

## STOCHASTIC MODELS OF WORK AND REST SCHEDULES

**Purpose.** To develop stochastic models for determining the duration of work and rest schedules that ensure protection of workers from occupational hazards and a high level of labor productivity during a work shift.

**Methodology.** Analysis of scientific literature was applied to determine the purpose and tasks of the research; methods of formalization – to describe the characteristics, dynamics and states of random processes of accumulation and elimination of the consequences of the negative impact of harmful production factors occurring in a worker within the work and rest schedules; methods of semi-Markov processes theory, theories of reliability and recovery – to build stochastic models of work and rest schedules, determine the conditions and probability of a stable mode of their operation.

**Findings.** Stochastic models were developed to determine the duration of work schedules, at the end of which the level of accumulation of the consequences of the negative impact of harmful production factors in the worker will not exceed the set maximum permissible values with a high probability. And such durations of rest schedules, at the end of which this level will be equal to zero (also with a high probability). The condition for ensuring the stable operation of these schedules throughout the entire period of work experience at the workplace was determined by the author.

**Originality.** For the first time an approach to the development of work and rest schedules at workplaces is proposed, which, contrary to others, is based on determining the probabilities of the levels of accumulation of the consequences of the negative impact of harmful production factors in the worker at the end of these schedules, given the actual random and dynamic characteristics of such impact. This, in turn, allows ensuring the protection of the worker both from accidents caused by the fatigue and from professional illness.

**Practical value.** The proposed approach allows increasing the economic efficiency of the enterprise by ensuring a high level of labor productivity, which is achieved by simultaneous development of the maximum possible duration of work and the minimum possible duration of rest schedules, which exclude the worker's development and occurrence of fatigue and professional illness with a high probability.

**Keywords:** *occupational health and safety, work-rest schedule, professional illness, negative production factors*

**Introduction.** The creation and maintenance of safe, healthy and comfortable working conditions at workplaces is achieved by developing and implementing efficient and effective measures and means for occupational health and safety within the occupational health and safety management system in the workplace. These measures and means are aimed at minimizing or eliminating (if possible) the negative impact of hazardous and harmful factors on the worker of the “man – machine – environment” system, the consequences of which can be occupational injuries, as well as occupational diseases.

According to the statistics of the International Labor Organization (ILO), about 2.8 million people worldwide die annually from these occupational hazards [1]. Moreover, the mortality of workers from occupational diseases is more than 6.5 times higher than the mortality from the consequences of occupational injuries [1]. These statistics show, in particular, that the existing approaches to the development of measures and means for the prevention of occupational diseases are ineffective. This applies primarily to measures and means to minimize harmful production factors (HPF), which are the main causes of these occupational hazards.

The main measures and means to minimize (eliminate) the negative impact of the HPF on the worker during operation of the “man – machine – environment” systems are the development and implementation of scientifically based work – rest schedules (WRS) for each workplace (taking into account the nomenclature of the HPF that have a negative impact on the worker at the relevant workplace), as well as the use (if possible) of personal (collective) protective equipment (PCPE) [2, 3]. At the same time, depending on the nature of the HPF, either only such a measure as the WRS or the PCPE and the WRS together are used to protect the worker. Thus, for example, to protect the worker from the impact of such HPF as static, dynamic, emotional, visual overstrain, monotony of work and others, the WRS are used as the main measure to minimize the relevant occupational risks (due to the objective impossibility of applying other effective measures and means). And for protection against such HPF as chemicals and com-

pounds, dust, noise, vibration, increased or decreased air temperature of the working area and others – the PCPE and the WRS are used together.

The development and implementation of the WRS involves establishing relevant scientifically based durations of working hours and breaks for rest for each workplace (regardless of the possibility of using the PCPE), which ensure so-called “protection by time” of the worker from the negative impact of the identified (at this workplace) HPF. The principle of “protection by time” is that, by limiting the duration of the worker's work schedule (taking into account the use of the PCPE at the workplace which can only reduce – not eliminate – the intensity of accumulation of the consequences of the HPF negative impact) it is possible to limit the accumulation level of the HPF negative impact in the worker (to the maximum permissible values). And by establishing relevant duration of the rest schedule, it is possible to achieve such a state in which (after the end of the rest schedule) the consequences of accumulation of this impact in the worker are highly likely to be completely absent (due to the natural physiological processes of recovery of the worker). That is, implementing the principle of “protection by time”, within the implementation of the WRS, it is possible to minimize the risk of relevant occupational hazards by minimizing the accumulation level of the consequences of the HPF negative impact in the worker during working hours and removal of the consequences of this accumulation during non-working hours.

In this case, the development of the WRS is a non-trivial task, since both the processes of accumulation of the consequences of the HPF negative impact in the worker and the processes of removal of the consequences of this accumulation from the worker are complex, random, dynamic processes that depend on many factors (random characteristics of the HPF impact on the worker in time during the work shift, physiological features of the worker, qualitative characteristics of the rest schedule, etc.) Accordingly, the objectivity of obtaining the results, namely required duration of the WRS, on which, in turn, depends the risk level of the appropriate occupational hazards depends on how objectively these processes can be studied (within applying an appropriate methodological ap-

proach) [4]. Taking into account these statistics on the number of cases of occupational hazards associated with the HPF impact on the worker, there is a need to analyze the existing methodological foundations of development of the WRS and determine the expediency and possible ways of their improvement based on the results of the analysis.

**Literature review.** Modern methodological approaches to the development of the WRS are represented by a set of methods for researching changes in the physiological state of workers of various professions during their work process, as well as during rest at work [5]. This research was aimed at determining the recovery time of the worker after beginning the work schedule to fatigue, under the HPF impact on the worker. However, the state of fatigue of the worker, as well as the state of recovery, is determined by certain physiological signs (blood pressure, state of the respiratory system, etc.) through the use of appropriate laboratory equipment or the subjective assessment of the worker (subjective assessment of health, for example, by using the Occupational Fatigue Exhaustion Recovery (OFER) Scale, Samn-Perelli Fatigue Scale, Chalder Fatigue Scale, etc.) [5, 6].

Depending on the signs of the research of the physiological state of the worker, all existing methods of development of the WRS can be divided into the following main groups, namely [5]: methods for assessing the state of the central nervous system, mental processes and functions of the worker; methods for assessing the state of the neuromuscular system; methods for assessing the dynamics of the cardiovascular, respiratory system; method for researching the subjective state of health, activity and mood of the worker; method for assessing the degree of fatigue, etc. However, regardless of what signs of the physiological state of the worker are researched, within this or that method, the criteria for determining duration of the WRS and the purpose of developing the WRS themselves are common.

Accordingly, the criterion for the end of the work schedule is fatigue of the worker, then the rest schedule, the criterion for the end of which is recovery of the worker's working capacity according to defined (within a certain method) physiological or subjective signs. The results obtained from the research on duration of the WRS are collected for each member of the control group of workers (taking into account the professional direction, personal and other characteristics) and processed using mathematical methods of processing experimental data (statistical, mathematical modeling methods, etc.) (Silverstein B.A., Fine L.J., & Armstrong T.J., 1986). According to the results of data processing, an average result is determined, which is the basis for recommendations for establishing the WRS of workers of certain professions.

Moreover, the main purpose of the development of the WRS is not "protection by time" of the worker from the HPF negative impact of the work process, but determination of such duration of the WRS that would ensure a high level of labor productivity during the work shift (by maintaining an optimal level of worker's working capacity) and prevent accidents due to the occurrence and development of fatigue in the worker.

Traditional methodological approaches to the development of the WRS (as noted in Rohmert W., 1973 and Leonova A. B., 1984) included the involvement of a sufficiently large control group of workers for the research, usually from 1,000 to 1,500 people, and processing of the results obtained of duration of the WRS (for each of the group members) using the methods of mathematical statistics. This made it possible to increase the objectivity of determining the recommended duration of the WRS (by increasing the number of members of the control group) for an average worker (in professional direction) due to obtaining and processing large arrays of experimental data that take into account the real characteristics of the HPF impact of the work process on the worker, as well as the worker's personal characteristics. At the same time, since the involvement of such a large number of workers for

the research is not always possible (especially at small and medium-sized enterprises) and also requires significant time, there is a need to improve these methodological approaches by applying methods of mathematical modeling of random processes for the development of the WRS.

These methods make it possible to determine the probabilistic characteristics of duration of the WRS based on the processing of experimental (or statistical) data of the research of small control groups (in comparison with previous methodological approaches).

Thus, in the study [7] the development of the WRS was conducted by researching a control group of rebar workers (19 people) who worked in the work-to-exhaustion-then-take-a-rest mode. The time characteristics of duration of the WRS were determined by monitoring such signs of the physiological state of the body as the state of the respiratory, cardiovascular system, taking into account the meteorological conditions of the working environment, as well as personal characteristics (overweight, age, bad habits, etc.). The data obtained were processed using the Monte Carlo simulation technique and scenario analysis, which made it possible to obtain probabilistic characteristics of duration of the WRS taking into account the criterion of maximizing the productive time and to develop the results of previous similar studies [8, 9].

However, if to analyze the results of this study through a comparison of the data on duration of the rest schedule obtained by modeling and experimentally, it is clear that, for example, in reality (according to the results of experimental research) only 78 % of workers can recover after the work schedule for the recommended 15-minute breaks (according to the results of modeling) [7]. That is, 22 % of workers, with a high degree of probability, have the consequences of residual level of accumulation of the consequences of the HPF negative impact in the worker, which significantly increases the risk of relevant occupational hazard (occupational disease).

An approach that allows determining duration of the WRS by processing experimental data of the performance of  $n$  tasks by the worker within  $m$  cycles of working time using mixed-integer linear programming (MILP) is presented in the study [10]. However, the authors themselves do not consider this approach to be perfect for the objective determination of required durations, due to the impossibility to take into account the real random and dynamic characteristics of the processes of fatigue and recovery of the worker. The authors note that the impossibility to take into account such characteristics does not allow to objectively assess both the accumulation level of the consequences of the HPF negative impact, which affect the occurrence of fatigue during the work schedule and the residual level of accumulation at the end of the rest schedule. This, in turn, can significantly increase the risk of occupational hazards for the worker.

The possibility to take into account the random characteristics of the work process, which affect the occurrence of worker's fatigue, by stochastic modeling of the worker's work schedule is shown in the study [11]. In this case, the work schedule is proposed to be developed taking into account the random characteristics of the worker's working time which depend on the reliability of machines. The machines may fail at random time periods during the work shift that affects duration of the WRS. Applying this approach, it is possible to objectively determine the time characteristics of the WRS, taking into account the random characteristics of the reliability of machines, but without taking into account the objective (random and dynamic) characteristics of the HPF negative impact on the worker, which appear during the operation of such machines (during the work schedule). This approach is aimed exclusively at determining the duration of the WRS that ensures the prevention of fatigue (to ensure a high level of labor productivity), due to taking into account these objective characteristics of working capacity of the equipment. But, as in previous studies, the presented approach does not take into

account the consequences of the negative impact of the HPF in the worker both at the end of the work schedule and at the end of the rest schedule, which, again, can lead to the occurrence of an occupational disease in the worker (over time).

It should be noted that the existing methodological approaches to the development of the WRS are not limited to the results of the analysis of relevant methods and research. There are other approaches based on the use of the mathematical tools of fuzzy logic methods (Dumitru V., Luban F., 1982), the construction of interval mathematical models (Matsveichuk N. M., Sotskov Yu. N., Egorova N. G., Lai T. C., 2009), scenario models (Aloulou M. A., Della Croce F., 2008) and others to the development of the WRS (including as part of production process management planning) [12].

However, regardless of which approach is used to the development of the WRS, their common disadvantage is that they do not take into account the risk of either excessive level of accumulation of the consequences of the HPF negative impact in the worker at the end of the work schedule, or residual level of accumulation of this impact at the end of the rest schedule. The existence of this problem significantly increases the risk of irreversible negative changes to the worker, which eventually can lead to the occurrence of relevant occupational diseases [13].

**Unsolved aspects of the problem.** Thus, as can be seen from the analysis, through implementing the WRS at a workplace developed within the existing methodological approaches, an employer can only rely on maintaining the optimal level of the worker's working capacity during the work shift and only minimizing the risk of an accident that may occur due to fatigue, but cannot effectively protect the worker from the occurrence (over time) of the relevant occupational disease. This fact may explain the above statistics on the significant part of fatalities from occupational diseases in the overall structure of occupational hazards [1].

The urgency of the problem of occupational diseases and related mortality due to improper organization of the WRS is also emphasized by the WHO/ILO Joint Estimates [14]. The authors note that the main risk factor for the occurrence of occupational diseases is the improper organization of the WRS, namely, excessive duration of the work schedule and, accordingly, reduced duration of the rest schedule, as a result of which there is gradual accumulation of the consequences of the HPF negative impact in the worker, which eventually leads to the occurrence of occupational disease [14]. At the same time, it is noted that the number of occupational diseases cases due to improper organization of the

WRS has a steady increase that indicates the ineffectiveness of existing approaches to the development of the WRS at workplaces in terms of prevention of relevant occupational hazards [14].

And although this study analyzed the relationship between relevant occupational diseases and the WRS, it can be assumed that the relevant conclusions are valid for other occupational diseases, since their occurrence is also associated with gradual accumulation of the consequences of the HPF negative impact in the worker, due to improper organization of the WRS which is confirmed by other researchers [15].

Thus, given that the existing methodological approaches to the development of the WRS are aimed solely at maintaining a high level of labor productivity (by maintaining the optimal level of worker working capacity) during a work shift and solely at minimizing the risks of accidents resulting from fatigue, there is *an unresolved urgent problem* of developing such the WRS that would be able to protect the worker also from the occurrence of occupational diseases; in other words, to ensure protection of the worker from the occurrence of these occupational hazards.

That is, an improved approach to the development of the WRS should be aimed at developing such WRS which, with a sufficiently high probability, will prevent the occurrence of the

relevant occupational disease due to gradual accumulation of the HPF negative impact in the worker while maintaining a high level of labor productivity. At the same time, the WRS developed in this way will also be aimed at minimizing the risks of accidents due to fatigue, since its occurrence is also the cause of accumulation of the consequences of the HPF negative impact in the worker.

It should be noted that the author of this research does not question the expediency and objectivity of the existing methodological approaches intended for the development of the WRS, but only strives for their improvement in terms of ensuring the principle of priority of the worker's occupational safety.

**Justification of the reasons for the existence of the identified problem and ways to solve it.** The reason for the existence of the identified urgent scientific and practical problem is precisely the purpose of the development of the WRS and the related criteria of duration of the WRS within the existing methodological foundations. As noted above, the purpose of the development of the WRS within the existing methodological foundations is to determine the durations of the WRS that would ensure a high level of labor productivity during a work shift and prevent accidents due to fatigue. Accordingly, the criterion for determining the end of the work schedule is fatigue of the worker, and the criterion for the end of the rest schedule is physiological or subjective signs of recovery from fatigue. The WRS developed in this way prevent, with a certain probability (depending on which methodological approach is used), the occurrence and development of fatigue, but do not solve the problem of the need to take into account the risk of excessive (residual) level of accumulation of the consequences of the HPF negative impact in the worker.

In order to minimize such risk, the development of the WRS should pursue the goal of preventing (with a probability close to 1) accumulation of the HPF negative impact in the worker at the end of the work schedule to exceed the established (for example, maximum permissible) values as well as removal of the consequences of this accumulation (also with a high probability) during the rest schedule to the established (lower than the maximum permissible) values.

The latter is necessary, since, if the maximum permissible values (MPV) are established as the criterion for the end of the rest schedule, the accumulation level of the HPF negative impact in the worker will gradually increase and exceed them (the accumulation level will become excessive) during the next work schedule; it is unacceptable. The expediency of applying such an approach to determining the criteria for duration of the WRS is also emphasized in [10, 16].

However, it should be noted that, if the establishment of the MPV of accumulation of the consequences of the HPF negative impact in the worker as a criterion for the end of the work schedule is not only objective but also practically possible (however, so far, not for all the HPF), the establishment of the values of the residual level of accumulation of the consequences of the HPF negative impact (less than the MPV) as a criterion for the end of the rest schedule is currently objectively impossible [4].

This is explained by the fact that, unlike MPV, the characteristics of which are regulated within the current legislation of any country in the world, the residual level of accumulation of the HPF negative impact is not a standardized indicator. The possibility of its standardization is a complex scientific and practical task, similar to determining the characteristics of MPV. Therefore, taking this into account, as well as the need to ensure the principle of safety priority within this research, it is proposed to establish a zero residual level of accumulation of consequences of the HPF negative impact in the worker as an objective criterion for the end of the rest schedule.

Accordingly, it can be proposed to establish the MPV of accumulation of the consequences of the HPF negative impact in the worker inherent in the relevant work process (the most dangerous in rank) as an objective criterion for the end of

the work schedule; and the zero residual level of accumulation of the consequences of the HPF negative impact in the worker as a criterion for the end of the rest schedule.

In this case, during the development of the WRS, it is also important that duration of the relevant schedule (within the proposed criteria) could ensure a high level of labor productivity while maintaining a high level of occupational safety. This can be achieved, firstly, by ensuring the possibility of controlling the parameters of the intensity of the accumulation of the consequences of the HPF negative impact in the worker and the removal of such impact (on which the duration of the WRS depends). And, secondly, by taking into account the objective random and dynamic characteristics of accumulation of the consequences of the HPF negative impact in the worker during the work schedule [11], as well as by taking into account the relevant characteristics of removal of the consequences of this accumulation during the rest schedule (after hours) [11].

The methodological approaches analyzed above (within which this or that methods for researching random processes are used to develop the WRS) do not allow taking into account these characteristics. Meanwhile, the neglect of the fact that the real intensity of impact on the worker during the work schedule is not constant but random (respectively, accumulation of the consequences of the HPF negative impact is not constant but random) can lead to underestimation (decrease) of duration of the work schedule during modeling [17]. And the neglect of the fact that the consequences of this accumulation are also removed with different intensity (in particular due to the possibility of activating the physiological processes of recovery of the worker, for example, by applying relevant qualitative characteristics of the rest schedule) can lead to overestimation (increase) of duration of the rest schedule. This, in turn, can significantly reduce the economic efficiency of an enterprise due to the irrational use of workers' working time.

It is possible to control these parameters and take into account the relevant characteristics within the mathematical tools of the theory of Markov processes, which is a flexible tool for researching random and dynamic processes. The practical possibility of constructing stochastic models based on the use of Markov processes to model the hybrid HPF negative impact in the worker (when some processes can change continuously over time, and the other part – discretely, i.e., abruptly) is shown in the author's previous research [18].

But, in contrast, in our case, within modeling the WRS, we should consider not hybrid, but only continuous-time random processes of accumulation of the consequences of the negative impact of the HPF in the worker and their removal, which occur with random intensity. That is, the processes of transition of a worker from the state of accumulation of the consequences of the HPF negative impact (within the work schedule) to the state of recovery from this impact (during the rest schedule) and vice versa. It is convenient to research these processes using the mathematical tools of semi-Markov processes, which combine the properties of Markov processes and recovery processes.

**Purpose.** Thus, given that the main problem of existing approaches to the development of the WRS is the impossibility (within their framework) to ensure the protection of workers (from accidents caused by fatigue and occupational diseases), **the purpose** of this research is to develop stochastic models for determining the duration of work and rest schedules that ensure protection of workers from the occurrence of these occupational hazards and a high level of labor productivity during a work shift.

**Methods.** Within the purpose, to mathematically describe the processes associated with the random and dynamic characteristics of the accumulation of the consequences of the HPF negative impact in the worker during the work schedule, as well as the removal of the consequences of this impact from the worker during the rest schedule, the research used methods of the theory of semi-Markov processes, methods of the

theory of reliability and the theory of recovery. It should be noted that, based on the analysis of scientific research, the author is not aware of the use of the mathematical tools of the theory of semi-Markov processes to develop the WRS.

On objective bases related to the consideration of real processes that take place within the functioning of work and rest schedules, the following assumptions were made in the development of stochastic models.

Assume that during the work schedule, as a result of the operation of a certain unit of production equipment, the worker is impacted by a certain HPF. As a result, the consequences of the negative impact of the given HPF accumulate in the worker with intensity  $A$ . During the rest schedule, as a result of natural processes occurring in the worker, the consequences of the given negative impact are removed from the worker with intensity  $B$ . The period after the end of the current work shift (within the current day) before the start of a new work shift (the next day) is also assumed to be a rest schedule.

Assume also that, both during the working and rest schedules, a sudden failure of the given production equipment (regardless of previous failures) may occur in a short time interval  $(t, t + \Delta t)$  with a probability  $\alpha_0 \Delta t + \nu(\Delta t)$  and  $\alpha_1 \Delta t + \nu(\Delta t)$ , respectively. During the failure, the production equipment immediately begins to be repaired (restored), and the worker immediately transitions to the rest schedule if the failure occurred during the work schedule; or remains in it if the failure occurred during the rest schedule. In both cases, the consequences of the accumulation of the HPF negative impact are removed from the worker with intensity  $B$ .

At the same time, work and rest schedules should be considered as alternating, since the time of the end of the current rest schedule is the beginning of the work schedule (following it) and vice versa. Thus, the duration of the work schedule is the time interval between the end of the current rest schedule and the end of the work schedule following it. Accordingly, the duration of the rest schedule is the time interval between the end of the current work schedule (or the end of the current work shift) and the end of the work schedule following it (or the start of a new work shift).

Taking into account the fact that the times of the end of the WRS are characterized by the appropriate levels of accumulation of the consequences of the HPF negative impact in the worker (see the criteria for the duration of the WRS justified above), for the practical purposes of developing the WRS, it is sufficient to know two indicators, namely:

1. The probability that at the end of the current work schedule (at time  $t$ ) the level of accumulation of the consequences of the HPF negative impact in the worker will not exceed the MPV, taking into account the random and dynamic characteristics of the intensity of the accumulation of the consequences of this impact.

2. The probability that at the end of the current rest schedule, the consequences of the HPF negative impact in the worker will be completely absent, taking into account the random and dynamic characteristics of the intensity of the removal of the consequences of this impact from the worker.

In other words, at the end of the current work schedule (the end of the current work shift) the following condition must be fulfilled

$$\lim_{t \rightarrow \infty} \mathbf{P}\{\eta(t) \leq \omega\} > 1 - \sigma, \quad (1)$$

where  $\eta(t)$  is the level of accumulation of the consequences of the HPF negative impact in the worker;  $\sigma$  is given small probability;  $\omega$  is the maximum permissible value of the relevant HPF.

And at the end of the current rest schedule (the beginning of a new work shift) the following condition must be fulfilled

$$\lim_{t \rightarrow \infty} \mathbf{P}\{\eta(t) = 0\} > 1 - \sigma. \quad (2)$$

For the formalized description of the random processes that occur during work and rest schedules, their states are

characterized and the following conventional symbols are introduced:

1. *Work schedule:*

1.1. At time  $t$ , the worker is within the workplace (he works); production equipment operates, producing the HPF, the consequences of the negative impact of which accumulate in the worker with intensity  $A$ . The consequences of this impact are removed from the worker with intensity  $B$  (given that the natural processes of removing the consequences of negative impact from the body, in particular with sweat, urine, etc. occur constantly, including during the work schedule) [15, 19]. The variables describing the state of the worker ( $\zeta(t)$ ) and production equipment ( $v(t)$ ) at time  $t$  take the following values during the work schedule:  $\zeta(t) = 1$ ;  $v(t) = 0$ .

2. *Rest schedule:*

2.1. At time  $t$ , the worker is in the rest schedule (outside the workplace), the production equipment works properly. The consequences of the HPF negative impact are removed from the worker with intensity  $B$ ;  $A = 0$ ;  $\zeta(t) = 0$ ;  $v(t) = 0$ .

2.2. During the rest schedule, at time  $t$ , a sudden failure of production equipment occurs. The worker continues to be in the current schedule until the production equipment is restored, with  $A = 0$ ;  $\zeta(t) = 0$ ;  $v(t) = 1$ ; the consequences of the HPF negative impact are removed from the worker with intensity  $B$ .

Given that the duration of a work shift and non-working time (after the end of a work shift) are fixed values in practice, assume that the periods of work and rest schedules (within a work shift) are also fixed ones in time. Based on the same considerations, assume that the total period of working time ( $T_0$ ) is less than the relevant period of non-working time ( $T_0$ ) (during the current day). In this case, the distribution functions of the duration of work and rest schedules  $Y_i(t)$ ,  $i = 0, 1$ , will be the following

$$Y_i(t) = \begin{cases} 0, & 0 \leq t \leq T_i \\ 1, & t > T_i \end{cases}, \quad (3)$$

where  $i = 0$  is for the period of work schedules,  $i = 1$  is for the period of rest schedules.

In order to prevent a worker from developing an occupational disease (over time) caused by a possible gradual exceeding of the maximum permissible values of the relevant HPF in worker, it is necessary to ensure meeting the condition of stable operation of the WRS. Namely, the condition when during a specified (long) period of time of the worker's work activity at the relevant workplace (for example, the entire work experience period), the average value of  $A$  would not exceed the average value of  $B$ . In mathematical form (taking into account the above conventional symbols and characteristics of the WRS states (1.1.)), this condition will be the following

$$AP_{10} < A, \quad (4)$$

where the probability  $P_{10}$  is defined as follows

$$P_{10} = \int_0^{\infty} \int_0^{\infty} \tilde{\delta}_{10}(\tau, u)(1 - Y_1(\tau))d\tau du, \quad (5)$$

where  $\tau$  is the time from the beginning of the current work schedule to its end (time  $t$ );  $u$  is the level of accumulation of the consequences of the HPF negative impact in the worker at time  $t$ ;  $x_{10}$  is the probability density of the joint distribution  $\eta(t)$  and the time remained until the change in the current work schedule to the rest schedule.

Based on the above assumptions, conventional symbols and introduced state characteristics, the accumulation process  $\eta(t)$  within the WRS operation can be described by the following differential equation (with probability 1)

$$\begin{aligned} \eta'(t) &= AI(\zeta(t) = 1; v(t) = 0) - BI(\zeta(t) = 0; v(t) = 0 \vee 1); \\ \eta(t) &> 0, \end{aligned} \quad (6)$$

where  $I(Y)$  is the indicator of event  $Y$ .

As can be seen, solving the differential equation (6) requires finding the distribution of a random process ( $\eta(t)$ ,  $\zeta(t)$ ,  $v(t)$ ), which in the presented form is not Markov one. To be able to find this distribution, by further operating with the Markov process, taking into account the function (3), an additional continuous component  $\psi(t)$ , which characterizes the time remaining from moment  $t$  to the change in the state of the current schedule (work or rest) to the one following it (rest or work, respectively) are introduced [20, 21]. Thus, as a result, a random Markov process which will be used in the future is obtained

$$H(t) = (\eta(t), \zeta(t), v(t), \psi(t)). \quad (7)$$

Based on the theory of semi-Markov processes, to be able to build stochastic models of the WRS, it is necessary to characterize the dynamics and find the marginal distribution of the introduced random process (7) [20, 21]. The latter, in turn, implies the need to derive a system of partial differential equations and the relevant boundary conditions.

To characterize the dynamics of the process (7), taking into account the accepted conventional symbols, the relevant probability densities are introduced

$$\begin{aligned} P\{\zeta(t) = i, v(t) = j, \tau < \psi(t) < \tau + d\tau, u < \eta(t) < u + du\} &= \\ = x_{ij}(u, \tau, t)(1 - Y_i(\tau))d\tau du, i = 0, 1; j = 0, 1; u > 0, \tau > 0; \\ P\{\zeta(t) = 0, v(t) = 0, \tau < \psi(t) < \tau + d\tau, \eta(t) = 0\} &= \\ = x_{00}^-(\tau, t)(1 - Y_0(\tau))d\tau, \tau > 0; \\ P\{\zeta(t) = i, v(t) = 1, \tau < \psi(t) < \tau + d\tau, \eta(t) = 0\} &= \\ = x_{i1}^-(\tau, t)(1 - Y_i(\tau))d\tau, i = 0, 1; \tau > 0, \end{aligned} \quad (8)$$

where  $u$  is the level of accumulation of the consequences of the HPF negative impact in the worker at time  $t$ ;  $\tau$  is the time from the beginning of the current schedule (work or rest schedule) to time  $t$  (the time of the end of the relevant current schedule);  $x$  is the probability density of the joint distribution of  $\eta(t)$  and the time remaining until the change in work schedule to rest schedule and vice versa;  $i$  and  $j$  are discrete variables describing the values of  $\zeta$  and  $v$ , respectively, at time  $t$ .

The marginal distribution of the random process (7), relative to the introduced probability densities, is as follows [20, 21]

$$\begin{aligned} x_{ij}(u, \tau) &= \lim_{t \rightarrow \infty} x_{ij}(u, \tau, t), i = 0, 1, j = 0, 1; \\ x_{00}^-(\tau) &= \lim_{t \rightarrow \infty} x_{00}^-(\tau, t); \\ x_{i1}^-(\tau) &= \lim_{t \rightarrow \infty} x_{i1}^-(\tau, t), i = 0, 1. \end{aligned} \quad (9)$$

**Results.** Using the standard approach of semi-Markov process theory, which is used to find the marginal distribution of a random process, as well as the full probability formula, the following system of partial differential equations is derived for functions (9) [20, 21]

$$\begin{aligned} \left(-B \frac{\partial}{\partial u} + \frac{\partial}{\partial \tau}\right) x_{00}(u, \tau) &= -\alpha_0 x_{00}(u, \tau) + \beta_0 x_{01}(u, \tau); \\ \left(-B \frac{\partial}{\partial u} + \frac{\partial}{\partial \tau}\right) x_{01}(u, \tau) &= -\beta_0 x_{01}(u, \tau) + \alpha_0 x_{00}(u, \tau), \\ u > 0, \tau > 0; \end{aligned} \quad (10)$$

$$\begin{aligned} \left(S \frac{\partial}{\partial u} + \frac{\partial}{\partial \tau}\right) x_{10}(u, \tau) &= -\alpha_1 x_{10}(u, \tau) + \beta_1 x_{11}(u, \tau); \\ \left(-B \frac{\partial}{\partial u} + \frac{\partial}{\partial \tau}\right) x_{11}(u, \tau) &= -\beta_1 x_{11}(u, \tau) + \alpha_1 x_{10}(u, \tau), \\ u > 0, \tau > 0, \end{aligned} \quad (11)$$

where  $\alpha_0$  and  $\alpha_1$  are the intensities of the flow of equipment failures during rest and work schedules, respectively;  $\beta_0$  and  $\beta_1$  are the intensities of the flow of removal of the consequences of the HPF negative impact from the worker (during rest and work schedules, respectively);  $S$  is an indicator characterizing

the dynamics of the intensity of accumulation of  $\eta(t)$  in the worker (the difference between intensities  $A$  and  $B$ ).

For the system of differential equations (10) and (11), the following necessary boundary conditions were derived by the standard method [20, 21].

Conditions describing the state of the production equipment and the worker during the work and rest schedules

$$\begin{aligned} \frac{d}{d\tau}x_{00}^-(\tau) - Bx_{00}(0, \tau) &= -\alpha_0x_{00}^-(\tau) + \beta_0x_{01}^-(\tau); \\ \frac{d}{d\tau}x_{01}^-(\tau) - Bx_{01}(0, \tau) &= \alpha_0x_{00}^-(\tau) - \beta_0x_{01}^-(\tau); \\ \frac{d}{d\tau}x_{11}^-(\tau) - Bx_{11}(0, \tau) &= -\beta_1x_{11}^-(\tau), \quad \tau > 0. \end{aligned} \quad (12)$$

Boundary conditions of changing work and rest regimes are

$$\begin{aligned} x_{00}(u, 0) &= \int_0^\infty x_{10}(u, \tau) dY_1(\tau); \\ x_{01}(u, 0) &= \int_0^\infty x_{11}(u, \tau) dY_1(\tau); \\ x_{10}(u, 0) &= \int_0^\infty x_{00}(u, \tau) dY_0(\tau); \\ x_{11}(u, 0) &= \int_0^\infty x_{01}(u, \tau) dY_0(\tau); \end{aligned} \quad (13)$$

$$\begin{aligned} x_{01}^-(0) &= \int_0^\infty x_{11}^-(\tau) dY_1(\tau); \\ x_{11}^-(0) &= \int_0^\infty x_{01}^-(\tau) dY_0(\tau); \\ x_{00}^-(0) &= 0. \end{aligned} \quad (14)$$

Boundary conditions describing the worker's transition from the rest schedule to the work schedule are

$$S \int_0^\infty x_{10}(0, \tau)(1 - Y_1(\tau)) d\tau = \beta_1 \int_0^\infty x_{11}^-(\tau)(1 - Y_1(\tau)) d\tau. \quad (15)$$

Normalization condition is

$$\begin{aligned} \int_0^\infty (x_{00}^-(\tau) + x_{01}^-(\tau))(1 - Y_0(\tau)) d\tau + \int_0^\infty x_{11}^-(\tau)(1 - Y_1(\tau)) d\tau + \\ + \int_0^\infty [(x_{00}(u, \tau) + q_{01}(u, \tau))(1 - Y_0(\tau)) + \\ + (x_{10}(u, \tau) + x_{11}(u, \tau))(1 - Y_1(\tau))] d\tau = 1. \end{aligned} \quad (16)$$

As a result of solving problems (10–16), expressions for finding required duration for the end of the work schedule and rest schedule, respectively are obtained (1 and 2), namely:

- the probability that, at the end of the current work schedule (at time  $t$ ), the accumulation level of the consequences of the HPF negative impact in the worker will not exceed the MPV

$$\begin{aligned} \int_0^\infty \int_0^\infty [(x_{00}(u, \tau) + x_{01}(u, \tau))(1 - Y_0(\tau)) + \\ + (x_{10}(u, \tau) + x_{11}(u, \tau))(1 - Y_1(\tau))] d\tau du, \end{aligned}$$

where  $\omega$  is the established MPV of accumulation of the consequences of the negative impact of the relevant HPF in the worker;

- the probability that at the end of the current rest schedule (at time  $t$ ) the accumulation level of the consequences of the negative impact of the relevant HPF in the worker will be zero

$$\int_0^\infty (x_{00}^-(\tau) + x_{01}^-(\tau))(1 - Y_0(\tau)) d\tau + \int_0^\infty x_{11}^-(\tau)(1 - Y_1(\tau)) d\tau.$$

As noted earlier, for the stable operation of the WRS, in order to prevent the occurrence of a certain occupational dis-

ease in the worker (over time), it is necessary (within the WRS development) to ensure meeting condition (4). At the same time, probability (5) can be calculated using the methods of recovery theory as follows (Cox D. R., Smith W. L., 1967)

$$P_{10} = \frac{\lambda_1 \beta}{(\lambda_1 + \lambda_2)(\alpha + \beta)}, \quad (17)$$

where the average duration of work and rest schedules are

$$\lambda_i = \int_0^\infty (1 - Y_i(\tau)) d\tau < \infty, \quad i = 0, 1. \quad (18)$$

Taking into account (17 and 18), condition (4) for the stable operation of the WRS will take the following form

$$\frac{\lambda_1 \beta}{(\lambda_1 + \lambda_2)(\alpha + \beta)} < \frac{B}{A}. \quad (19)$$

The approach presented within this research has a fundamental difference from other similar methodological approaches which are intended to develop the WRS in the workplace.

Thus, the existing methodological approaches are intended to develop the WRS which are aimed solely at maintaining a high level of the worker's working capacity during the work shift and preventing accidents due to fatigue of the worker (which occurs due to gradual accumulation of the consequences of the HPF negative impact of the work process in the worker). At the same time, a significant common disadvantage of the existing approaches is that the appropriately developed WRS do not protect the worker from the other occupational hazards which are also associated with gradual accumulation of the consequences of the HPF negative impact of the work process in the worker (specifically, from relevant occupational diseases).

Instead, the approach presented in this research is aimed at developing the WRS that are capable of ensuring protection of the worker both from accidents associated with the occurrence and development of fatigue and from occupational diseases while ensuring a high level of labor productivity. At the same time, protection of the worker from the occurrence of these occupational hazards is done by ensuring compliance with the principle of "protection by time".

In this case, the principle of "protection by time" is ensured both by establishing duration of the work schedule at the end of which the consequences of the HPF negative impact in the worker will not exceed the relevant MPV with a probability close to 1; and by ensuring the duration of the rest schedule at the end of which the consequences of the given negative impact in the worker will be completely absent with a probability close to 1.

At the same time, ensuring a high level of labor productivity (within the developed approach) can be achieved by controlling duration of both work and rest schedules during a work shift by changing the characteristics of the parameters  $A$  and  $B$ .

The fact is that in the formal description of the process (7) the parameters of the intensity of accumulation of the consequences of the HPF negative impact in the worker ( $A$ ) and the intensity of removal of these consequences ( $B$ ), on which, respectively, duration of the work schedule and duration of the rest schedule depend, are actually manageable, since their characteristics can be changed through the implementation of organizational, technical, preventive and other occupational safety measures and means.

For example, applying rational modes of filtration and ventilation of the air of the working area is able to reduce the intensity of the release of the relevant HPF into the air of the working area and its concentration during the operation of the equipment that produces it, thereby reducing the characteristics of the parameter  $A$ . Accordingly, using certain complexes of preventive or rehabilitation exercises during the rest sched-

ule, which are intended to intensify the removal of the consequences of the negative impact of the given HPF from the worker, is able to positively influence the change in the characteristics of the parameter  $B$ .

Thus, it is possible to maximize the duration of the work schedule and minimize the duration of the rest schedule while maintaining a high level of occupational safety (by meeting the conditions (1, 2, 19)) by simultaneously changing the characteristics of the parameters  $A$  and  $B$ . This, in turn, makes it possible to increase the level of economic efficiency of enterprise, both by increasing the level of labor productivity and by expected decreasing the relevant compensation (social) payments associated with the consequences of occupational hazards (disability, rehabilitation, treatment, etc.).

In this case, in terms of the need to meet condition (19), it is very important to be able to ensure the constant maintenance of the relevant characteristics of the parameters  $A$  and  $B$  during a work shift in the workplace. Taking into account the fact that the characteristics of these parameters depend on the characteristics of the intensity of the HPF negative impact on the worker (during the work schedule), as well as the qualitative characteristics of the rest schedule, the process of their maintenance should ensure for the possibility of:

- continuous monitoring of the parameters of the HPF negative impact on the worker and their prompt correction (within the characteristics of the parameter  $A$  determined by the modeling results) during the work schedule;

- continuous monitoring of workers' compliance with the established qualitative characteristics of the rest schedule.

The task of maintaining these parameters  $A$  and  $B$  can be realized, in particular, within the automated occupational health and safety management system (SAOHSM) developed in the previous study by the author [22]. This system is designed, in particular, to continuously monitor and promptly correct the parameters of the HPF negative impact on the worker of the "human-machine-environment" systems, as well as to monitor the workers' compliance with the established time and quality characteristics of the WRS. The system involves the technical capabilities to maintain parameters  $A$  and  $B$  according to the identified nomenclature of the HPF of the relevant work process. The practical implementation of the maintenance of these parameters within the SAOHSM system is a *promising direction* for the development of this research.

However, it should be noted that maintenance of these parameters can also be ensured within the traditional SAOHSM. Thus, the proposed approach to the development of the WRS can be applied within any existing occupational health and safety management systems at enterprises.

In general, the practical implementation of the process of developing a DMP (within the presented approach) to obtain the necessary quantitative results can be carried out by numerically solving the system of differential equations and boundary conditions (10–16) using special computer software packages, such as Matlab, or analytically. In the latter case, to solve (10–16), an algorithm developed by the author in the previous research can be used, which involves simplifying the mathematical structure of this task by using the Erlang phase method [18, 23].

### Conclusions.

1. Based on the results of the analysis, it was established that the existing methodological approaches that are used to develop work and rest schedules are aimed solely at preventing accidents caused by fatigue of the worker and ensuring a high level of labor productivity during the work shift. At the same time, they are ineffective in preventing the occurrence of occupational diseases caused by the gradual accumulation of the consequences of the negative impact of the harmful production factors (HPF) in the worker. Given that the number of deaths from occupational diseases in the world is 6.5 times higher than the similar mortality rate from accidents, there is an urgent scientific and practical problem of developing an approach to determine work and rest schedules that ensure pro-

tection of workers from the occurrence of these occupational hazards and a high level of labor productivity.

2. It is determined that the problem of the impossibility of ensuring protection of workers within the existing methodological approaches to the development of work and rest schedules is associated with the fact that none of them takes into account either the risk of excessive accumulation of the consequences of the HPF negative impact in the worker at the end of the work schedule or the risk of the residual level of accumulation of the consequences of this impact at the end of the rest schedule. This, in turn, creates prerequisites for the development of an occupational disease in the worker over time. To solve this problem, within the research, it is proposed to use the maximum permissible values of accumulation of the consequences of the negative impact of the identified HPF at a certain workplace in the worker as a criterion of the end of the work schedule; and zero residual level of accumulation of the consequences of the relevant impact as a criterion for the end of the rest schedule.

3. According to the results of the research, based on the use of the mathematical tools of the theory of semi-Markov processes, stochastic models that allow determining the duration of the work schedule, at the end of which the level of accumulation of the consequences of the HPF negative impact in the worker will not exceed (with a probability close to 1) the established maximum permissible values and the duration of the rest schedule, at the end of which, also with a high probability, the given level of accumulation of the consequences will be zero, taking into account the random and dynamic characteristics of the accumulation and removal of such impact were developed. A condition that allows ensuring the stable operation of these schedules for a long time (the work experience of the worker at the relevant workplace) was determined. These results make it possible to ensure protection of the worker against the occurrence of both accidents caused by fatigue and occupational diseases.

4. Ensuring a high level of labor productivity during a work shift, within the presented approach, is achieved by maximizing the duration of the work schedule and simultaneously minimizing the duration of the rest schedule with simultaneous maintaining the established level of occupational safety (within the accepted criteria for the end of these schedules). In turn, maximisation and minimisation of the duration of work and rest schedules is achieved by changing the values of the controlled parameters of the intensity of accumulation of the consequences of the HPF negative impact in the worker and the intensity of the removal of the consequences of this impact, which are parts of the models. Since the stability of the operation of work and rest schedules, within the presented approach, depends on the need to constantly maintain the selected (in the process of modeling) relevant values of the controlled parameters during the work shift, a *promising direction for the development of the presented research* is the development of measures that will allow for such maintenance.

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## Стохастичні моделі режимів праці та відпочинку

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**Мета.** Розробити стохастичні моделі для визначення тривалості режиму праці та відпочинку, що забезпечують захист працівника від виникнення професійних небезпек і високий рівень продуктивності праці протягом робочої зміни.

**Методика.** Аналіз наукової літератури – для визначення мети і завдань дослідження; методи формалізації – для опису характеристики, динаміки та станів випадкових процесів накопичення й позбавлення від наслідків негативного впливу шкідливих виробничих факторів, що відбуваються в організмі працівника в рамках режимів праці та відпочинку; методи теорії напівмарковських процесів, теорій надійності й відновлення – для побудови стохастичних моделей режимів праці й відпочинку, визначення умови та ймовірності сталого режиму їх функціонування.

**Результати.** Розроблені стохастичні моделі, що дозволяють визначати такі тривалості режимів праці, у моменти завершення яких, з високою ймовірністю, рівень накопичення наслідків негативного впливу шкідливих виробничих факторів в організмі працівника не перевищить установлених гранично-допустимих значень. Та такі тривалості режимів відпочинку, у моменти завершення яких зазначений рівень (теж із високою ймовірністю) буде нульовим. Визначені умови забезпечення сталого функціонування зазначених режимів протягом усього терміну трудової діяльності працівника на робочому місці.

**Наукова новизна.** Уперше запропоновано підхід для розробки режимів праці й відпочинку на робочих місцях, що, на відміну від інших, базується на визначенні ймовірності рівнів накопичення наслідків негативного впливу шкідливих виробничих факторів в організмі працівника в моменти завершення цих режимів, з урахуванням реальних випадкових і динамічних характеристик такого впливу. Що, у свою чергу, дозволяє забезпечити захист працівника як від виникнення нещасних випадків, обумовлених настанням стану втоми, так і від професійних захворювань.

**Практична значимість.** Запропонований підхід дозволяє підвищити економічну ефективність роботи підприємства шляхом забезпечення високого рівня продуктивності праці, що досягається за рахунок можливості одночасної розробки максимально можливих за тривалістю режимів праці та мінімально можливих за тривалістю режимів відпочинку, які, із високою ймовірністю, включають розвиток і настання у працівника стану втоми та професійних захворювань.

**Ключові слова:** охорона праці, режим праці та відпочинку, професійні захворювання, шкідливі виробничі фактори

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