UDC 519.86

A METHOD FOR CONSTRUCTING A MATHEMATICAL MODEL OF THE COMPANY'S VALUE

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Семенов А.С. Про метод побудови математичної моделі вартості компанії.

Побудовано математичну модель економічної динаміки вартості компанії у вигляді диференційного рівняння другого порядку. Рівняння містить параметри управління, що дозволяють домагатися оптимального розвитку і максимізації вартості. Використаний кореляційно-регресійний аналіз для конкретизації функції вартості акціонерного капіталу і реалізації побудованої моделі на конкретному прикладі.

Ключові слова: вартість, диференційне рівняння, керуючі параметри, регресійний аналіз

Семенов А.С. О методе построения математической модели стоимости компании.

Построена математическая модель экономической динамики стоимости компании в виде дифференциального уравнения второго порядка. Уравнение содержит параметры управления, позволяющие добиваться оптимального развития и максимизации стоимости. Использован корреляционно-регрессионный анализ для конкретизации функции стоимости акционерного капитала и реализации построенной модели на конкретном примере.

Ключевые слова: стоимость, дифференциальное уравнение, управляющие параметры, регрессионный анализ

Semenov A.S. A method for constructing a mathematical model of the company's value.

A mathematical model of the dynamics of the economic value of the company in the form of a second order differential equation. The equation contains the control parameters allowing to achieve optimum development and maximize value. Use correlation and regression analysis in order to specify the cost of the share capital of the functions and the implementation of the model constructed on a concrete example.

Keywords: cost, differential equation, control parameters, regression analysis

Along with such indicator of the company's performance as a profit, the company's value is traditionally used as a measure of its performance. Domestic and foreign authors, for example [1-4], it is shown that company's effective functioning is often based on the management of the index. Value enters into market relations as one of the main indicators, reflecting the production process as well.

Analysis of recent researches and publications

Assessing the value of a company and creating a model for the dynamic change of this one of the main business indicators, the characteristics of the company's business that are quantified, such as assets, profit, cash flow, etc., are used. In classical static and dynamic models, for example, in the Keynesian model, in the Harrod-Domar model these parameters are interrelated and in optimal proportions. It should be noted that the cost-forming factors include exogenous factors that are independent of the company itself, as well as factors that the company can influence.

There are a number of approaches to creating a change model and the possibility of managing the company's value. Models are based, among other things, on the factor approach, the aim of which is to try to assess the impact of each factor, both external and internal, on the integral indicator – on the value of company. For the purpose of analyzing the dynamic behaviour of value, it is more convenient to use the apparatus of differential equations.

In work [5, 6], a mathematical model of t company's value with the use of the theory of differential equations was proposed. Two variants of the total company's value, net assets value and the cost of debt dependence are proposed. In addition, the method of finding the function of net assets and the debt function, included as part of the differential equation of the model, is proposed in [6].

The aim of the article is offering a variant of creating a mathematical model of the companies value based on the theory of differential equations, which, in particular, can complement and develop the models of works [5, 6].

The main part

We believe that the value of the company consists of two components: the cost of equity and the cost of debt:

$$X(t)=C(t)-Z(t), \quad (1)$$

where C (t) is the cost of share capital,

Z (t) is the total amount of debt repayment and replenishment of the social and insurance reserve of the enterprise.

The cost of equity capital, taking into account the lag in time, would be written in the form:

$$C(t)=\int_0^t e^{-\delta\tau}F[X(t-\tau)]d\tau, \quad (2)$$

where the functional dependence of F[X(t)]as one of the cost factors is determined on the basis of regression analysis of the data, depends on X (t) and implicitly on time, δ – discount factor. One of the difficulties in calculating the cost according to formula (1) is the absence of a universal and accurate way of calculating the function F[X(t)] based on financial reporting indicators. For evaluation, each company uses its sometimes-unique calculation methods, including correlation-repression analysis.

We consider the function of the value of debt depending on the rate of change in the company's total value, the faster the value changes, the faster the debt changes:

$$Z(t)=\beta(t)X(t), \quad (3)$$

where β (t) is the proportionality function. Strictly speaking, it varies within the limits $0\leq\beta\leq1$. However, taking into account the fact, that X (t) may have different signs and magnitude, this restriction can be removed. The function can have a certain form depending on the market situation and the consistent or liquidation of the debt by the company or the build-up of debt. In addition, correlation-regression analysis can also be applied to the definition of this function.

Substituting relations (2) and (3) into (1), we obtain the following equation of company value:

$$X(t)=\int_0^t e^{-\delta\tau}F[X(t-\tau)]d\tau-\beta(t)X(t). \quad (4)$$

Next, we make a replacement with a change:

$$t-\tau=s; \quad ds=-d\tau;$$

$$\int_0^t e^{-\delta\tau}F[X(t-\tau)]\,d\tau=e^{-\delta t}\int_0^t e^{\delta s}F[X(s)]ds.$$

This brings equation (4) to the form:

$$X(t)=e^{-\delta t}\int_0^t e^{\delta s}\,F[X(s)]ds-\beta(t)\dot{X}(t). \quad (5)$$

Multiply equation (5) by exp(δt) and go from equation (5) to the next integral-differential equation of company value:

$$X(t)e^{\delta t}=\int_0^t e^{\delta s}F[X(s)]ds-e^{\delta t}\,\beta(t)X(t). \quad (6)$$

Differentiating the left and right sides of equation (6) with respect to the parameter t (time), we pass from equation (6) to the ordinary differential equation of the second order: Differentiating the left and right sides of equation (6) with respect to the parameter t (time), we pass from equation (6) to the ordinary differential equation of the second order:

$$\beta(t)X(t)+(\beta+\beta\delta+1)X(t)+\delta X(t)=F\big(X(t)\big). \quad (7)$$

Adding the initial conditions to equation (7), we obtain the standard Cauchy problem. The parameters of the model δ and β(t) can serve as controlling parameters for the functioning of the economic system. Depending on the nature of the functioning of the economic system, the right-hand side of equation (7) also changes, i.e. function F(X(t)), which can also be attributed to the value management parameter of the company.

For β(t)=0, (3) and (1) imply the relation $X(t)=C(t)$, that is, We have a case of simple reproduction.

At β(t)=1, the debt becomes proportional to the rate of change in value: the faster the value changes (the greater the derivative), the more or less the company's debt becomes, which also looks like an exotic case. Equation (7) takes the form:

$$X(t)+(\delta+1)X(t)+\delta X(t)=-F\big(X(t)\big). \quad (8)$$

The equation is the equation of oscillations with allowance for the forces of resistance. The solution is a damped oscillation with a frequency of oscillations and a coefficient of resistance damping, determined, in particular, by the value of the discount factor δ Thus, with this value of β(t), we arrive at a certain stationary value of the company's value with time.

The right-hand side of equation (8) depends on time implicitly; the argument is the cost X(t). As a result of the application of regression analysis, this function can be represented, including, in the form of a polynomial of the third degree. Thus, equation (8) becomes nonlinear with the cubic nonlinearity characteristic of the Duffing equation. In addition, to the analysis of the change in the value of X(t), one can apply the theory of the Duffing equation taking into account all the characteristic features of the behavior of the solution of this equation.

It is possible to reduce the equation (7) to the oscillation equation without considering the resistance forces formally by setting the parameters of the problem. Assuming β+βδ+1=0, we obtain:

$$\beta(t)=-\frac{1}{\delta}+\left(\beta_0+\frac{1}{\delta}\right)e^{-\delta t}.$$

From a practical point of view, with the rise in value, the derivative of the cost function must be positive. In accordance with formula (3), we should expect a decrease in debt, which requires the function β(t) to be inversely proportional to the time. The assumed dependence satisfies this requirement. Equation (7) will then have the form:

$$X(t)+\frac{\delta}{\beta}X(t)=\frac{1}{\beta}F[X(t)].$$

Regardless of the type of the right side, i.e. from F[X(t)] the solution of the equation has an oscillatory character.

Practice shows that the development of the economic system, as well as the society as a whole, has an oscillatory character. However, one should not speak of periodicity in the strict mathematical understanding of this term. For a number of problems of modeling economic processes, we do not mean periodicity, but cyclist, including when the cyclic process is superimposed on the overall trend of development. The cyclical nature of the changes in economic phenomena occurs from several years to several decades, it makes sense to consider equation (7), in particular, as an equation of oscillations.

In order to reduce equation (7) to the unified form of the oscillation equation with allowance for the driving force and resistance forces, we set:

$$\beta+\delta\beta+1=2\beta k.$$

Here the introduced coefficient k characterizes the degree of influence of certain conditions on the damping, on the decrease in the amplitude of the oscillations with time (the coefficient of "viscous" resistance, i.e., resistance, depending on the rate of change of the function).

Solving the obtained differential equation of the first degree, we find:

$$\beta(t)=-\frac{1}{\delta-2k}+C\,e^{-(\delta-2k)t}$$

and equation (7):

$$X(t)+2kX(t)+\frac{\delta}{\beta(t)}X(t)=\frac{F(X(t))}{\beta t}. \qquad (9)$$

To illustrate the practical application of the derived company value equation, you can specify the input parameters and examine the change in value over time.

Assuming the form of the function $\beta(t)$ is already known, we must choose a function that determines the cost of equity F[X(t)].

To determine the form of the function F[X(t)], it is quite possible to apply the correlation-regression analysis used in the comparative (market) approach to business valuation, as well as to develop models of the dependence of the market value of equity on a number of indicators of its economic activities.

An example of the successful use of correlation-regression methods in the economy can be found in [7]. The author studied as an example of a telecommunications industry enterprise. It is noted that the method is applicable to companies with a share of private capital.

We note some features of the application of the method. In establishing the patterns and trends of development, the larger the data array is being processed, the larger the statistical aggregate (profit, revenue, dividends, net cash flow, book value of assets, net assets, etc.), the closer the theoretical regularity is built to reality. However, in this case it is advisable to consider not the whole set, but the part that most significantly reflects the process. Two goals are pursued in the analysis of statistical data: the establishment of patterns and forecasting. The constructed trend more accurately reflects the actual forecast if data close to the forecast period is used.

The volume of information processed dictates the choice of the form of the equation. The more parameters in the equation, the more the number of observed phenomena is investigated for the same degree of reliability of the estimation. The type of the trend curve can be selected based on the quality criterion of the regression equation, when the minimum sum of squares of deviations of the actual values of the level of the series from the values of the levels calculated by the trend equation. Another statistical criterion is the multiple determination coefficient R2, the values of which for a sufficiently strong bond exceed the value of 0.7.

To analyze the value model of the company built in the work, a trend was used for the function of the cost of equity capital, built and studied in detail in [7].

The equation of linear regression constructed in [7] corresponds to the value R ^ 2 = 0.812, where the dependence of the company's market value on the value of assets is accepted.

Polynomial regression also has a high degree of correlation; so the equation of the second degree, for example, has the form:

$$F[X(t)]=0.36X(t)^2+5.15X(t)-8.75. \qquad (10)$$

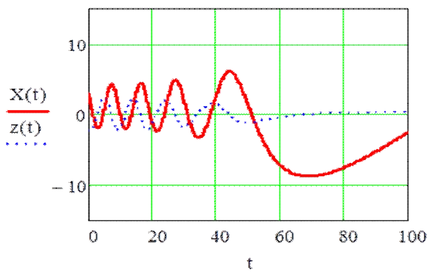
Relation (10) describes a characteristic case of economic growth with constant acceleration. The function well reflects the development tendencies of many economic processes, when absolute.

Increments increase, and growth rates decrease. Using the second-order regression equation (10), the parameter values equal to $\delta = 0.3$, $2k = 0.4$, $C = 1$, we determine the nature of the company's value change using equation (9). Where in:

$$\beta(t)=10+\exp(0.1t).$$

For the numerical account we use the software product Matkad. We open the block of solving differential equations with the Given command and solve equation:

$$\frac{d^2}{dt^2}X(t)+0.4\left(\frac{d}{dt}X(t)\right)+\left(\frac{0.3}{10+e^{0.1t}}\right)X(t)=$$
$$=\left(\frac{1}{10+e^{0.1t}}\right)(0.36X(t)^2+5.15X(t)-8.75).$$



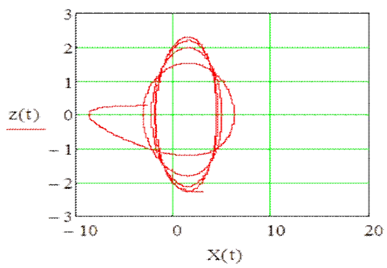


Fig. 1. The change in cost for a quadratic regression function F [X (t)]
Source: Own elaboration

Under the following initial conditions: X(0)=3; X(0)=-2. Using the operator X:=Odesolve (t, 100, 100) at t:=0,0.01..100 and introducing the notation $z(t):=\frac{d}{dt}X(t)$, we represent the solution in the form of a graph of the change in cost and the behaviour of the solution on the phase plane.

After insignificant increasing fluctuations and falling to the minimum value, the cost increases. A characteristic cyclicity appears with a subsequent output to the "stationary" value. The phase trajectory shows the convergence of the process to a certain nonzero value.

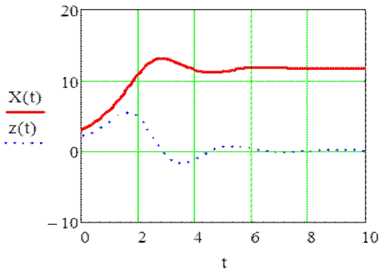
Using the same regression equation (10), setting β(t)=1, i.e. Rapid decrease in debt, δ=0.2, we use equation (8). Again, use the software product Matkad and get the following results:

$$\frac{d^2}{dt^2}X(t)+1.2\left(\frac{d}{dt}X(t)\right)-4.95\,X(t)-0.36\,(X(t))^2=-8.$$

$$X(0)=3;\ X(0)=-2$$

X=Odesolve (t,15,150), t=0,0.01..150,

$$z(t)=\frac{d}{dt}X(t).$$



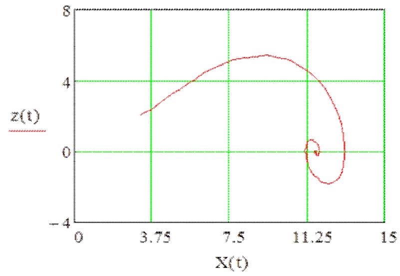


Fig. 2. The change in cost for a quadratic regression function F [X (t)] and β (t) = 1.
Source: Own elaboration

The change in the cost function is of the nature of rapidly damped oscillations, and from the moment of time about 6 it takes a stationary value close to ten (a value exceeding the initial one). The debt function becomes zero from the same moment in time. The phase trajectory shows the convergence of the process to a stationary value. The coefficient before the first power of X (t) in the equation affects the overall location of the entire curve over the abscissa axis: the larger it is the lower the curve, the lower the stationary value of the value. With a coefficient close to unity, the oscillations occur near the initial value of the value equal to three.

Conclusions

Thus, using the received equation of company value and giving the controlling parameters certain values, one can observe the process of changing the value in time, i.e., a dynamic model of the company's value change is constructed.

The equation admits a certain modification. For example, if we take into account the inflow of external investments I (t), then the equation takes the form:

$$X(t)=C(t)+I(t)-Z(t). \quad (11)$$

Moreover, since the investment comes in batches at certain times $t_1,t_2,t_3,\ldots,t_n=T$, where T is the period under consideration, it is natural to specify the inflow of investments using the delta Dirac time function δ(t), for example in the form:

$$I(t)=b_k\delta(t-t_k),\ \ t_k-\varepsilon<t<t_k+\varepsilon.$$

Equation (11) becomes a differential equation with impulse actions, which requires a special integration method.

So, the equation of the dynamic model of the company's value with control parameters is obtained. By adjusting the parameters, it is possible, therefore, to adjust the value, achieving stable development with the maximization of value.

Abstract

The value approach to the disclosure of the functioning of the company is the most popular way to assess its real state and development. Functioning of the company cannot be divorced from the cost criteria, since it is the price is the most important characteristic of economic processes, especially in the conditions of market transformations. One of the most effective approaches and fundamental valuation of the company is a profitable method to justify the cost of using the planned indicators of the financial statements of the company.

The paper presents a version of the model of economic dynamics of the company's value in the form of second-order nonlinear differential equation. Estimates in this case is a standard solution of the Cauchy problem. The equation contains the parameters that are actually controlling parameters of the problem: the discount of the share capital cost factor of proportionality function in the formula the company's debt, the functional dependence of the equity value of F [X (t)].

Function F [X (t)], as a factor of cost is calculated based on the regression analysis. For the purposes of the practical application of the model constructed as a productive change was chosen as the market value of shares in the telecommunications sphere, adopted by the dependence of the market value of the value of assets. The cases of linear and quadratic regression.

Analysis of the simulation results, in particular, for the quadratic regression showed that starting from a certain initial value after a minor at the time and amplitude of the damped oscillations of the price tends to a steady-state value.

Thus, a mathematical model of the dynamics of the economic value of the company, which allows using the control parameters to achieve sustainable development and maximize value.

JEL Classification: C20.

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Надано до редакційної колегії 09.03.2017

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Посилання на статтю / Reference a Journal Article:
A method for constructing a mathematical model of the company's value. [Електронний ресурс] / A. S. Semenov // Економіка: реалії часу. Науковий журнал. – 2017. – № 2(30). – С. 133-137. – Режим доступу до журн.: http://economics.opu.ua/files/archive/2017/No2/133.pdf