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METHODS OF MATHEMATICAL DESIGN AND MACHINE AUTHENTICATION OF NON - STATICS ANOMALOUS DIFFUSIVE PROCESSES

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For a class of the non-statics abnormal diffusive processes which mathematical models are formalized in the form of variation inequalities in private derivatives, the method of mathematical modeling based on optimizing procedure is offered. Thus the problem of realization of mathematical models of non-statics abnormal diffusive processes is reduced to search of a maximum of function of Hamilton defined in space of conditions of studied processes. The method of parametrical identification of mathematical models of non-statics abnormal diffusive processes in case of an induction problem definition of research is also offered. The method is reduced to use of optimizing procedure of a method of a projection of a gradient. Possibility of the solution of a problem of parametrical identification, as for linear, and nonlinear mathematical models of non-statics abnormal diffusive processes is proved.

Keywords: abnormal diffusive process, mathematical model, variation, variation inequality, optimization, principle of a maximum functionality, gradient, parametrical identification.

Introduction

In a number of important applied tasks technological (or naturally natural) processes are characterized by deviations from well-known physical laws. In this regard these processes received in special literature the name anomalous (in particular, abnormal diffusive) [1 — 4]. First of all, the rheological processes connected with mining can be an example of such processes. For the description of abnormal diffusive processes, as the adequate mathematical models (MM) it was offered to use the device of variation inequalities in private derivatives [5 — 8].

As it was shown in work [9], in practical appendices it is most convenient to use the following formalization of abnormal diffusive processes.

Let the function $\psi(t,\bar{z})$, defined on a bounded open set Ω of the space $\Re^n$, $n=1,2$, with smooth boundary Γ and the time interval $(0,t_k)$ for $t_k<\infty$, $Q=\Omega\times(0,t_k)$, $\Sigma=\Gamma\times(0,t_k)$ is the solution of the varitional inequality

$$\psi \in K:\left(m(t)\frac{\partial\psi}{\partial t}, v-\psi\right)+\big(B(t)\psi, v-\psi\big)+j(v)-j(u)\geq\big(f, v-\psi\big)\quad \forall\, v\in H^1(\Omega), \tag{1}$$

$$\psi(0,\bar{z})=\psi_0(\bar{z}), \tag{2}$$

where the operator $B(\gamma)$ specifies a linear transformation $B(\gamma): H^1(\Omega)\to H^1(\Omega)$ and is defined by the bilinear form:

$$\left(B(t)\psi, v-\psi\right) = \int_{\Omega}\left(\sum_{i=1}^{n}\frac{\partial\psi}{\partial z_i}\cdot\frac{\partial(v-\psi)}{\partial z_i}\right)d\bar{z}\,, \tag{3}$$

f – the driving function of the process, for which the operation $\left(f, v-\psi\right)$ coincides with the scalar product in $L^2(\Omega)$, i.e. $\left(f, v-\psi\right) = \int_{\Omega}\left[f(\bar{z}), v-\psi\right]d\Omega$ or $\left(f, v-\psi\right) = \int_{\Gamma}\left[f(\bar{z}), v-\psi\right]d\Gamma$ (hereinafter, for simplicity, restrict ourselves to the tasks at the border Γ); $j(\cdot)$ — convex functionals defining the kind of physical process in rheology and which are specified as follows

$$j(\cdot) = \int_{\Gamma}\varphi(\psi, \bar{z})\cdot\lambda(\psi)\,d\Gamma\,, \quad j(\cdot) = \int_{\Omega}\varphi(\psi, \bar{z})\cdot\lambda(\psi)\,d\Omega\,. \tag{4}$$

In the relation (4) accept that $\varphi(\cdot)$ – is a continuous function, $\lambda(\cdot)$ – is continuous differentiable or not having the properties of differentiable functions.

Space of admissible functions $\varphi(\cdot)$ and $\lambda(\cdot)$ are defined as $\Delta \in L^{\infty}(\overline{Q}),\ \Lambda \in L^{\infty}(\overline{Q})$ where it is assumed that $\varphi(\cdot),\ \lambda(\cdot) \in L^{\infty}(\overline{Q})$, $\overline{Q} = \overline{\Omega}\times(0, t_k)$ and the spaces Δ and Λ are Banach with respect to the norm $\left\|\varphi(\psi, \bar{z})\right\|_{\Delta} = \left\|\varphi(\psi, \bar{z})\right\|_{L^{\infty}(\overline{Q})}$.

Method of mathematical modelling of abnormal diffusive processes

The proposed method for solving varitional inequalities of the form (1), (2) is based on the proof of the following statements.

To find the optimal solution $\psi(t, \bar{z})$ of the varitional inequality (1), (2) there must exist a nonzero continuous function $p(t, \bar{z})$, so that at any time t in the interval $0 \le t \le T$ (T — time of physical processes) the Hamiltonian function $\tilde{H}$ in the spatial domain Ω (or on its boundary Γ) would take the maximum value, where

$$\tilde{H} = \left\langle\left(\left(B(t)\tilde{\psi}, \tilde{v}-\tilde{\psi}\right) + \phi(\tilde{v}) - \phi(\tilde{\psi}) - \left(\theta(\tilde{\psi}, \tilde{v}), \tilde{v}-\tilde{\psi}\right) - \left(f, \left(\tilde{v}-\tilde{\psi}\right)\right)\right), \tilde{p}\right\rangle.$$

Carry out a preliminary series of reforms to simplify the original formulation of the problem. Introduce the notation

$$\varphi(t, \bar{z})\cdot\lambda(\psi) = \Phi(\psi), \quad \varphi(t, \bar{z})\cdot\lambda(v) = \Phi(v),$$

and

$$\phi(\psi) = \int_{\Gamma}\Phi(\psi)\,d\Gamma\,, \quad \phi(v) = \int_{\Gamma}\Phi(v)\,d\Gamma\,.$$

In addition, introduce an additional unknown function $\theta(\psi, v)$, the structure corresponding to the functionals $j(\cdot)$, such that

$$\left(\theta(\psi, v), v-\psi\right) \ge 0 \quad \forall v \in K\,.$$

Taking into account the executed transformations introduce the relations (1), (2) in the form

$$\psi \in K : \left(m(t)\frac{\partial \psi}{\partial t}, v - \psi \right) + \left(B(t), v - \psi \right) + \phi(v) - \phi(\psi) - \\ - \left(\theta(\psi, v), v - \psi \right) = \left(f, v - \psi \right) \quad \forall v \in K \tag{5}$$

$$\psi(0, \bar{z}) = \psi_0(\bar{z}). \tag{6}$$

To solve the problem of finding a state function $\psi(t, \bar{z})$, use an optimization procedure of the Pontryagin maximum principle [10], for which choose the following performance criterion

$$J = \min \int_{\Gamma} \int_{0}^{T} |v - \psi| dt d\Gamma \tag{7}$$

The physical meaning of this criterion follows from the next. The trial function $v(t, \bar{z})$ is some approximation of the unknown function $\psi(t, \bar{z})$, reflecting only the essence of physics in the specific process. Therefore, the adequacy of physical processes caused by the action of functions $v(t, \bar{z})$ and $\psi(t, \bar{z})$, is provided up to the accuracy within the difference between these functions. In this case, the integral difference between the trial $v(t, \bar{z})$ and the unknown $\psi(t, \bar{z})$ functions can be regarded as a quantitative measure or a penalty for the deviation of the actual flow of the process from its true value.

Obtain the necessary optimality conditions of the problems (5) (6), (7).

According to [6], introduce a new coordinate

$$\frac{\partial^2 \sigma}{\partial t \partial z} = |v - \psi|^2 \Big|_{z \in \Gamma}. \tag{8}$$

Thus, the original problem will be considered in $(n+1)$-dimensional space with the equation of dynamics

$$\tilde{\psi} \in K : \left(m(t)\frac{\partial \tilde{\psi}}{\partial t}, \tilde{v} - \tilde{\psi} \right) + \left(B(t), \tilde{v} - \tilde{\psi} \right) + \phi(\tilde{v}) - \phi(\tilde{\psi}) -$$

$$- \left(\theta(\tilde{\psi}, \tilde{v}), \tilde{v} - \tilde{\psi} \right) = \left(f, \tilde{v} - \tilde{\psi} \right) \quad \forall \tilde{v} \in K, \tag{9}$$

where $\tilde{\psi} = (\sigma, \psi_1, ..., \psi_n)$, $\tilde{v} = (\sigma, v_1, ..., v_n)$, with the initial conditions $\tilde{\psi}(0, \bar{z}) = [0, \psi_0(\bar{z})]$.

Assume that we have found $\psi(t, \bar{z})$. This condition corresponds to the relation

$$\min \int_{\Gamma} \int_{0}^{T} |\tilde{v} - \tilde{\psi}|^2 \, dt \, d\Gamma \to J_{\min} = J^*.$$

At $t = \tau$ $(0 \le \tau \le T)$ perform a needle-shaped variation with the duration ε. As a result of the variation performed the value of the functional J (7) changes $\hat{J} = \int_{\Gamma} \int_{0}^{T} |\tilde{v} - \tilde{\psi}| \, dt \, d\Gamma > J_{\min}$.

Write down the detailed result of the variation

$$\delta\tilde{v} = \tilde{v} - \tilde{\psi} = \varepsilon\{[(B(t)\tilde{\psi}, \tilde{v} - \tilde{\psi}) + \phi(\tilde{v}) - \phi(\tilde{\psi}) - (\theta(\tilde{\psi}, \tilde{v}), \tilde{v} - \tilde{\psi}) - (f, (\tilde{v} - \tilde{\psi}))] - (B(t)\tilde{\psi}, \psi) +$$

$$+ \phi(\tilde{\psi}) - (\theta(\tilde{\psi}), \tilde{\psi}) - (f, \tilde{\psi})\}_{t=\tau}. \qquad (10)$$

Express $\tilde{v}$ through the variation and optimal function of the state

$$\tilde{v} = \tilde{\psi} + \delta\tilde{v}. \qquad (11)$$

Substituting (11) into (9), obtain

$$\tilde{\psi} \in K : \left(m(t)\frac{\partial\tilde{\psi}}{\partial t}, (\tilde{\psi} + \delta\tilde{v}) - \tilde{\psi} \right) = (B(t)\tilde{\psi}, (\tilde{\psi} + \delta\tilde{v}) - \tilde{\psi}) + \phi(\tilde{\psi} + \delta\tilde{v}) - \phi(\tilde{\psi}) -$$

$$- (\theta(\tilde{\psi}, (\tilde{\psi} + \delta\tilde{v})), (\tilde{\psi} + \delta\tilde{v}) - \tilde{\psi}) - (f, (\tilde{\psi} + \delta\tilde{v}) - \tilde{\psi}) \quad \forall\tilde{v} \in K. \qquad (12)$$

For further transformations use the coordinate-wise analog (12)

$$\tilde{\psi}_i \in K : \left(m(t_i)\frac{\partial\tilde{\psi}_i}{\partial t}, (\tilde{\psi}_i + \delta\tilde{v}_i) - \tilde{\psi}_i \right) = (B(t)\tilde{\psi}_i, (\tilde{\psi}_i + \delta\tilde{v}_i) - \tilde{\psi}_i) + \phi(\tilde{\psi}_i + \delta\tilde{v}_i) - \phi(\tilde{\psi}_i) -$$

$$- (\theta(\tilde{\psi}_i, (\tilde{\psi}_i + \delta\tilde{v}_i)), (\tilde{\psi}_i + \delta\tilde{v}_i) - \tilde{\psi}_i) - (f, (\tilde{\psi}_i + \delta\tilde{v}_i) - \tilde{\psi}_i) \quad \forall\tilde{v}_i \in K \quad i = 0,1,...,n. \qquad (13)$$

$$- (\theta(\tilde{\psi}_i, (\tilde{\psi}_i + \delta\tilde{v}_i)), (\tilde{\psi}_i + \delta\tilde{v}_i) - \tilde{\psi}_i) - (f, (\tilde{\psi}_i + \delta\tilde{v}_i) - \tilde{\psi}_i) \quad \forall\tilde{v}_i \in K \quad i = 0,1,...,n.$$

Expand (13) in Taylor series and restrict the consideration with the quantities of 1-th order of infinitesimally

$$m(t)\left(\frac{\partial\tilde{\psi}_i}{\partial t} + \frac{\partial\tilde{v}_i}{\partial t} \right) = (B(t)\tilde{\psi}_i, \tilde{\psi}_i) + \phi(\tilde{\psi}_i) - (f, \tilde{\psi}_i) + \sum_{i=0}^{n} \frac{\partial[(B(t)\tilde{\psi}_i, \tilde{\psi}_i)_i + \phi(\tilde{\psi}_i) - (f, \tilde{\psi}_i)]}{\partial\tilde{v}_i}\delta\tilde{v}_i,$$

$$i = 0,1,...,n. \qquad (14)$$

From (14) it follows that

$$m(t)\frac{\partial\tilde{v}_i}{\partial t} = \sum_{i=0}^{n} \frac{\partial[(B(t)\tilde{\psi}_i, \tilde{\psi}_i) + \phi(\tilde{\psi}_i) - (f, \tilde{\psi}_i)]}{\partial\tilde{v}_i}\delta\tilde{v}_i, \quad i = 0,1,...,n. \qquad (15)$$

Now turn to $t = T$. Define a variation of the functional at $t = T$

$$\delta J_{t=T} = \hat{J} - J_{\min} > 0 \ \text{ or } \ -\delta J_{t=T} = -\delta\,\sigma_{Ib=\Gamma} \le 0.$$

Introduce the variable $\tilde{p}(t,\bar{z})$ so that when $t = T$ this condition is satisfied

$$-\delta J_{t=T} = -\delta\,\sigma(T) = \langle\delta\tilde{v}, \tilde{p}\rangle_{t=T}. \qquad (16)$$

Coordinate wise analog (16) is as follows: $-\delta J_{t=T} = -\delta\sigma(T) = \langle\delta\tilde{v}_i, \tilde{p}_i\rangle_{t=T}$, $i = 0,1,...,n$.

Since $\delta\,\sigma(T) > 0$, in order to satisfy this relation there should take place:

$$p^0(T,\bar{z}_i) = -1;\ p_j(T,\bar{z}) = 0, \text{ where } i = 0,1,...,n;\ j = 1,...,n.$$

Thus, if the optimal solution is not found, then $-\delta J < 0$, and for the optimal solution $-\delta J = 0$ is valid, since the variation of functional must be zero for the optimal solution.

Associate a variable $\tilde{p}(t,\bar{z})$ to the dynamic equation of the process observed through trial function $v(t,\bar{z})$. Find a variable $\tilde{p}(t,\bar{z})$ which satisfies

$$\left\langle \delta\tilde{v}(t,\bar{z}), \tilde{p}(t,\bar{z}) \right\rangle = \left\langle \delta\tilde{v}(T,\bar{z}), \tilde{p}(T,\bar{z}) \right\rangle_{\tau+\varepsilon\le t\le T} = const.$$

Then we have

$$\frac{\partial}{\partial t}\left\langle \delta\tilde{v}(t,\bar{z}), \tilde{p}(t,\bar{z}) \right\rangle = \left\langle \frac{\partial\,\delta\tilde{v}(t,\bar{z})}{\partial t}, \tilde{p}(t,\bar{z}) \right\rangle + \left\langle \frac{\partial\, v\tilde{p}(t,\bar{z})}{\partial t}, \delta\tilde{v}(t,\bar{z}) \right\rangle_{\tau+\varepsilon\le t\le T} = 0. \tag{17}$$

Coordinate wise analog (17) is

$$\sum_{i=0}^{n} \frac{\partial\,\delta\tilde{v}_i(t,\bar{z})}{\partial t}, \tilde{p}_i(t,\bar{z}) + \sum_{i=0}^{n} \delta\tilde{v}_i(t,\bar{z}) \frac{\partial\,\tilde{p}_i\tilde{v}_i(t,\bar{z})}{\partial t} = 0,\ i = 0,1,...,n. \tag{18}$$

Substitute in (18) the value of the derivative $\frac{\partial\,\delta\tilde{v}(t,\bar{z})}{\partial t}$ from (15)

$$m(t)\sum_{i=0}^{n}\tilde{p}_i \times \sum_{i=0}^{n} \frac{\partial\left[\left(B(t)\tilde{\psi}_i,\tilde{\psi}_i\right) + \phi(\tilde{\psi}_i) - \left(f,\tilde{\psi}_i\right)\right]}{\partial\tilde{v}_i}\delta\tilde{v}_i + \sum_{i=0}^{n}\delta\tilde{v}_i \frac{\partial\tilde{p}}{\partial t} = 0,\ i = 0,1,...,n. \tag{19}$$

Change the order of summation in (19)

$$m(t)\sum_{i=0}^{n}\delta\tilde{v}_i + \left[\sum_{i=0}^{n}\tilde{p}_i \frac{\partial\left[\left(B(t)\tilde{\psi}_i,\tilde{\psi}_i\right) + \phi(\tilde{\psi}_i) - \left(f,\tilde{\psi}_i\right)\right]}{\partial\tilde{v}_i} + \frac{\partial\tilde{p}_i}{\partial t}\right] = 0,\ i = 0,1,...,n.$$

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Finally get

$$\frac{\partial\tilde{p}_i}{\partial t} = -\sum_{i=0}^{n} \frac{\partial\left[\left(B(t)\tilde{\psi}_i,\tilde{\psi}_i\right) + \phi(\tilde{\psi}_i) - \left(f,\tilde{\psi}_i\right)\right]}{\partial\tilde{v}_i}\tilde{p}_i,\ i = 0,1,...,n.$$

Note that this equation is the dual of (5), and the variable $\tilde{p}(t,\bar{z})$ is expressed through the function of phase.

Again turn to the variation of functional (7) at $t = T$: $-\delta J_{t=T} = \left\langle \delta\tilde{v}(t,\bar{z}), \tilde{p}(t,\bar{z}) \right\rangle_{t=T} = 0$

Replace the variation $\delta\tilde{v}$ with the value of (10), reduce by ε and, since τ can be arbitrary, obtain

$$\begin{gathered}\left\langle \left(\left(B(t)\tilde{\psi}, \tilde{v}-\tilde{\psi}\right) + \phi(\tilde{v}) - \phi(\tilde{\psi}) - \left(\theta(\tilde{\psi},\tilde{v}), \tilde{v}-\tilde{\psi}\right) - \left(f,(\tilde{v}-\tilde{\psi})\right)\right), \tilde{p} \right\rangle_{t=\tau} - \\ -\left\langle \left(\left(B(t)\tilde{\psi},\tilde{\psi}\right) + \phi(\tilde{\psi}) - \left(f,\tilde{\psi}\right)\right), \tilde{p} \right\rangle_{t=T} = 0.\end{gathered} \tag{20}$$

From (20) it follows that the second summand in it corresponds to the optimal solution of the varitional inequality (5). In the case when the optimal solution $\psi(t,\bar{z})$ is found, variation of functional J will be zero, i.e. $\delta J = 0$. Given this, the first summand in (20), defined by the Hamiltonian function

$$\tilde{H} = \langle ((B(t)\tilde{\psi}, \tilde{v} - \tilde{\psi}) + \phi(\tilde{v}) - \phi(\tilde{\psi}) - (\theta(\tilde{\psi}, \tilde{v}), \tilde{v} - \tilde{\psi}) - (f, (\tilde{v} - \tilde{\psi}))), \tilde{p} \rangle, \tag{21}$$

should take the maximum value. Thus, the above statement is proven. Let's show the possibility of determining the maximum value of Hamiltonian function.

Coordinate wise analog (21) is defined by

$$\tilde{H} = \langle ((B(t)\tilde{\psi}_i, \tilde{v}_i - \tilde{\psi}_i) + \phi(\tilde{v}_i) - \phi(\tilde{\psi}_i) - (\theta(\tilde{\psi}_i, \tilde{v}_i), \tilde{v}_i - \tilde{\psi}_i) - (f, (\tilde{v}_i - \tilde{\psi}_i))), \tilde{p}_i \rangle,$$

$$i = 0, 1, \ldots, n. \tag{22}$$

To maximize the value of the function $\tilde{H}$, it's necessary to set all the partial derivatives of this function to zero by a testing variable $v(t, \bar{z})$, that taking into account (22) gives the system of equations

$$\frac{\partial \tilde{H}}{\partial v_i} = 0, \quad i = 0, 1, \ldots, n. \tag{23}$$

Coordinate wise analog (22) contains $(n+1)$ of v_i functions, $(n+1)$ of θ_i functions and $(n+1)$ of p_i functions. Since the equations (23) are only $(n+1)$, and the unknown are $(3n+3)$, then the system (23) cannot be solved. To solve (23) define also the partial derivatives

$$\frac{\partial \tilde{H}}{\partial \theta_i} = \tilde{p}_i, \qquad i = 0, 1, \ldots, n. \tag{24}$$

$$\frac{\partial \tilde{H}}{\partial p_i} = \left[m(t_i) \frac{\partial \tilde{\psi}_i}{\partial t}, \tilde{v}_i - \tilde{\psi}_i \right], \quad i = 0, 1, \ldots, n. \tag{25}$$

In this case, the solution of (23) can be obtained.

As a result of the reasoning done, the scheme of the algorithm for solving varitional inequality (5) using the maximum principle can be represented as follows:

1. The dynamic equation (9), subject to the additional coordinate σ is written down.
2. An auxiliary function (Hamilton) $\tilde{H}$ in accordance with the expression (22) is compiled.
3. A test function $v(t, \bar{z})$ that delivers maximum $\tilde{H}$ functions in accordance with the expression (23) is determined. For the redefinition of the independent variables θ and p the system (23) is supplemented with equations (24) and (25).
4. The unknown variable $\psi(t, \bar{z})$ is determined by the test variable $v(t, \bar{z})$, which gives the maximum value of function $\tilde{H}$.

Method of parametrical identification of abnormal diffusive processes

At statement of an inductive task - (1) — (4), the method focused on numerical machine realization can be offered parametrical identification of MM of a look. The essence of a method consists in the following.

It agrees [11], to MM (1) — (4) (in increments) it is possible to present in a look

$$-\frac{m\partial(\Delta\psi)}{\partial t}-\int_{\Omega}\sum_{i=1}^{n}\left[B(t)\frac{\partial^2(\Delta\psi)}{\partial z_i^2}|\Delta v|\right]dz \ge \sum_{j=1}^{k}\zeta_j(z)f_j, \quad \forall\psi, v\in K, \tag{26}$$

$$\Delta\psi(0,z)=\Delta\psi_0(z), \tag{27}$$

where $\psi=\psi(t,z)$ — sought function; $v=v(t,z)$ — trial function; K — a lot, of that is defined functions $\psi=\psi(t,z)$ and $v=v(t,z)$; f — exciting function; k — number of exciting functions; $\zeta(z)$ - Dirac's function; $m=m(\cdot)$ and $B=B(\cdot)$ — identified parameters.

As criterion of quality of the solution of a problem of identification we will accept functionality of a look

$$J[m(\cdot),B(\cdot)]=\sum_{j=1}^{k}\left\{\int_{T_j}\left[\psi'(t,z,m,B)-F_j^{\psi}(t)\right]^2 dt\right\}, \tag{28}$$

where $\psi'(t,z,m,B)$ — exact values sought functions; $F_j^{\psi}(t)$ — measured values of the sought function; T — time of measurements.

Let's show that the accepted criterion of quality will be differentiable in any point of spatial area $\bar{z}\in\Omega$ (including and its border Γ), i.e. an increment (28) equal

$$\Delta J = J\left[(m+h^m),(B+h^B)\right] - J(m,B)$$

represent able in a look

$$\Delta J = \int_{\Omega}\left\{\left[J'(m,B)h^m\right]dz+\left[J'(m,B)h^B\right]dz\right\}+\left[O\left(\left\|h^m\right\|_{L^2}\right)+O\left(\left\|h^B\right\|_{L^2}\right)\right], \tag{29}$$

where $J'(m,B)$ — some function from $L^2(\Omega)$; $O\left(\left\|h^m\right\|_{L^2}\right)$ and $O\left(\left\|h^B\right\|_{L^2}\right)$ — residual members such, that $\lim\limits_{\alpha^m\to 0}\left[O(\alpha^m)(\alpha^m)^{-1}\right]=0$, $\lim\limits_{\alpha^m\to 0}\left[O(\alpha^m)(\alpha^m)^{-1}\right]=0$.

Let's write down formally a functionality increment

$$\Delta J=\sum_{j=1}^{k}\left\{\int_{\Omega}\left\{\left[\psi(t,z_j,v,B)+\Delta\psi(t,z_j)-F_j^{\psi}(t)\right]^2-\left[\psi(t,z,m,B)-F_j^{\psi}(t)\right]\right\}dz\right\}=$$

$$=\sum_{j=1}^{k}\left\{\int_{\Omega}\left\{\left\{\left[\psi(t,z,m,B)-F_j^{\psi}(t)\right]+\Delta\psi(t,z_j)\right\}^2-\left[\psi(t,z_j,v,B)-F_j^{\psi}(t)\right]\right\}dz\right\}= \tag{30}$$

$$=\sum_{j}^{k}\left\{\left\{\int_{\Omega}2\left[\psi(t,z,m,B)-F_j^{\psi}(t)\right]\Delta\psi(t,z)dz+\int_{\Omega}\Delta\psi^2(t,z)\right\}.\right.$$

Let's transform this expression to a look (29). For this purpose we will enter into consideration of function $p_{\psi}^{*}(t,z)\equiv p_{\psi}^{*}(t,z,m,B)$ as the solution of the following regional task

$$-\frac{m\partial p_{\psi}^{*}}{\partial t}(v-\psi)-\int_{\Omega}\sum_{i=1}^{n}\left[B(t)\frac{\partial^2 p_{\psi}^{*}}{\partial z_i^2}|\Delta v|\right]dz \ge \sum_{j=1}^{k}\zeta_j(z)f_j, \quad \forall\psi, v\in K, \tag{31}$$

$$p_{\psi}^{*}\Big|_{t=t_k} = 2\left[\psi(t_k, z, m, B) - F_j^{\psi}(t)\right] p_{\psi}^{*}\Big|_{t=t_k} \quad \forall z \in \Omega . \tag{32}$$

The first integral in the first composed in the right part of equality (30) taking into account (26), (27), (31), (32) it will be transformed so

$$I = 2\left[\psi(t, z, m, B) - F_j^{\psi}(t)\right]\Delta\psi(t, z)dz = \int_{\Omega} p_{\psi}^{*}(t_k, z_j)\Delta\psi(t_k, z_j) =$$

$$= \int_{\Omega}\left[\int_0^{t_k} \frac{\partial}{\partial t}\left(p_{\psi}^{*}, \Delta\psi\right)dt\right]dz = \int_{\Omega}\int_0^{t_k}\left[\frac{\partial p_{\psi}^{*}}{\partial \psi}\Delta\psi + p_{\psi}^{*}\frac{\partial(\Delta\psi)}{\partial t}\right]dt\, dz =$$

$$= \int_{\Omega}\int_0^{t_k}\left\{\frac{1}{m(\cdot)}\left\{\sum_{i=1}^{n}\left[B(\cdot)\frac{\partial^2 p_{\psi}^{*}}{\partial z_i^2}|v|\right]\right\}\Delta\psi + p_{\psi}^{*}\left\{\sum_{i=1}^{nk}\left[B(\cdot)\frac{\partial^2 p_{\psi}^{*}}{\partial z_i^2}|\Delta v|\right]\right\}\right\}dt\, dz .$$

Integrating the last expression in spatial area, we will receive the following result

$$I_{\psi}' = \int_0^{t_k}\left\{\frac{1}{m(\cdot)}\left\{\sum_{i=1}^{n}\left[B(\cdot)\frac{\partial^2 p_{\psi}^{*}}{\partial z_i^2}|v|\right]\right\}\Delta\psi + p_{\psi}^{*}\left\{\sum_{i=1}^{n}\left[B(\cdot)\frac{\partial^2 p_{\psi}^{*}}{\partial z_i^2}|\Delta v|\right]\right\}\right\}\Bigg|_{\Omega} dt =$$

$$= \int_0^{t_r}\frac{1}{m(\cdot)}\left[\left(\sum_{i=1}^{n}B(\cdot)p_{\psi}^{*}|v|\right)\right]\Delta\psi\, dt .$$

Here, and further, designation $(\cdot)$ determines as the linear (from space), and non-linear (from required function) parameter. The second integrals in composed in the right part (30) the members of the look $\left[O\left(\|h\|_{L_2}^{\psi}\right)\right]$, presented in (29) and written down for spatial problem definition define. Let's have in this case: $\Delta J = \int_{\Omega}\frac{1}{m(\cdot)}\left[\left(\sum_{i=1}^{n}B(\cdot)p_{\psi}^{*}|v|\right)\right]h^{\psi}dz + O\left(\|h\|_{L_2}^{\psi}\right)$ and, the step h^{ψ} determines cooperative value by steps h^{m} and h^{B}. As a result we will receive that the increment of functionality (28) is represented in the form of expression

$$\Delta J = \int_{\Omega}\frac{1}{m(\cdot)}\left[\left(\sum_{i=1}^{n}B(\cdot)p_{\psi}^{*}|v|\right)\right]h^{\psi}dz + O\left(\|h\|_{L_2}^{\psi}\right).$$

Thus, required representation (29) for functionality (28) is received, and the gradient of this functionality looks like

$$J'[m(\cdot), B(\cdot)] \equiv \frac{1}{m(\cdot)}\left[\left(\sum_{i=1}^{n}B(\cdot)p_{\psi}^{*}|v|\right)\right], \quad \forall z \in \Omega,\ t \in [0, t_k]. \tag{33}$$

Further, having a gradient (33) and using procedure of a method of a projection of the gradient [11], defined by ratios

$$Q = \{q(t) : q(t) \in L_2[0,t_k],\ a \le q(t) \le b, \quad \forall t \in [0,t_k]\} \qquad \mathrm{Pr}_q[q(t)] = \begin{cases} q(t), & a \le q(t) \le b, \\ a, & q(t) < a, \\ b, & q(t) > b. \end{cases}$$

For identified functions $m(\cdot)$ and $B(\cdot)$ also we will receive final ratios on an offered method of parametrical identification

$$m_{u+1}(\cdot) = \begin{cases} m_r - \dfrac{\alpha_m}{m(\cdot)}\left[\left(\sum_{i=1}^{n} B(\cdot) p_\psi^* |v|\right)\right]; & m_{\min} \le m_r - \dfrac{\alpha_m}{m(\cdot)}\left[\left(\sum_{i=1}^{n} B(\cdot) p_\psi^* |v|\right)\right] \le m_{\max}\,; \\ m_{\min}\,, & m_r - \dfrac{\alpha_m}{m(\cdot)}\left[\left(\sum_{i=1}^{n} B(\cdot) p_\psi^* |v|\right)\right] < m_{\min}\,; \\ m_{\max}\,, & m_r - \dfrac{\alpha_m}{m(\cdot)}\left[\left(\sum_{i=1}^{n} B(\cdot) p_\psi^* |v|\right)\right] > m_{\max}\,, \end{cases}$$

$$B_{u+1}(\cdot) = \begin{cases} B_r - \dfrac{\alpha_B}{m(\cdot)}\left[\left(\sum_{i=1}^{n} B(\cdot) p_\psi^* |v|\right)\right], & B_{\min} \le B_r - \dfrac{\alpha_B}{m(\cdot)}\left[\left(\sum_{i=1}^{n} B(\cdot) p_\psi^* |v|\right)\right] \le B_{\max}\,; \\ B_{\min}\,, & B_r - \dfrac{\alpha_B}{m(\cdot)}\left[\left(\sum_{i=1}^{n} B(\cdot) p_\psi^* |v|\right)\right] < B_{\min}\,; \\ B_{\max}\,, & B_r - \dfrac{\alpha_B}{m(\cdot)}\left[\left(\sum_{i=1}^{n} B(\cdot) p_\psi^* |v|\right)\right] > B_{\max}\,, \end{cases}$$

where α_m and α_B — method parameters, defined by practical consideration, r — step of the numerical decision.

Conclusion

The conducted numerical researches showed that the offered methods of mathematical model operation and parametrical identification of the abnormal diffusion processes, based on iterative procedures of optimization possess good convergence (the decision is reached no more, than for 8 - 10 iterations) at accuracy of the decision 0.2% are not lower.

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МЕТОДИ МАТЕМАТИЧНОГО МОДЕЛЮВАННЯ ТА МАШИННОЇ ІДЕНТИФІКАЦІЇ НЕСТАЦІОНАРНИХ АНОМАЛЬНИХ ДИФУЗІЙНИХ ПРОЦЕСІВ

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Для класу нестаціонарних аномальних дифузійних процесів, математичні моделі яких формалізуються у вигляді варіаційних нерівностей у часткових похідних,, запропоновано метод математичного моделювання, заснований на оптимізаційній процедурі. При цьому задача реалізації математичних моделей нестаціонарних аномальних дифузійних процесів зводиться до відшукання максимуму функції Гамільтона, яку визначено у просторі стану досліджуваних процесів. Також запропоновано метод параметричної ідентифікації математичних моделей нестаціонарних аномальних дифузійних процесів на випадок індукційної постановки задачі дослідження. Метод зводиться до застосування оптимізаційної процедури методу проекції градієнта. Обгрунтовано можливість розв’язання задачі ідентифікації, як для лінійних, так і для нелінійних математичних моделей нестаціонарних аномальних дифузійних процесів.

Ключові слова: аномальний дифузійний процес, математична модель, варіація, варіаційна нерівність, оптимізація, принцип максимуму, функціонал, градієнт, екстремум, параметрична ідентифікація.

МЕТОДЫ МАТЕМАТИЧЕСКОГО МОДЕЛИРОВАНИЯ И МАШИННОЙ ИДЕНТИФИКАЦИИ НЕСТАЦИОНАРНЫХ АНОМАЛЬНЫХ ДИФФУЗИОННЫХ ПРОЦЕССОВ

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Для класса нестационарных аномальных диффузионных процессов, математические модели которых формализуются в виде вариационных неравенств в частных производных, предложен метод математического моделирования, основанный на оптимизационной процедуре. При этом задача реализации математических моделей нестационарных аномальных диффузионных процессов сводится к отысканию максимума функции Гамильтона, определенной в пространстве состояний исследуемых процессов. Также предложен метод параметрической идентификации математических моделей нестационарных аномальных диффузионных процессов в случае индукционной постановки задачи исследования. Метод сводится к использованию оптимизационной процедуры метода проекции градиента. Обоснована возможность решения задачи параметрической идентификации, как для линейных, так и нелинейных математических моделей нестационарных аномальных диффузионных процессов.

Ключевые слова: аномальный диффузионный процесс, математическая модель, вариация, вариационное неравенство, оптимизация, принцип максимума функционал, градиент, параметрическая идентификация.