee x_4)\left(x_1 \vee x_2 \vee x_3\right) \wedge \wedge\left(x_2 \vee x_3 \vee x_4\right)\left(x_3 \vee x_4 \vee x_5\right)\left(x_4 \vee x_5 \vee x_6\right) \wedge \wedge\left(x_5 \vee x_6 \vee x_7\right)\left(x_6 \vee x_7 \vee x_8\right)\left(x_7 \vee x_8 \vee x_9\right) \wedge \wedge\left(x_8 \vee x_9 \vee x_{10}\right);$$

$$f_7' = (x_7 \vee x_8 \vee x_9 \vee x_{10})(x_1 \vee x_2 \vee x_3) \wedge \\ \wedge (x_2 \vee x_3 \vee x_4)(x_3 \vee x_4 \vee x_5)(x_4 \vee x_5 \vee x_6) \wedge \\ \wedge (x_5 \vee x_6 \vee x_7)(x_6 \vee x_7 \vee x_8)(x_7 \vee x_8 \vee x_9) \wedge \\ \wedge (x_8 \vee x_9 \vee x_{10}).$$

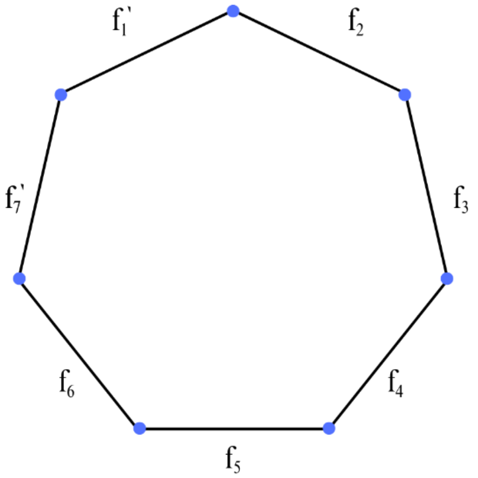

Fig. 3. **The graph of the GL-model *K'(4, 10)* with modified edges**
Source: **compiled by the authors**

Another way to modify the model is described in [34]. It is sufficient to change the structure of the graph by adding two edges. Assign functions f_8 and f_9 to these edges (Fig. 4).

$$f_8 = f_9 = (x_1 \vee x_2 \vee x_3)(x_2 \vee x_3 \vee x_4)(x_3 \vee x_4 \vee x_5) \wedge \\ \wedge (x_4 \vee x_5 \vee x_6)(x_5 \vee x_6 \vee x_7)(x_6 \vee x_7 \vee x_8) \wedge \\ \wedge (x_7 \vee x_8 \vee x_9)(x_8 \vee x_9 \vee x_{10}).$$

When a vector with three consecutive zeros appears, the graph will lose the edges f_8 and f_9, and the graph connectivity will be broken.

The structure of the graph can also be changed, for example, by adding an additional vertex and an edge. Assign a new edge function f_8 (Fig. 5).

$$f_8 = (x_1 \vee x_2 \vee x_3)(x_2 \vee x_3 \vee x_4)(x_3 \vee x_4 \vee x_5) \wedge \\ \wedge (x_4 \vee x_5 \vee x_6)(x_5 \vee x_6 \vee x_7)(x_6 \vee x_7 \vee x_8) \wedge \\ \wedge (x_7 \vee x_8 \vee x_9)(x_8 \vee x_9 \vee x_{10}).$$

In all cases, we obtain the non-basic model *K'(4, 10)*, which fully corresponds to the behavior of the given (n, f, k) system in a failure flow. For example, the model loses edge f_4 on the vector 0011111100, meaning that the graph maintains connectivity when 4 non-consecutive processors fail. At the same time, on the vectors 0001111111, 1000111111, 1100011111, 1110001111, 1111000111, 1111100011, 1111110001, 1111111000, the model loses connectivity, which corresponds to the condition of system failure when 3 consecutive processors fail.

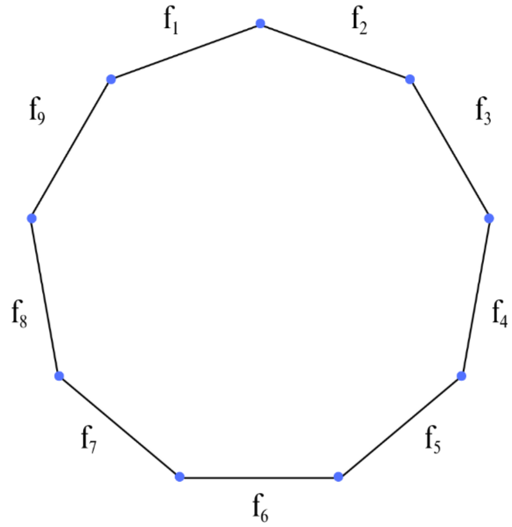

Fig. 4. **The graph of the GL-model *K'(4, 10)* with two additional edges**
Source: **compiled by the authors**

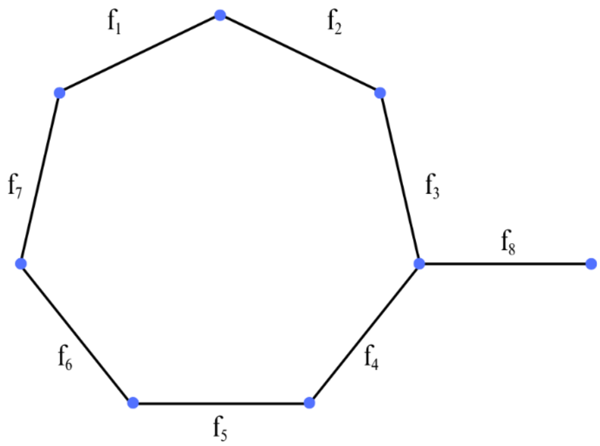

Fig. 5. **The graph of the GL-model *K'(4, 10)* with an additional vertex and edge**
Source: **compiled by the authors**

Example 2. For a linear consecutive-k-out-of-n system, an example can be a system similar to the one from the example for (n, f, k) systems. The only difference will be the absence of the condition regarding system failure when any *f* processors fail.

Let there be a given linear consecutive-k-out-of-n system, where $k = 2$ and $n = 11$. We will start by determining the maximum allowable number of failures m_l under which the system remains operational.

$$m_l = 11 - \left\lfloor \frac{11}{2} \right\rfloor = 6.$$

Accordingly, the graph of the GL-model will not lose connectivity on vectors with m_l or less zeros.

Let's construct the basic MLE-model *K (6, 11)*. The number of functions will be:

$$\rho(6,11) = 11 - 6 + 1 = 6.$$

We will define the edge functions for the MLE-model *K(6, 11)* according to [32]:

$$f_1 = x_1 \vee x_2 \vee x_3 \vee x_4 \vee x_5 \vee x_6;$$

$$f_2 = (x_1 \vee x_2 \vee x_3 \vee x_4 \vee x_5)\big((x_1 \vee x_2 \vee x_3 \vee x_4) \wedge \\ \wedge\big((x_1 \vee x_2 \vee x_3)\big((x_1 \vee x_2)(x_1x_2 \vee x_3) \vee x_4\big) \vee \\ \vee x_5\big) \vee x_6\big) \vee x_7x_8x_9x_{10}x_{11};$$

$$f_3 = (x_1 \vee x_2 \vee x_3 \vee x_4)((x_1 \vee x_2 \vee x_3)\big((x_1 \vee x_2) \wedge \\ \wedge(x_1x_2 \vee x_3) \vee x_4\big) \vee x_5x_6)\big((x_1 \vee x_2)(x_1x_2 \vee \\ \vee x_3x_4)(x_3 \vee x_4)\big) \vee (x_7 \vee x_8)(x_7x_8 \vee x_9) \wedge \\ \wedge(x_7x_8x_9 \vee x_{10}x_{11})(x_{10} \vee x_{11});$$

$$f_4 = (x_1 \vee x_2 \vee x_3)\big((x_1 \vee x_2)(x_1x_2 \vee x_3) \vee x_4x_5x_6) \wedge \\ \wedge\big(x_1x_2x_3 \vee (x_4 \vee x_5)(x_4x_5 \vee x_6)(x_4 \vee x_5 \vee x_6)\big) \vee \\ \vee(x_7 \vee x_8 \vee x_9)\big((x_7 \vee x_8)(x_7x_8 \vee x_9) \vee x_{10}x_{11}\big) \wedge \\ \wedge(x_7x_8x_9 \vee x_{10} \vee x_{11});$$

$$f_5 = (x_1 \vee x_2)(x_1x_2 \vee x_3x_4)(x_3 \vee x_4)(x_1x_2x_3x_4 \vee x_5x_6) \wedge \\ \wedge(x_5 \vee x_6) \vee \big((x_7 \vee x_8 \vee x_9 \vee x_{10})\big((x_7 \vee x_8 \vee x_9) \wedge \\ \wedge(x_7 \vee x_8)(x_7x_8 \vee x_9) \vee x_{10}\big) \vee x_{11}\big);$$

$$f_6 = x_1x_2x_3x_4x_5x_6 \vee x_7 \vee x_8 \vee x_9 \vee x_{10} \vee x_{11}.$$

Next, we will identify all the vectors that need to be blocked by weakening. Their number can be calculated using the formula from the example for (n, f, k) systems. For the given system, their number will be:

$$c = 11 - 2 + 1 = 10.$$

All ten vectors with two consecutive zeros are listed in Table 4.

Let's modify functions f_1 and f_6:

$$f_1' = (x_1 \vee x_2 \vee x_3 \vee x_4 \vee x_5 \vee x_6)(x_1 \vee x_2)(x_2 \vee x_3) \wedge \\ \wedge(x_3 \vee x_4)(x_4 \vee x_5)(x_5 \vee x_6)(x_6 \vee x_7)(x_7 \vee x_8) \wedge \\ \wedge(x_8 \vee x_9)(x_9 \vee x_{10})(x_{10} \vee x_{11});$$

$$f_6' = (x_1x_2x_3x_4x_5x_6 \vee x_7 \vee x_8 \vee x_9 \vee x_{10} \vee x_{11}) \wedge \\ \wedge(x_1 \vee x_2)(x_2 \vee x_3)(x_3 \vee x_4)(x_4 \vee x_5) \wedge \\ \wedge(x_5 \vee x_6)(x_6 \vee x_7)(x_7 \vee x_8)(x_8 \vee x_9) \wedge \\ \wedge(x_9 \vee x_{10})(x_{10} \vee x_{11}).$$

Table 4. **All vectors with two consecutive zeros**

No.	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}
1	**0**	**0**	1	1	1	1	1	1	1	1	1
2	1	**0**	**0**	1	1	1	1	1	1	1	1
3	1	1	**0**	**0**	1	1	1	1	1	1	1
4	1	1	1	**0**	**0**	1	1	1	1	1	1
5	1	1	1	1	**0**	**0**	1	1	1	1	1
6	1	1	1	1	1	**0**	**0**	1	1	1	1
7	1	1	1	1	1	1	**0**	**0**	1	1	1
8	1	1	1	1	1	1	1	**0**	**0**	1	1
9	1	1	1	1	1	1	1	1	**0**	**0**	1
10	1	1	1	1	1	1	1	1	1	**0**	**0**

Source: **compiled by the authors**

The graph of the resulting model will lose connectivity on the vectors 00111111111, 10011111111, 11001111111, 11100111111, 11110011111, 11111001111, 11111100111, 11111110011, 11111111001, 11111111100, and on other vectors with two consecutive zeros. Meanwhile, the graph maintains connectivity on the vector 01010101010.

Experimental evidence has shown that the resulting GL-model matches the behavior of the given linear consecutive-k-out-of-n system in a failure flow.

Example 3. For a circular consecutive-k-out-of-n system, consider a system that includes processors connected by a common bus. Each processor is connected to sensors of different types. Let each type of sensor be denoted by a number. Let *n* = 9 and *k* = 4 (Fig. 6).

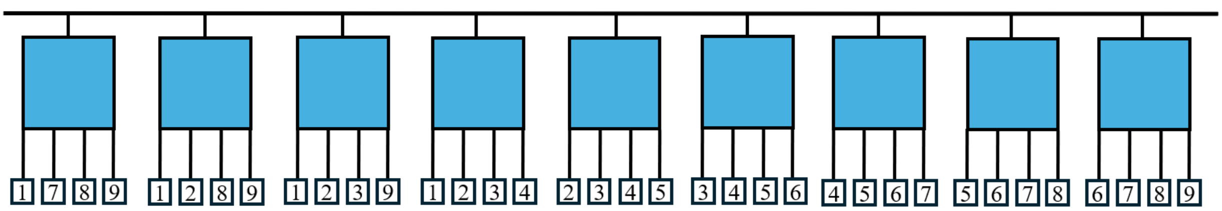

Fig. 6. **Example of a circular system for *n* = 9, *k* = 4**
Source: **compiled by the authors**

As in the previous example, let's start by determining the maximum allowable number of failures under which the system remains operational - the number m_c:

$$m_c = 9 - \left\lceil \frac{9}{4} \right\rceil = 6.$$

The next step is to construct the basic MLE-model *K(6, 9)*. Let's find the number of functions in the model:

$$\rho(6,9) = 9 - 6 + 1 = 4.$$

Let's define the edge functions for the MLE-model *K(6, 9)* according to [32]:

$$f_1 = x_1 \vee x_2 \vee x_3 \vee x_4 \vee x_5 \vee x_6;$$

$$f_2 = (x_1 \vee x_2 \vee x_3 \vee x_4 \vee x_5)\big((x_1 \vee x_2 \vee x_3 \vee x_4) \wedge \wedge\big((x_1 \vee x_2 \vee x_3)\big((x_1 \vee x_2)(x_1x_2 \vee x_3) \vee x_4\big) \vee \vee x_5\big) \vee x_6\big) \vee x_7x_8x_9;$$

$$f_3 = (x_1 \vee x_2 \vee x_3 \vee x_4)((x_1 \vee x_2 \vee x_3)\big((x_1 \vee x_2) \wedge \wedge(x_1x_2 \vee x_3) \vee x_4\big) \vee x_5x_6)\big((x_1 \vee x_2)(x_1x_2 \vee \vee x_3x_4)(x_3 \vee x_4) \vee (x_5 \vee x_6)\big) \vee (x_7 \vee x_8) \wedge \wedge(x_7x_8 \vee x_9);$$

$$f_4 = (x_1 \vee x_2 \vee x_3)\big((x_1 \vee x_2)(x_1x_2 \vee x_3) \vee x_4x_5x_6\big) \wedge \wedge\big(x_1x_2x_3 \vee (x_4 \vee x_5)(x_4x_5 \vee x_6)\big)(x_4 \vee x_5 \vee x_6) \vee \vee(x_7 \vee x_8 \vee x_9).$$

Now let's identify the vectors on which the model needs to be weakened to correspond to the given system. Since the system is circular, the sequence of *k* failures can start from any element. Therefore, the number of all vectors with consecutive-*k* failures will be *n*. All the vectors on which the graph should lose connectivity are listed in Table 5.

Table 5. **All vectors with four consecutive zeros**

No.	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
1	**0**	**0**	**0**	**0**	1	1	1	1	1
2	1	**0**	**0**	**0**	**0**	1	1	1	1
3	1	1	**0**	**0**	**0**	**0**	1	1	1
4	1	1	1	**0**	**0**	**0**	**0**	1	1
5	1	1	1	1	**0**	**0**	**0**	**0**	1
6	1	1	1	1	1	**0**	**0**	**0**	**0**
7	**0**	1	1	1	1	1	**0**	**0**	**0**
8	**0**	**0**	1	1	1	1	1	**0**	**0**
9	**0**	**0**	**0**	1	1	1	1	1	**0**

Source: **compiled by the authors**

As in the previous examples, we will weaken the model *K(6, 9)* on all vectors with *k* zeros. We will modify functions f_1 and f_2:

$$f_1 = (x_1 \vee x_2 \vee x_3 \vee x_4 \vee x_5 \vee x_6)(x_1 \vee x_2 \vee x_3 \vee x_4) \wedge \wedge(x_2 \vee x_3 \vee x_4 \vee x_5)(x_3 \vee x_4 \vee x_5 \vee x_6) \wedge \wedge(x_4 \vee x_5 \vee x_6 \vee x_7)(x_5 \vee x_6 \vee x_7 \vee x_8) \wedge \wedge(x_6 \vee x_7 \vee x_8 \vee x_9)(x_7 \vee x_8 \vee x_9 \vee x_1) \wedge \wedge(x_8 \vee x_9 \vee x_1 \vee x_2)(x_9 \vee x_1 \vee x_2 \vee x_3);$$

$$f_2 = \big((x_1 \vee x_2 \vee x_3 \vee x_4 \vee x_5)\big((x_1 \vee x_2 \vee x_3 \vee x_4) \wedge \wedge\big((x_1 \vee x_2 \vee x_3)\big((x_1 \vee x_2)(x_1x_2 \vee x_3) \vee x_4\big) \vee \vee x_5\big) \vee x_6\big) \vee x_7x_8x_9\big)(x_1 \vee x_2 \vee x_3 \vee x_4) \wedge \wedge(x_2 \vee x_3 \vee x_4 \vee x_5)(x_3 \vee x_4 \vee x_5 \vee x_6) \wedge \wedge(x_4 \vee x_5 \vee x_6 \vee x_7)(x_5 \vee x_6 \vee x_7 \vee x_8) \wedge \wedge(x_6 \vee x_7 \vee x_8 \vee x_9)(x_7 \vee x_8 \vee x_9 \vee x_1) \wedge \wedge(x_8 \vee x_9 \vee x_1 \vee x_2)(x_9 \vee x_1 \vee x_2 \vee x_3).$$

The graph of the resulting model will maintain connectivity on the vector 000100011, while on vectors containing four consecutive zeros, such as 000011111, 011111000, 00111100, 010000110, connectivity will be lost.

CONCLUSIONS

The paper proposes methods for constructing GL-models for (n, f, k) systems, linear consecutive-k-out-of-n systems, and circular consecutive-k-out-of-n systems. A distinctive characteristic of these non-basic types of systems is the failure of the entire system when *k* consecutively connected processors fail, whereas, for the failure of a basic system, it is sufficient for any *m*+1 processors to fail. A universal method for calculating the number of vectors that need to be blocked has been defined for all types of systems. A method for calculating the allowable number of failures for all types of systems has also been determined. It is demonstrated that the methods for constructing GL-models for (n, f, k) systems and both types of consecutive-k-out-of-n systems are similar, except for the calculation of the allowable number of non-consecutive failures. For (n, f, k) systems, the number *f* indicates the number of non-consecutive processor failures that cause the system to fail, so calculating the maximum allowable number of non-consecutive failures under which the system remains operational is straightforward. In turn, for both types of consecutive-k-out-of-n systems, calculating the maximum allowable number of failures is more complex, as the only specified failure condition is the failure of *k*

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consecutive components. Therefore, calculating the maximum allowable number of failures reduces to finding the minimum required number of functioning processors needed for the system to operate. It is enough to subtract this number from the total number of processors to determine the maximum allowable number of failures.

The model construction methods are based on the use of MLE-models. The resulting MLE-model is weakened on previously identified vectors containing *k* consecutive zeros. The MLE-model can be weakened by modifying the edge functions, changing the graph structure, or combining these two approaches. Examples of model construction for (n, f, k), linear consecutive-k-out-of-n systems, and circular consecutive-k-out-of-n systems are provided. It is experimentally demonstrated that the obtained models correspond to the given systems.

The paper describes the construction of GL-models for FTMS, where the system elements are processors. However, the described methods can also be applied to other types of systems whose components may include memory, network devices, and so on.

Further research may involve optimizing the approach to modifying edge functions or constructing GL-models for other types of non-basic k-out-of-n systems.

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Графо-логічні моделі для (n, f, k) – та послідовних-k-out-of-n – систем

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АНОТАЦІЯ

Стаття присвячена методам побудови GL-моделей (графо-логічні) відмовостійких багатопроцесорних систем. Зокрема розглянуті системи типу (n, f, k), лінійні послідовні k-out-of-n та кругові послідовні k-out-of-n, особливістю яких є вихід з ладу системи при відмові деякої кількості послідовних процесорів. GL-моделі можуть бути використані для оцінки параметрів надійності відмовостійких багатопроцесорних систем методом проведення статистичних експериментів із моделями їх поведінки в потоці відмов. В основі GL-моделей, що будуються лежать базові моделі з мінімальним числом ребер, що втрачаються. Визначено, що для побудови GL-моделі систем такого типу достатньо розрахувати максимально можливу допустиму кількість процесорів, що відмовили, при якій система залишається у робочому стані. Будується GL-модель базової системи, що витримує таку кількість відмов, без урахування послідовності цих відмов. Наступним кроком визначаються всі можливі послідовні відмови, при яких система виходить з ладу. Далі, базова модель модифікується таким чином, щоб відобразити на ній вихід з ладу системи при появі послідових відмов. Тобто, послабити базову модель на вище визначених векторах. Запропоновані методи побудови моделей можна використовувати як для лінійних та кругових послідовних k-out-of-n систем, так і для (n, f, k) систем. Незначна відмінність буде полягати в розрахунку деяких параметрів.

У роботі описані розрахунки таких параметрів, як максимально допустима кількість відмов при якій система залишається у робочому стані, а також розрахунок кількості всіх комбінацій послідовних відмов при яких система виходить з ладу. Проведені експерименти, що підтверджують відповідність моделі поведінці системи в потоці відмов. Наведені приклади, що демонструють процес побудови GL-моделей для лінійних послідовних k-out-of-n, кругових послідовних k-out-of-n та (n, f, k) систем запропонованими методами.

Ключові слова: GL-моделі (графо-логічні); МВР-моделі; небазові відмовостійкі багатопроцесорні системи; k-out-of-n системи

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