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A TECHNIQUE TO SOLVE THE PROBLEM OF PARAMETRIC IDENTIFICATION OF MATHEMATICAL MODELS OF ANOMALOUS FLUIDS FILTRATION PROCESSES IN POROUS MEDIA

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The problem of parametric identification of mathematical models (MM) of reservoir systems for the porosity and permeability functions of ground rock (medium) was defined. A technique to solve the problem of parametric identification of the MM class under study was proposed based on the gradient projection method with minimization of the squared quality criterion.

Keywords: anomalous liquids, mathematical model, parametric identification, functional, quality criterion, gradient projection method.

Introduction

In some important industries such as construction industry, mining and geological prospecting, modeling of filtration fluid flows is required. Here the complexity of mathematical models (MM) of filtration flows is determined, among other factors, by a number of fluids subjected to filtration (i.e., number or phases), their fractional make-ups, and geological soil structure. The MM proposed in [1,2] as a variational inequality with proper boundary and initial conditions may be considered as an adequate MM for mutual filtration of a viscous (ideal) fluid and viscoplastic (anomalous) fluid. It should be noted that the porosity $m(\bar{z})$ and permeability functions $k_i(\bar{z}), i = 1,2$ of the medium (reservoir rock) are used as the parameters of this MM. Hereinafter, indices $i = 1,2$ define fluids subjected to filtration. On frequent occasions, the values of these parameters are not known, and the problem of their determination is a complex autonomous problem of investigation of a reservoir system, the problem of parametric identification.

The *purpose* of this work was to develop a constructive computational procedure for identification of the porosity and permeability functions of the formation.

Statement of a problem

According to [1], an incremental MM for mutual filtration of a viscous (ideal) liquid and viscoplastic (anomalous) fluid may be represented as

$$-\frac{m\partial(\Delta S_2)}{\partial t}-\int_\Omega\sum_{i=1}^{n}\left[k_1\frac{\partial^2(\Delta P)}{\partial z_i^2}|\Delta v|\right]dz+\int_\Omega\sum_{i=1}^{n}\left[k_1\frac{\partial^2(\Delta P)}{\partial z_i^2}|\Delta S_2|\right]dz\geq\frac{1}{h}\sum_{j=1}^{K_1}\zeta_j(z)Q_{1_j}, \quad \forall v,\, S_2\in K, \tag{1}$$

$$-\frac{m\partial(\Delta S_2)}{\partial t}-\int_\Omega\sum_{i=1}^{n}\left(k_2\frac{\partial^2(\Delta P)}{\partial z_i^2}\right)dz=\frac{1}{h}\sum_{j=1}^{K_2}\zeta_j(z)Q_{2_j}, \tag{2}$$

$$\Delta P(0,z)=\Delta P_0(z),\ \Delta S_2(0,z)=\Delta S_{2_0}(z), \tag{3}$$

$$\frac{\partial[\Delta S_2(t,z)]}{\partial\eta}\geq 0;\ S_2(t,z)<S_{2_{\max}}, \tag{4}$$

$$\frac{\partial[\Delta S_2(t,z)]}{\partial\eta}=0;\ S_2(t,z)\geq S_{2_{\max}}, \tag{5}$$

where $P=P(t,z)$ is the intrastratal pressure, $S_2=S_2(t,z)$ is the reservoir saturation with displacing fluid, $v=v(t,z)$ is the test function, h is bulk of reservoir rock, K is the set at which $S_2=S_2(t,z)$ and $v=v(t,z)$ functions are defined, K_1,K_2 are a number of production and injection wells, respectively; Q_1,Q_2 are the debits of corresponding wells, $\zeta(z)$ is the Dirac function, η is the normal to the boundary. Let us assign the following functional as a quality criterion for the solution of identification problem:

$$J_1[m(\cdot),k_1(\cdot)]=\sum_{j=1}^{K_1+K_2}\left\{\int_{T_j}\left[P'(t,z_j,m,k_1)-F_j^P(t)\right]^2dt+\int_{T_j}\left[S_2'(t,z_j,m,k_1)-F_j^S(t)\right]^2dt\right\}, \tag{6}$$

where $P'(\cdot)$ and $S_2'(\cdot)$ are the exact values of the functions of intrastratal pressure and the reservoir saturation with displacing fluid, respectively, F^P and F^S are the measured values of the specified functions, and T is the period of time when the measurement is done.

Proof of the differentiability of the quality criterion

Let us show that the quality criterion assigned will be differentiable in any point of the spatial domain $\bar{z}\in\Omega$ (including its boundary G), i.e, that the increment (6) equal to

$$\Delta J_1=J_1\left[(m+h^m),(k_1+h^k)\right]-J_1(m,k_1)$$

is presentable as

$$\Delta J_1=\int_\Omega\left\{\left[J'(m,k_1)h^m\right]dz+\left[J'(m,k_1)h^k\right]dz\right\}+\left[O\left(\|h^m\|_{L^2}\right)+O\left(\|h^k\|_{L^2}\right)\right], \tag{7}$$

where $J'(m,k_1)$ is a function from $L^2(\Omega)$, $O\left(\|h^m\|_{L^2}\right)$ and $O\left(\|h^k\|_{L^2}\right)$ are the remainders such that $\lim_{\alpha^m\to+0} O(\alpha^m)(\alpha^m)^{-1}=0$, $\lim_{\alpha^k\to+0} O(\alpha^k)(\alpha^k)^{-1}=0$.

Let us re-write the functional increment ΔJ_1 in a formal way as follows:

$$\Delta J_1 = \sum_{j=1}^{K_1+K_2}\left\{\int_\Omega \left\{\left[P(t,z_j,m,k_1)+\Delta P(t,z_j)-F_j^P(t)\right]^2 - \right.\right.$$

$$\left.-\left[P(t,z_j,m,k_1)-F_j^P(t)\right]^2\right\}dz + \left\{\left[S_2(t,z_j,m,k_1)+\Delta S_2(t,z_j)-F_j^S(t)\right]^2 -\right.$$

$$\left.\left.+\left\{\left[S_2(t,z_j,m,k_1)+\Delta S_2(t,z_j)-F_j^S(t)\right]^2-\left[S_2(t,z_j,m,k_1)-F_j^S(t)\right]^2\right\}dz\right\}=$$

$$= \sum_{j=1}^{K_1+K_2}\left\{\int_\Omega\left\{\left\{\left[P(t,z_j,m,k_1)-F_j^P(t)\right]+\Delta P(t,z_j)\right\}^2-\right.\right.$$

$$\left.-\left[P(t,z_j,m,k_1)-F_j^P(t)\right]\right\}dz+\left\{\left\{\left[S_2(t,z_j,m,k_1)-F_j^S(t)\right]+\Delta S_2(t,z_j)\right\}^2-\right. \tag{8}$$

$$\left.\left.-\left[S(t,z_j,m,k_1)-F_j^S(t)\right]\right\}dz\right\}=$$

$$=\sum_{j=1}^{K_1+K_2}\left\{\left\{\int_\Omega 2\left[P(t,z_j,m,k_1)-F_j^P(t)\right]\Delta P(t,z)dz+\int_\Omega \Delta P^2(t,z_j)dz\right\}+\right.$$

$$\left.+\left\{\int_\Omega 2\left[S_2(t,z_j,m,k_1)-F_j^S(t)\right]\Delta S_2(t,z)dz+\int_\Omega \Delta S_2^2(t,z_j)dz\right\}\right\}.$$

Now let us convert this expression to the form of (7). To this end, let us introduce functions $p_P^*(t,z)\equiv p_P^*(t,z,m,k_1)$ and $p_S^*(t,z)\equiv p_S^*(t,z,m,k_1)$ into consideration as solutions of the following boundary-value problem

$$-\frac{m\partial p_S^*}{\partial t}(v-S_2)-\int_\Omega\sum_{i=1}^n\left[k_1\frac{\partial^2 p_P^*}{\partial z_i^2}|v|\right]dz+\int_\Omega\sum_{i=1}^n\left[k_1\frac{\partial^2 p_P^*}{\partial z_i^2}|S_2|\right]dz\geq\geq\frac{1}{h}\sum_{j=1}^{K_1}\zeta_j(z)Q_{1_j}, \quad \forall v,S_2\in K, \tag{9}$$

$$-\frac{m\partial p_S^*}{\partial t}-\int_\Omega\sum_{i=1}^n\left(k_2\frac{\partial^2 p_P^*}{\partial z_i^2}\right)dz=\frac{1}{h}\sum_{j=1}^{K_2}\zeta_j(z)Q_{2_j}, \tag{10}$$

$$\left.\frac{\partial p_S^*}{\partial\eta}\right|_{z=0}\geq 0; 0\leq t\leq t_k, \tag{11}$$

$$\left.p_P^*\right|_{t=t_k}=2\left[P(t_k,z_j,m,k_1)-F^P(t)\right], \left.p_S^*\right|_{t=t_k}=2\left[S_2(t_k,z_j,m,k_1)-F^S(t)\right],\ \forall z\in\Omega. \tag{12}$$

While taking (1) – (5), (9) – (12) into account, the first integral in the first component of right-hand side of equation (8) is converted as follows

$$I_P=\int_\Omega 2\left[P(t,z_j,m,k_1)-F_j^P(t)\right]\Delta P(t,z_j)dz=\int_\Omega p_P^*(t_k,z_j)\Delta P(t,z_j)dz=$$

$$= \int_{\Omega} \left[\int_{0}^{t_k} \frac{\partial}{\partial t} \left(p_P^* \Delta P \right) dt \right] dz = \int_{\Omega} \int_{0}^{t_k} \left[\frac{\partial p_P^*}{\partial t} \Delta P + p_P^* \frac{\partial (\Delta P)}{\partial t} \right] dt dz =$$

$$= \int_{\Omega} \int_{0}^{t_k} \left\{ \frac{1}{m(z)} \left\{ \sum_{i=1}^{n} \left[k_1(z_i) \frac{\partial^2 p_P^*}{\partial z_j^2} |v| \right] + \sum_{i=1}^{n} \left[k_1(z_i) \frac{\partial^2 p_P^*}{\partial z_j^2} |S_2| \right] \right\} \Delta P + \right.$$

$$\left. + p_P^* \left\{ \sum_{i=1}^{n} \left[k_1(z_i) \frac{\partial^2 p_P^*}{\partial z_j^2} |\Delta v| \right] + \sum_{i=1}^{n} \left[k_1(z_i) \frac{\partial^2 p_P^*}{\partial z_j^2} |\Delta S_2| \right] \right\} \right\} dt dz \ .$$

Neglecting the last expression in spatial domain, we will obtain the following

$$I_P' = \int_{0}^{t_k} \left\{ \frac{1}{m(z)} \left\{ \sum_{i=1}^{n} \left[k_1(z_i) \frac{\partial^2 p_P^*}{\partial z_j^2} |v| \right] + \sum_{i=1}^{n} \left[k_1(z_i) \frac{\partial^2 p_P^*}{\partial z_j^2} |S_2| \right] \right\} \Delta P + \right.$$

$$\left. \left. + p_P^* \left\{ \sum_{i=1}^{n} \left[k_1(z_i) \frac{\partial^2 p_P^*}{\partial z_j^2} |v| \right] + \sum_{i=1}^{n} \left[k_1(z_i) \frac{\partial^2 p_P^*}{\partial z_j^2} |S_2| \right] \right\} \right\} \right|_{\Omega} dt = \qquad (13)$$

$$= \int_{0}^{t_k} \frac{1}{m(z)} \left[\left(\sum_{i=1}^{n} k_1(z_i) p_P^* |v| \right) + \left(\sum_{i=1}^{n} k_1(z_i) p_P^* |S_2| \right) \right] \Delta P dt \ .$$

Similarly, for the expression in (8) written with respect to function $S_2(t,z)$ (i.e., the first integral in the second component of right-hand side of equation (8)), one may write the following

$$I_S = \int_{\Omega} 2 \left[S_2(t, z_j, m, k_1) - F_j^S(t) \right] \Delta S_2(t, z_j) dz = \int_{\Omega} p_P^*(t_k, z_j) \Delta S_2(t, z_j) dz$$

or, omitting obvious conversions, when using integration in the spatial domain, finally we obtain the following

$$I_S' = \int_{0}^{t_k} \frac{1}{m(z)} \left[\left(\sum_{i=1}^{n} k_1(z_i) p_P^* |v| \right) + \left(\sum_{i=1}^{n} k_1(z_i) p_P^* |S_2| \right) \right] \Delta S_2 dt \ . \qquad (14)$$

Second integrals in the components of right-hand side of equation (8) are determined mostly by the terms of $\left[O\left(\|h\|_{L^2}^P \right) + O\left(\|h\|_{L^2}^S \right) \right]$ form presented in (7) and written for spatial problem statement. In this case,

$$\Delta J_1 = \int_{\Omega} \frac{1}{m(z)} \left[\left(\sum_{i=1}^{n} k_1(z_i) p_P^* |v| \right) + \left(\sum_{i=1}^{n} k_1(z_i) p_P^* |S_2| \right) \right] h dz + O\left(\|h\|_{L^2} \right). \qquad (15)$$

Determining the functional increment and forming the calculated relationships

As a result, the increment of functional (6) is represented as the following expression

$$\Delta J_1 = \int_0^{t_k} \frac{1}{m(z)} \left[\left(\sum_{i=1}^{n} k_1(z_i) p_P^* |v| \right) + \left(\sum_{i=1}^{n} k_1(z_i) p_P^* |S_2| \right) \right] h dz + O\left(\|h\|_{L^2} \right).$$

Therefore, the desired presentation (7) for functional (6) has been obtained, and the gradient of this functional has the following form

$$J_1' [m(z), k_1(z)] \equiv \frac{1}{m(z)} \left\{ \sum_{i=1}^{n} \left[k_1(z_i) p_P^* |v| \right] + \sum_{i=1}^{n} \left[k_1(z_i) p_P^* |S_2| \right] \right\}, \quad \forall z \in \Omega, \forall t \in [0, t_k]. \tag{16}$$

Then, with the gradient (16) available and using the procedure of gradient projection method [3] determined by relationships $U = \{ u(t) : u(t) \in L^2[0, t_k],\ a \le u(t) \le b,\ \forall t \in [0, t_k] \}$,

$$\mathrm{Pr}_u [u(t)] = \begin{cases} u(t),\ a \le u(t) \le b, \\ a,\ u(t) < a, \\ b,\ u(t) > b, \end{cases}$$

we obtain the final calculated relationships for identifiable functions $m(z)$ and $k_1(z)$

$$m_{q+1}(z) = \begin{cases} m_q(z) - \dfrac{\alpha_m}{m(z)} \left\{ \sum\limits_{i=1}^{n} \left[k_1(z_i) p_P^* |v| \right] + \sum\limits_{i=1}^{n} \left[k_1(z_i) p_P^* |S_2| \right] \right\}, \\ m_{\min} \le m_q(z) - \dfrac{\alpha_m}{m(z)} \left\{ \sum\limits_{i=1}^{n} \left[k_1(z_i) p_P^* |v| \right] + \sum\limits_{i=1}^{n} \left[k_1(z_i) p_P^* |S_2| \right] \right\} \le m_{\max}, \\ m_{\min}, \\ m_q(z) - \dfrac{\alpha_m}{m(z)} \left\{ \sum\limits_{i=1}^{n} \left[k_1(z_i) p_P^* |v| \right] + \sum\limits_{i=1}^{n} \left[k_1(z_i) p_P^* |S_2| \right] \right\} < m_{\min}, \\ m_{\max}, \\ m_q(z) - \dfrac{\alpha_m}{m(z)} \left\{ \sum\limits_{i=1}^{n} \left[k_1(z_i) p_P^* |v| \right] + \sum\limits_{i=1}^{n} \left[k_1(z_i) p_P^* |S_2| \right] \right\} > m_{\max}. \end{cases}$$

$$k_{1_{q+1}}(z) = \begin{cases} k_{1_q}(z) - \dfrac{\alpha_k}{m(z)} \left\{ \sum\limits_{i=1}^{n} \left[k_1(z_i) p_P^* |v| \right] + \sum\limits_{i=1}^{n} \left[k_1(z_i) p_P^* |S_2| \right] \right\}, \\ k_{1_{\min}} \le k_{1_q}(z) - \dfrac{\alpha_k}{m(z)} \left\{ \sum\limits_{i=1}^{n} \left[k_1(z_i) p_P^* |v| \right] + \sum\limits_{i=1}^{n} \left[k_1(z_i) p_P^* |S_2| \right] \right\} \le k_{1_{\max}}, \\ m_{\min}, \\ k_{1_q}(z) - \dfrac{\alpha_k}{m(z)} \left\{ \sum\limits_{i=1}^{n} \left[k_1(z_i) p_P^* |v| \right] + \sum\limits_{i=1}^{n} \left[k_1(z_i) p_P^* |S_2| \right] \right\} < k_{1_{\min}}, \\ m_{\max}, \\ k_{1_q}(z) - \dfrac{\alpha_k}{m(z)} \left\{ \sum\limits_{i=1}^{n} \left[k_1(z_i) p_P^* |v| \right] + \sum\limits_{i=1}^{n} \left[k_1(z_i) p_P^* |S_2| \right] \right\} > k_{1_{\max}} \end{cases}$$

Conclusions

The numerical investigations performed showed that the procedure proposed for sequential search of the porosity $m(\bar{z})$ and permeability $k_i(\bar{z}), i = 1,2$ has a high convergence rate (thus, e.g., to identify the parameters of a reservoir defined by the spatial sampling grid $z_1 = 12,\ z_2 = 16$, a sequence of no more than 8 iterations is required), with the total problem solving time of no more than 120 s.

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МЕТОД ВИРІШЕННЯ ЗАДАЧІ ПАРАМЕТРИЧНОЇ ІДЕНТИФІКАЦІЇ МАТЕМАТИЧНИХ МОДЕЛЕЙ ПРОЦЕСІВ ФІЛЬТРАЦІЇ АНОМАЛЬНИХ РІДИН У ПОРИСТИХ СЕРЕДОВИЩАХ

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Виконано постановку задачі параметричної ідентифікації математичних моделей (ММ) пластових систем для функцій пористості та проникності грунтової породи (середовища). Запропоновано метод розв'язання задачі параметричної ідентифікації досліджуваного класу ММ, заснований на методі проекції градієнту при мінімізації квадратичного критерію якості.

Ключові слова: аномальна рідина, математична модель, параметрична ідентифікація, функціонал, критерій якості, метод проекції градієнта

МЕТОД РЕШЕНИЯ ЗАДАЧИ ПАРАМЕТРИЧЕСКОЙ ИДЕНТИФИКАЦИИ МАТЕМАТИЧЕСКИХ МОДЕЛЕЙ ПРОЦЕССОВ ФИЛЬТРАЦИИ АНОМАЛЬНЫХ ЖИДКОСТЕЙ В ПОРИСТЫХ СРЕДАХ

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Выполнена постановка задачи параметрической идентификации математических моделей (ММ) пластовых систем для функций пористости и проницаемости грунтовой породы (среды). Предложен метод решения задачи параметрической идентификации исследуемого класса ММ, основанный на методе проекции градиента при минимизации квадратичного критерия качества.

Ключевые слова: аномальная жидкость, математическая модель, параметрическая идентификация, функционал, критерий качества, метод проекции градиента