

# EFFECTIVENESS OF THE WAVELET FILTERING APPLICATION FOR IDENTIFICATION OF NONLINEAR SYSTEMS BASED ON VOLTERRA MODEL IN FREQUENCY DOMAIN

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The research results of the wavelet filtering efficiency using the interpolation method for nonlinear dynamical systems identification based on the Volterra model in the frequency domain are presented. The accuracy and noise immunity of the amplitude- and phase-frequency characteristics of the first, second and third order determining for nonlinear system using experimental data "input-output" using test polyharmonic signals are investigated. The noise immunity of the identification method have been increased using wavelet filtering of measurement noises of received responses and characteristics of the identifiable system.

**Keywords:** wavelet filtering, identification, efficiency, noise immunity, nonlinear dynamic systems, Volterra models, multidimensional characteristics, polyharmonic signals.

## Introduction

All natural objects in the real world are nonlinear with different level on nonlinearity. The presented method allows building linear and nonlinear models for different nonlinear dynamical systems (NDS). Most dynamical objects are nonlinear stochastic inertial systems. The model in the form of integral Volterra series used to identify them [3, 4]. The nonlinear and dynamic properties of such systems are completely characterized by a sequence of multidimensional weighting functions – Volterra kernels [11].

Building a model of nonlinear dynamic system in the form of a Volterra series lies in the choice of the characteristics for the test impacts. Also the developed algorithm is used and allows determining the Volterra kernels and their Fourier-images for the measured responses (multidimensional amplitude–frequency characteristics (AFC) and phase–frequency characteristics (PFC)) to simulate the NDS in the time or frequency domain, respectively [9].

## The formulation of the initial boundary value problem

There are no ideal conditions in real world. Thus the measurements during identification of the systems need to be modelled considering the errors of the measurements that are usually modelled by additive Gaussian noise [10].

The research of noise immunity of the interpolation method of nonlinear dynamical systems identification based on the Volterra model in the frequency domain is proposed. The developed identification toolkit is used to build information model of the test nonlinear dynamic system in the form of the first, second and third order model with respect of measurement noise.

The *aim* of this work is to identify the continuous NDS using Volterra model in the frequency domain, i.e. to determine its multi-frequency characteristics on the basis of the data of the “input-output” experiment [12], using test polyharmonic signals and interpolation method with obtained model coefficients and to research the noise immunity and potential filtering ability of final characteristics.

### The scheme of the problem solution

Generally, “input–output” type ratio for nonlinear dynamical system can be presented by Volterra series [8].

$$y[x(t)] = w_0(t) + \int_0^\infty w_1(\tau)x(t-\tau)d\tau + \int_0^\infty \int_0^\infty w_2(\tau_1, \tau_2)x(t-\tau_1)x(t-\tau_2)d\tau_1 d\tau_2 + \int_0^\infty \int_0^\infty \int_0^\infty w_3(\tau_1, \tau_2, \tau_3)x(t-\tau_1)x(t-\tau_2)x(t-\tau_3)d\tau_1 d\tau_2 d\tau_3 + \dots = w_0(t) + \sum_{n=1}^\infty y_n[x(t)], \quad (1)$$

where the  $n$ -th partial component of response of the system is

$$y_n[x(t)] = \int_0^t \dots \int_0^t \underset{n \text{ times}}{w_n(\tau_1, \dots, \tau_n)} \prod_{i=1}^n x(t-\tau_i) d\tau_i,$$

$x(t)$  and  $y(t)$  are input and output signals of system respectively;  $w_n(\tau_1, \tau_2, \dots, \tau_n)$  – weight function or  $n$ -order Volterra kernel;  $y_n[x(t)]$  –  $n$ -th partial component of system’s response;  $w_0(t)$  – denotes free component of the series (for zero initial conditions  $w_0(t) = 0$ ;  $t$  – current time).

Commonly, the Volterra series are replaced by a polynomial, with only taking several first terms of series (1) into consideration. Nonlinear dynamical system identification in a form of Volterra series consists in  $n$ -dimensional weighting functions determination  $w_n(\tau_1, \dots, \tau_n)$  for time domain or it’s Fourier transforms  $W_n(j\omega_1, \dots, j\omega_n)$  –  $n$ -dimensional transfer functions for frequency domain.

Multidimensional Fourier transform for  $n$ -order Volterra kernel (1) is written in a form:

$$W_n(j\omega_1, \dots, j\omega_n) = F_n \langle w_n(\tau_1, \dots, \tau_n) \rangle = \int_0^\infty \dots \int_0^\infty w_n(\tau_1, \dots, \tau_n) \exp\left(-j \sum_{i=1}^n \omega_i \tau_i\right) \prod_{i=1}^n d\tau_i,$$

where  $F_n \langle \rangle$  –  $n$ -dimensional Fourier transform;  $j = \sqrt{-1}$ . Then the model of nonlinear system based on Volterra model in frequency domain can be represented as:

$$y[x(t)] = \sum_{n=1}^\infty F_n^{-1} \left\langle W_n(j\omega_1, \dots, j\omega_n) \prod_{i=1}^n X(j\omega_i) \right\rangle_{t_1 = \dots = t_n = t},$$

where  $F_n^{-1} \langle \rangle$  – inverse  $n$ -dimensional Fourier transform;  $X(j\omega_i)$  – Fourier transform of input signal.

Identification of nonlinear system in frequency domain consists in determination of absolute value and phase of multidimensional transfer function at given frequencies –

multidimensional AFC  $|W_n(j\omega_1, j\omega_2, \dots, j\omega_n)|$  and PFC  $\arg W_n(j\omega_1, j\omega_2, \dots, j\omega_n)$  which are defined by formulas:

$$|W_n(j\omega_1, \dots, j\omega_n)| = \sqrt{[\operatorname{Re}(W_n(j\omega_1, \dots, j\omega_n))]^2 + [\operatorname{Im}(W_n(j\omega_1, \dots, j\omega_n))]^2}, \quad (2)$$

$$\arg W_n(j\omega_1, \dots, j\omega_n) = \operatorname{arctg} \frac{\operatorname{Im}[W_n(j\omega_1, \dots, j\omega_n)]}{\operatorname{Re}[W_n(j\omega_1, \dots, j\omega_n)]}, \quad (3)$$

where Re and Im – accordingly real and imaginary parts of a complex function of  $n$  variables respectively.

An interpolation method of identification of the nonlinear dynamical system based on Volterra series is used [7–8].

*Affirmation 1.* Let at input of system test signal of  $ax(t)$  kind is given, where  $x(t)$  – is arbitrary function and  $a$  – is scale coefficient (amplitude of signal), where  $0 < |a| \leq 1$ , then for the selection of a partial component of the  $n$ -th order  $\hat{y}_n(t)$  from measurement of the response nonlinear system  $y[ax(t)]$  in the form of Volterra series, it is necessary to determine  $n$ -th partial derivative of the total response amplitude  $a$  where  $a = 0$

$$\hat{y}_n(t) = \int_0^t \dots \int_0^t \underset{0 \text{ times}}{w_n(\tau_1, \dots, \tau_n)} \prod_{i=1}^n x(t - \tau_i) d\tau_i = \frac{1}{n!} \left. \frac{\partial^n y[ax(t)]}{\partial a^n} \right|_{a=0} \quad (4)$$

We use the method of extracting the partial components with the help of  $n$ -fold differentiation of the response  $y[ax(t)]$  with respect to parameter – amplitude  $a$  and the use of the derivative value at  $a = 0$ .

Injecting an input signal  $ax(t)$  where  $a$  is the scaling factor (signal amplitude), one has the following response of the nonlinear system:

$$\begin{aligned} y[a \cdot x(t)] &= a \int_0^t w(\tau) \cdot x(t - \tau) d\tau + a^2 \int_0^t \int_0^t w_2(\tau_1, \tau_2) x(t - \tau_1) x(t - \tau_2) d\tau_1 d\tau_2 + \\ &+ a^n \int_0^t \dots \int_0^t \underset{n \text{ times}}{w_n(\tau_1, \dots, \tau_n)} \prod_{r=1}^n x(t - \tau_r) d\tau_r + \dots \end{aligned}$$

To distinguish the partial component of the  $n$ -th order, differentiate the system response  $n$  times with respect to the amplitude:

$$\begin{aligned} \frac{\partial^n y[a \cdot x(t)]}{\partial a^n} &= n! \int_0^t \dots \int_0^t \underset{n \text{ times}}{w_n(\tau_1, \dots, \tau_n)} \prod_{r=1}^n x(t - \tau_r) d\tau_r + \\ &(n+1)! \cdot a \int_0^t \dots \int_0^t \underset{n+1 \text{ times}}{w_{n+1}(\tau_1, \dots, \tau_{n+1})} \prod_{r=1}^{n+1} x(t - \tau_r) d\tau_r + \dots \end{aligned}$$

Taking the value of the derivative at  $a = 0$ , we finally obtain the expression for the partial component (4).

*Formulas for numerical differentiation.* Partial derivative should be substituted by form of finite difference for calculation. Differentiation of function, which was set in discrete points, could be accomplished by means of numerical computing after preliminary smoothing

of measured results. Various formulas for the numerical differentiation are known, which differ from each other by means of error.

Let's use universal reception which allows to substitute a derivative of any  $n$  order for differential ratio so that the error from such replacement for function  $y(a)$  was any beforehand set order of  $p$  approximation concerning a step of  $h = \Delta a$  of computational mesh on amplitude. Method of undetermined coefficient for equality

$$\frac{d^n y(a)}{da^n} = \frac{1}{h^n} \sum_{r=-r_1}^{r_2} c_r y(a+rh) + O(h^p), \tag{5}$$

where the coefficients  $c_r$  are taken not depending on  $h$ ,  $r = -r_1, -r_1+1, \dots, -1, 0, 1, \dots, r_2-1, r_2$ , so that equality (5) was fair. The limits of summation  $r_1 \geq 0$  и  $r_2 \geq 0$  could be arbitrary, but so that the differential relation  $h^{-n} \sum c_r y(a+rh)$  of  $r_1 + r_2$  order have to satisfy to inequality  $r_1 + r_2 \geq n + p - 1$ .

To define the  $c_r$  it is necessary to solve the following set of equations

$$\begin{bmatrix} 1 & 1 & \dots & 1 \\ -r_1 & -r_1+1 & \dots & r_2 \\ \dots & \dots & \dots & \dots \\ (-r_1)^{n-1} & (-r_1+1)^{n-1} & \dots & r_2^{n-1} \\ (-r_1)^n & (-r_1+1)^n & \dots & r_2^n \\ (-r_1)^{n+1} & (-r_1+1)^{n+1} & \dots & r_2^{n+1} \\ \dots & \dots & \dots & \dots \\ (-r_1)^{n+p-1} & (-r_1+1)^{n+p-1} & \dots & r_2^{n+p-1} \end{bmatrix} \cdot \begin{bmatrix} c_{-r_1} \\ c_{-r_1+1} \\ \dots \\ c_{-1} \\ c_0 \\ c_1 \\ \dots \\ c_{r_2} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \dots \\ 0 \\ n! \\ 0 \\ \dots \\ 0 \end{bmatrix}. \tag{6}$$

If  $r_1 + r_2 = n + p - 1$  then inscribed in  $n + p$  equality forms linear system concerning the same number of  $c_r$  unknown. The determiner of this system is Vandermonde's determiner and differs from zero. Thus, there is only one set of  $n$  coefficients, satisfying the system.

If  $r_1 + r_2 \geq n + p$ , then there are many such sets of coefficients  $c_r$ .

On the basis of (6) in [7–8] the formulas of derivative calculation of the first, second and third orders are received at  $a = 0$  with use of the central differences for equidistant nodes of the computational grid.

The amplitudes of the test signals  $a_i^{(n)}$  and the corresponding coefficients  $c_i^{(n)}$  for responses are presented in [9].

The test polyharmonic effects for identification in the frequency domain representing by signals of such type:

$$x(t) = \sum_{k=1}^n A_k \cos(\omega_k t + \varphi_k), \tag{7}$$

where  $n$  – the order of transfer function being estimated;  $A_k$ ,  $\omega_k$  and  $\varphi_k$  – accordingly amplitude, frequency and a phase of  $k$ -th harmonics. In research, it is supposed every amplitude of  $A_k$  to be equal, and phases  $\varphi_k$  equal to zero.

The test polyharmonic signals are used for identification in frequency domain.

*Statement.* If test polyharmonic signal is used in form

$$x(t) = A \sum_{k=1}^n \cos \omega_k t = \frac{A}{2} \sum_{k=1}^n (e^{j\omega_k t} + e^{-j\omega_k t}),$$

then the  $n$ -th partial component of the response of test system can be written in form:

$$y_n(t) = \frac{A^n}{2^{n-1}} \sum_{m=0}^{E(n/2)} C_n^m \sum_{k_1=1}^n \dots \sum_{k_n=1}^n |W_n(-j\omega_{k_1}, \dots, -j\omega_{k_m}, j\omega_{k_{m+1}}, \dots, j\omega_{k_n})| \times \\ \times \cos\left(\left(-\sum_{l=0}^m \omega_{k_l} + \sum_{l=m+1}^n \omega_{k_l}\right)t + \arg W_n(-j\omega_{k_1}, \dots, -j\omega_{k_m}, j\omega_{k_{m+1}}, \dots, j\omega_{k_n})\right),$$

where  $E$  – function used to obtain the of integer part of the value.

The component with frequency  $\omega_1 + \dots + \omega_n$  is extracted from the response to test signal (7):

$$A^n |W_n(j\omega_1, \dots, j\omega_n)| \cos[(\omega_1 + \dots + \omega_n)t + \arg W_n(j\omega_1, \dots, j\omega_n)]. \quad (8)$$

Certain limitations should be imposed while choosing of frequency polyharmonic test signals in a process determining multidimensional AFC and PFC. This is the reason why the values of AFC and PFC in this unallowable points of multidimensional frequency space can be calculated using interpolation only. In practical realization of nonlinear dynamical systems identification it is needed to minimize number of such undefined points at the range of multidimensional frequency characteristics determination. This was performed to provide a minimum of restrictions on choice of frequency of the test signal. It is shown that existed limitation can be weakened. New limitations on choice of frequency are reducing number of undefined points.

After analyzing the (8) it is defined: to obtain Volterra kernels for nonlinear dynamical system in frequency domain the limitations on choice of frequencies of test polyharmonic signals have to be restricted. These restrictions provide inequality of combination frequencies in the test signal harmonics. The theorem about choice of test signals frequencies is proven.

*The theorem about choice of test signals frequencies.* For the definite filtering of a response of the harmonics with combination frequencies  $\omega_1 + \omega_2 + \dots + \omega_n$  within the  $n$ -th partial component it is necessary and sufficient to keep the frequency from being equal to another combination frequencies of type  $k_1\omega_1 + \dots + k_n\omega_n$ , where the coefficients  $\{k_i | i=1, 2, \dots, n\}$  must satisfy the conditions:

- number  $K$  of negative value coefficients ( $k_i < 0$ ) is in  $0 \leq K \leq E(n/2)$  (where  $E$  – function used to obtain the of integer part of the value);

- $\sum_{i=1}^n |k_i| \leq n$ ;

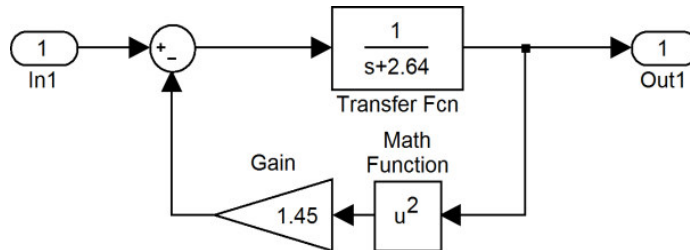
- $\sum_{i=1}^n |k_i| \equiv n \pmod{2}, n - \sum_{i=1}^n |k_i| = 2l, l \in N$ .

It was shown that during determination of multidimensional transfer functions of nonlinear systems it is necessary to consider the imposed constraints on choice of the test polyharmonic signal frequencies. This provides inequality of combination frequencies in output signal harmonics:  $\omega_1 \neq 0$ ,  $\omega_2 \neq 0$  and  $\omega_1 \neq \omega_2$  for the second order identification procedure, and  $\omega_1 \neq 0$ ,  $\omega_2 \neq 0$ ,  $\omega_3 \neq 0$ ,  $\omega_1 \neq \omega_2$ ,  $\omega_1 \neq \omega_3$ ,  $\omega_2 \neq \omega_3$ ,  $2\omega_1 \neq \omega_2 + \omega_3$ ,

$2\omega_2 \neq \omega_1 + \omega_3$ ,  $2\omega_3 \neq \omega_1 + \omega_2$ ,  $2\omega_1 \neq \omega_2 - \omega_3$ ,  $2\omega_2 \neq \omega_1 - \omega_3$ ,  $2\omega_3 \neq \omega_1 - \omega_2$ ,  $2\omega_1 \neq -\omega_2 + \omega_3$ ,  $2\omega_2 \neq -\omega_1 + \omega_3$  and  $2\omega_3 \neq -\omega_1 + \omega_2$  for the third order identification procedure.

Described method was tested using nonlinear test system (fig. 1) represented by Riccati equation:

$$\frac{dy(t)}{dt} + \alpha y(t) + \beta y^2(t) = u(t).$$

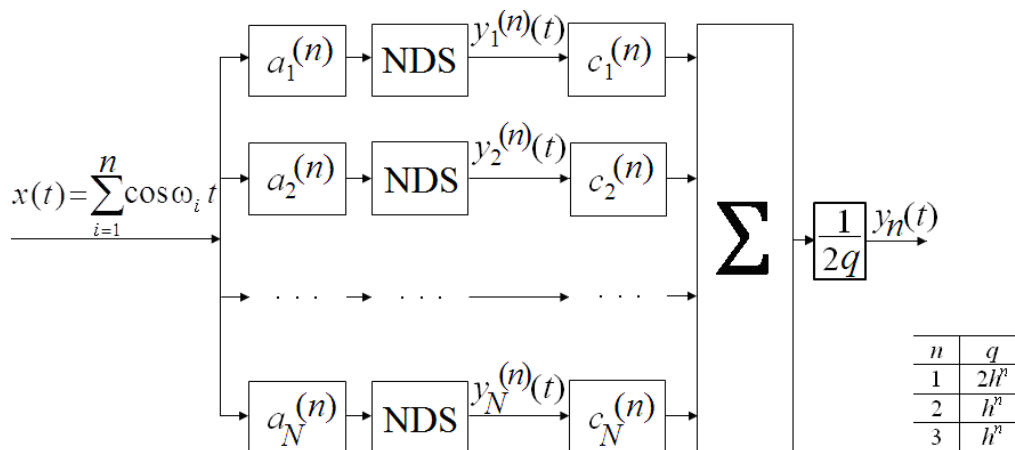


**Fig. 1.** Simulink–model of the test system

Analytical expressions of AFC and PFC for the first, second and third order model were received in [8].

The main purpose was to identify the multi–frequency performances characterizing nonlinear and dynamical properties of nonlinear test system using the methodic [10]. Volterra model in the form of the 1, 2 and 3 order polynomial is used. Thus, test system properties are characterized by transfer functions of  $W_1(j\omega)$ ,  $W_2(j\omega_1, j\omega_2)$ ,  $W_3(j\omega_1, j\omega_2, j\omega_3)$  – by Fourier-images of weight functions  $w_1(t)$ ,  $w_2(t_1, t_2)$  and  $w_3(t_1, t_2, t_3)$ .

Structure chart of identification procedure – determination of the  $n$ -th order AFC and PFC of NDS is presented in fig. 2.



**Fig. 2.** The structure chart of identification using  $n$ -th order Volterra model in frequency domain using interpolation method

The weighted sum is formed from received signals – responses of each group (fig. 2). As a result the partial components of NDS responses  $y_1(t)$ ,  $y_2(t)$  and  $y_3(t)$  are taken. For each partial component of response the Fourier transform (the FFT is used) is calculated and

only an informative harmonics (which amplitudes represent values of required characteristics of the 1, 2 and 3 order AFCs) are taken from received spectrum.

The first order AFC  $|W_1(j\omega)|$  and PFC  $\arg W_1(j\omega)$  are received by extracting the harmonics with frequency  $\omega$  from the spectrum of the CC partial response  $y_1(t)$  to the test signal  $x(t) = (A/2)\cos\omega t$ .

The second order AFC  $|W_2(j\omega, j(\omega + \Omega_1))|$  and PFC  $\arg W_2(j\omega, j(\omega + \Omega_1))$  having  $\omega_1 = \omega$  and  $\omega_2 = \omega + \Omega_1$  were received by extracting the harmonics with summary frequency  $\omega_1 + \omega_2$  from the spectrum of the NDS partial response  $y_2(t)$  to the test signal  $x(t) = (A/2)(\cos\omega_1 t + \cos\omega_2 t)$ .

The third order AFC  $|W_3(j\omega, j(\omega + \Omega_1), j(\omega + \Omega_2))|$  and PFC  $\arg W_3(j\omega, j(\omega + \Omega_1), j(\omega + \Omega_2))$  having  $\omega_1 = \omega$ ,  $\omega_2 = \omega + \Omega_1$  and  $\omega_3 = \omega + \Omega_2$ , were received by extracting the harmonics with summary frequency  $\omega_1 + \omega_2 + \omega_3$  from the spectrum of the NDS partial response  $y_3(t)$  to the test signal  $x(t) = (A/2)(\cos\omega_1 t + \cos\omega_2 t + \cos\omega_3 t)$ .

The results (first, second and third order AFC and PFC which had been received after procedure of identification) and the second and third order surfaces for AFC and PFC received after procedure of the test system identification are presented in [8].

The surfaces are built from sub-diagonal cross-sections which were received separately.  $\Omega_1$  was used as growing parameter of identification with different value for each cross-section in second order characteristics. Fixed value of  $\Omega_2$  and growing value of  $\Omega_1$  were used as parameters of identification to obtain different value for each cross-section in third order characteristics.

Numerical values of identification accuracy as a percentage RMSE using interpolation method for the test system are represented in Table 1, where:  $n$  – order of the estimated Volterra kernel,  $N$  – approximation order/number of interpolation knots (number of experiments). The comparisons to previous works are given in [9].

**Table 1.**

Numerical values of identification accuracy using interpolation method

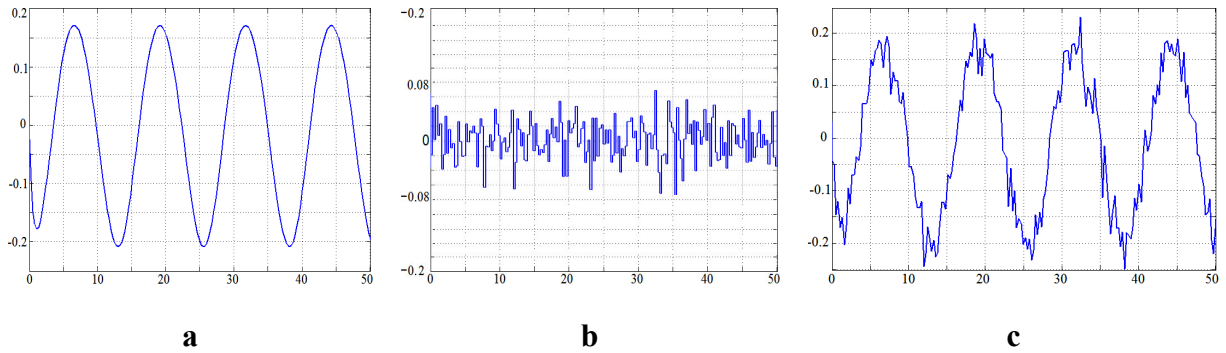
$n$	$N$	RMSE for AFC, %	RMSE for PFC, %
1	2	2.8853	2.1005
	4	0.5180	2.3964
	6	0.4075	2.3663
2	2	29.7526	89.3218
	4	2.0103	5.1023
	6	3.0488	7.8248
3	4	2.9299	11.0100
	6	11.0285	5.9724

Experimental researches of the noise immunity of the identification method were performed. The simulations with the test model were performed. Different noise levels were defined for different order of the Volterra model.

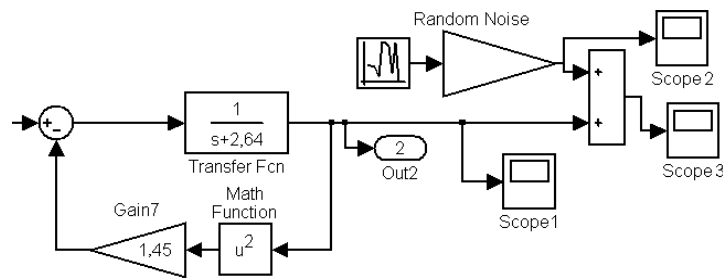
The main purpose was the studying of the noise impact (noise means the inexactness of the measurements) to the characteristics of the test system model using interpolation method of identification in frequency domain.

The first step was the measurement of the level of useful response signal (harmonic cosine test signal shown in fig. 3a) after test system (Out2 in fig. 4). The amplitude of this signal was defined as the 100% of the signal power.

After that procedure the Random Noise signal (with the form shown in fig. 3b) where added to the test system output signal. This steps where performed to simulate inexactness of the measurements in the model. The sum of these two signals for the linear test model signal is shown in fig. 3c.

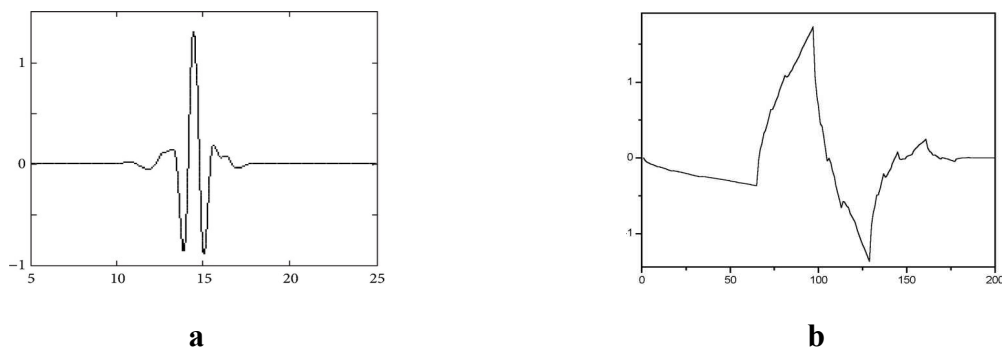


**Fig. 3.** The sum of two signals for the linear test model signal: a – the NDS subchannel response, b – random noise with 50% amplitude of max amplitude of the response, c – “noised” response of the test system



**Fig. 4.** The Simulink model of the test system with noise generator and osillosopes

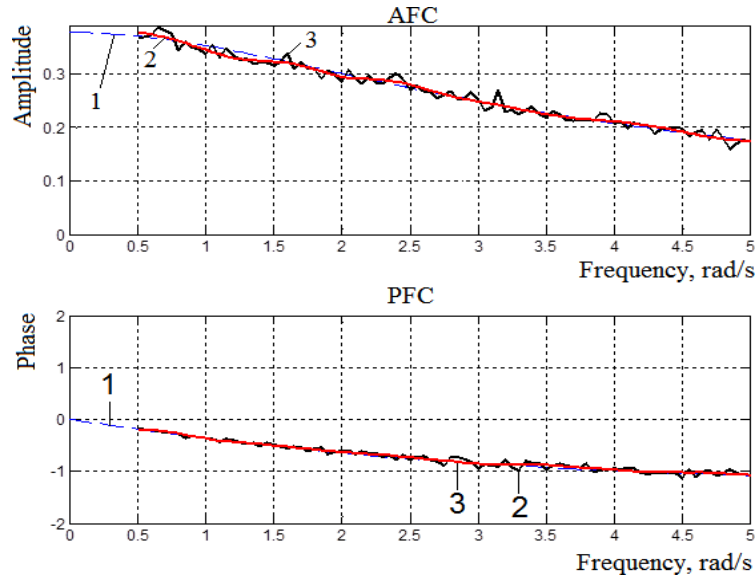
After series of test researches it was decided to use Coiflet and Daubechie wavelets [2, 5, 7] due to its minimal distortion impact on form of filtered signal with RMSE minimization. The wavelet filtering was used to reduce the noise impact on final characteristics of the test system. The Coiflet-4 and Daubechie-3 of the 3 level were chosen (fig. 5) and used for the AFC and PFC filtering respectively.



**Fig. 5.** The Coiflet (a) and Dubechie (b) and wavelet function

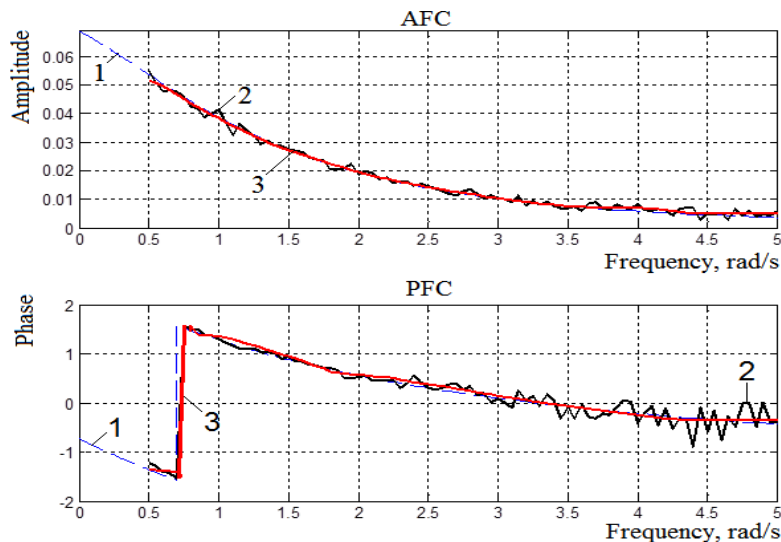


The first order (linear) model was tested with the level of noise 100% and 10% and showed excellent level of noise immunity. The standard, noised and de-noised (filtered) (fig. 6) characteristics (AFC and PFC) with level of noise 100% are presented.



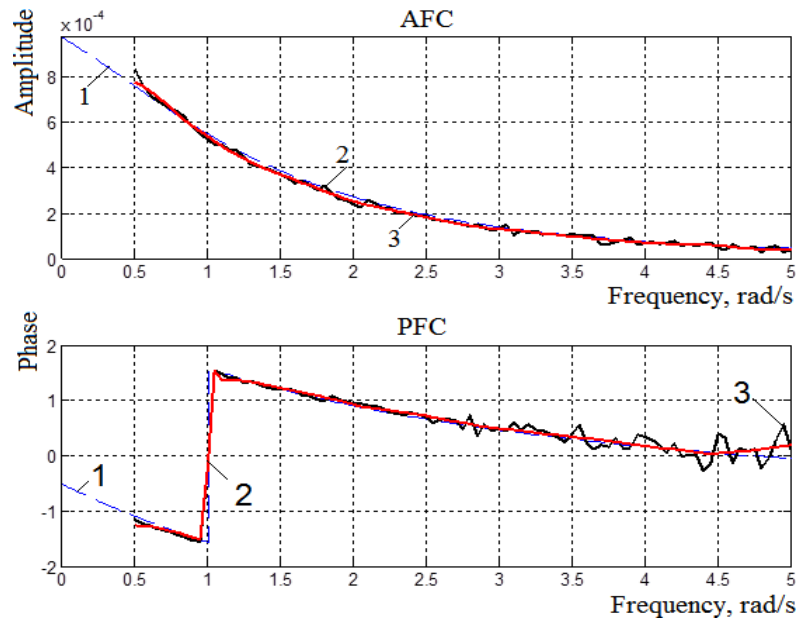
**Fig. 6.** The standard (1), noised (2) and denoised (3) characteristics (AFC – top, PFC – bottom) of the 1 order model of the test system with level of noise 100%

The second order (nonlinear) model was tested with the level of noise 10% and 1% and showed good level of noise immunity. The standard, noised and de-noised (filtered) (fig. 7) characteristics (AFC and PFC) with level of noise 10% are presented.



**Fig. 7.** The standard (1), noised (2) and denoised (3) characteristics (AFC – top, PFC – bottom) of the 2 order model of the test system with level of noise 10%

The third order (nonlinear) model was tested with the level of noise 10% and 1% and showed good level of noise immunity. The standard, noised and de-noised (filtered) (fig. 8) characteristics (AFC and PFC) with level of noise 1% are presented.



**Fig. 8.** The standard (1), noised (2) and denoised (3) characteristics (AFC – top, PFC – bottom) of the 3 order model of the test system with level of noise 0,1%

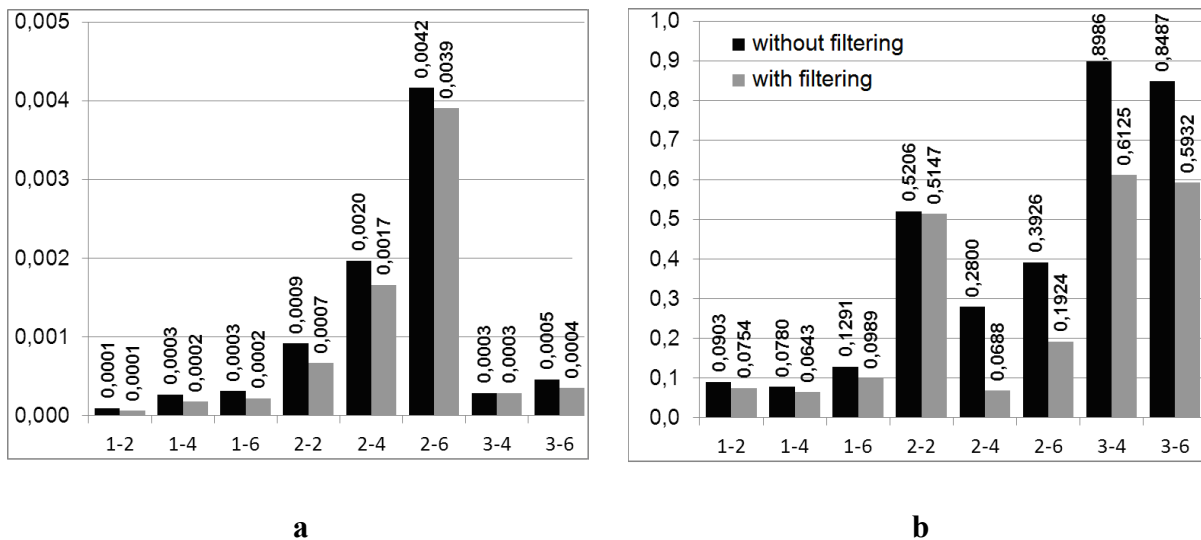
The numerical values of RMSE of the identification accuracy before and after wavelet filtering procedure are presented in Table 2.

**Table 2.**

Standard deviation for interpolation method with noise impact

<i>n</i>	<i>N</i>	Noise level = 10%		Noise level = 0,1%		Improvement	
		RMSE for AFC	RMSE for PFC	RMSE for AFC	RMSE for PFC	for AFC, times	for PFC, times
		(without / with filtering)					
1	2	0.000097 / 0.000063	0.09031 / 0.07541	–	–	1.540	1.198
	4	0.000271 / 0.000181	0.07804 / 0.06433	–	–	1.497	1.213
	6	0.000312 / 0.000223	0.12913 / 0.09889	–	–	1.399	1.306
2	2	0.000920 / 0.000670	0.52063 / 0.51465	–	–	1.373	1.012
	4	0.001972 / 0.001663	0.28004 / 0.06877	–	–	1.186	4.072
	6	0.004165 / 0.003908	0.39260 / 0.19237	–	–	1.066	2.041
3	4	–	–	0.000288 / 0.000288	0.89857 / 0.61251	1.003	1.467
	6	–	–	0.000461 / 0.000352	0.84868 / 0.59319	1.310	1.431

The diagrams showing the improvement of RMSE for identification accuracy using the wavelet filtering of the received characteristics for AFC and PFC are shown in fig. 9a and fig. 9b respectively.



**Fig. 9.** RMSE changing for AFC (a) and PFC (b) using wavelet-filtering

## Conclusion

The interpolation method used for nonparametric model building is based on Volterra model. The polyharmonic test signals are used for identification the nonlinear dynamical systems. The method based on linear combination of responses to the test signals with different amplitudes is used to differentiate the responses of system for partial components.

The provided results have confirmed nonlinearity of the test system. Selected values of test signal amplitudes and corresponding coefficients are raising greatly the accuracy of identification in compare to amplitudes and coefficients written in [1, 9]. The accuracy of identification of nonlinear part of the test system as RMSE is less than 3% for AFC and less than 6 % for PFC in best cases. This characterizes method as excellent one.

The noise immunity of the method is very high for the linear part and high enough for the nonlinear part of the model. The RMSE of noised characteristics doesn't overgrow 10% in best cases with 10% measurement inaccuracy. The wavelet filtering of the response and final characteristics of the nonlinear dynamical system is very effective and gives the possibility to improve the identification accuracy of the inexact measurements up to 1.54 and 4.07 times for the AFC and PFC respectively.

## References

1. Danilov, L.V. The theory of nonlinear electrical circuits / L.V. Danilov, P.N. Mathanov, E.S. Philipov // Published Energoatomizdat, Leningrad – 1990 – 396 p.
2. Donoho, D.L. Threshold selection for wavelet shrinkage of noisy data / D.L. Donoho, I.M. Johnstone // Proc. 16th Annual Conf. of the IEEE Engineering in Medicine and Biology Society, 24a–25a, IEEE Press – 1994 – PP.128–139.
3. Doyle, F.J. Identification and Control Using Volterra Models / F.J. Doyle, R.K. Pearson, B.A. Ogunnaikie // Published Springer Technology & Industrial Arts – 2001 – 420 p.
4. Giannakis, G.B. Bibliography on nonlinear system identification and its applications in signal processing, communications and biomedical engineering / G.B. Giannakis, E.A. Serpedin // Signal Processing, EURASIP, Elsevier Science B.V. 81(3) – 2001 – PP. 533-580.
5. Goswami, J.G. Fundamentals of Wavelets: Theory, Algorithms, and Applications / J.G. Goswami, A.K.Chan // Publishing John Wiley&Sons Inc – 1999 – 289 p.
6. Misiti, M. Wavelets Toolbox Users Guide / M. Misiti, Y. Misiti, G. Oppenheim, J-M. Poggi // The MathWorks Inc. Wavelet Toolbox, for use with MATLAB – 2000 – 137 p.

7. Pavlenko, V.D. Interpolation Method of Nonlinear Dynamical Systems Identification Based on Volterra Model in Frequency Domain / V.D. Pavlenko, S.V. Pavlenko, V.O. Speransky // Proceedings of the 7th IEEE International Conference on Intelligent Data Acquisition and Advanced Computing Systems: Technology and Applications (IDAACS'2013), 15–17 September 2013, Berlin, Germany – 2013. PP.173–178.
8. Pavlenko, V.D. Analysis of identification accuracy of nonlinear system based on Volterra model in frequency domain / V.D. Pavlenko, V.O. Speransky // American Journal of Modeling and Optimization – Vol.1, No.2 – 2013. PP.11–18. DOI: 10.12691/ajmo-1-2-2.
9. Pavlenko, V.D. Communication Channel Identification in Frequency Domain Based on the Volterra Model / V.D. Pavlenko, V.O. Speransky // Recent Advances in Computers, Communications, Applied Social Science and Mathematics. Proceedings of the International Conference on Computers, Digital Communications and Computing (ICDCC'11), Barcelona, Spain, September 15-17, 2011. Published by WSEAS Press – 2011. PP.218–222.
10. Sergeev, A.G. Metrologija / A.G. Sergeev, V.V. Krohin // М. : Logos, 2000. – 407 p.
11. Schetzen, M. The Volterra and Wiener Theories of Nonlinear Systems // Wiley&Sons, New York – 1980 – 245 p.
12. Westwick, D.T. Methods for the Identification of Multiple-Input Nonlinear Systems // Departments of Electrical Engineering and Biomedical Engineering, McGill University, Montreal, Quebec, Canada – 1995 – 312 p.

### ЕФЕКТИВНІСТЬ ЗАСТОСУВАННЯ ВЕЙВЛЕТ-ФИЛЬТРАЦІЇ ДЛЯ ІДЕНТИФІКАЦІЇ НЕЛІНІЙНИХ СИСТЕМ НА ОСНОВІ МОДЕЛЕЙ ВОЛЬТЕРРА В ЧАСТОТНІЙ ОБЛАСТІ

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Наведено результати досліджень ефективності вейвлет-фільтрації при застосуванні інтерполяційного методу ідентифікації нелінійних динамічних систем на основі моделі Вольєрра в частотній області. Досліджено точність та завадостійкість визначення амплітудно- і фазо-частотних характеристик першого, другого і третього порядків нелінійної системи за даними експерименту «вхід-вихід» за допомогою тестових полігармонічних сигналів. Підвищення завадостійкості методу ідентифікації досягнуто за допомогою вейвлет-фільтрації шумів вимірювань відгуків та отримуваних характеристик системи, що ідентифікують.

**Ключові слова:** вейвлет-фільтрація, ідентифікація, ефективність, завадостійкість, нелінійні динамічні системи, моделі Вольєрра, багатомірні АЧХ і ФЧХ, полігармонічні сигнали.

### ЭФФЕКТИВНОСТЬ ПРИМЕНЕНИЯ ВЕЙВЛЕТ-ФИЛЬТРАЦИИ ДЛЯ ИДЕНТИФИКАЦИИ НЕЛИНЕЙНЫХ СИСТЕМ НА ОСНОВЕ МОДЕЛЕЙ ВОЛЬТЕРРА В ЧАСТОТНОЙ ОБЛАСТИ

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Приводятся результаты исследования эффективности вейвлет-фильтрации при применении интерполяционного метода идентификации нелинейных динамических систем на основе модели Вольєрра в частотной области. Исследуется точность и помехоустойчивость определения амплитудно- и фазо-частотных характеристик первого, второго и третьего порядков нелинейной системы по данным эксперимента «вход-выход» с помощью тестовых полигармонических сигналов. Повышение помехоустойчивости метода идентификации достигается с помощью вейвлет-фильтрации шумов измерений откликов и получаемых характеристик идентифицируемой системы.

**Ключевые слова:** вейвлет-фильтрация, идентификация, эффективность, помехоустойчивость, нелинейные динамические системы, модели Вольєрра, многомерные АЧХ и ФЧХ, полигармонические сигналы.