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Interpolation Method of Nonlinear Dynamical Systems Identification Based on Volterra Model in Frequency Domain

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***Abstract*— The accuracy of the interpolation method of nonlinear dynamical systems identification based on the Volterra model in the frequency domain is studied in this paper. To extract the n–th partial component in the response of the system to the test signal the n–th partial derivative of the response using the test signal amplitude is found and its value is taken at zero. The polyharmonic signals are used as the test ones. The algorithmic and software toolkit in Matlab is developed for identification processes. This toolkit is used for constructing the informational model of test system. The model is built as a first, second and third order amplitude–frequency and phase–frequency characteristics. The comparison of obtained characteristics with previous works is given.**

Keywords—identification; nonlinear dynamic systems; Volterra models; multifrequency characteristics; polyharmonic signals; communication channels

I. INTRODUCTION

It is necessary to consider technical conditions of the communication channels (CC) operation for effective data transfer. Changes in environmental conditions cause reducing the transmission data rate: in the digital CC – up to a full stop of the transmission, in analog CC – to the noise and distortion of the transmitted signals. The new methods and supporting toolkit are developing to automate the measurement of parameters and taking into account the characteristics of the CC. This toolkit allows obtaining the informational and mathematical model of such nonlinear dynamic object, as the CC [1], i.e. to solve the identification problem.

Modern continuous CCs are nonlinear stochastic inertial systems. The model in the form of integro–power Volterra series used to identify them [2]–[5].

The nonlinear and dynamic properties of such system are completely characterized by a sequence of multidimensional weighting functions – Volterra kernels).

Building a model of nonlinear dynamic system in the form of a Volterra series lies in the choice of the test actions form. Also it uses the developed algorithm that allows determining the Volterra kernels and their Fourier–images for the measured responses (multidimensional amplitude–frequency characteristics (AFC) and phase–frequency characteristics (PFC)) to simulate the CC in the time or frequency domain, respectively. [7].

The additional research of new method of nonlinear dynamical systems identification, based on the Volterra model in the frequency domain is proposed. This method lies in n–fold differentiation of responses of the identifiable system by the amplitude of the test polyharmonic signals. The developed identification toolkit is used to build information model of the test nonlinear dynamic object in the form of the first, second and third order model. [8]

II. INTERPOLATION METHOD OF NONLINEAR DYNAMICAL SYSTEMS IDENTIFICATION

The presentation of the “input–output” type nonlinear dynamical system presented by Volterra series were given in previous work [6].

An interpolation method of identification of the nonlinear dynamical system based on Volterra series is used [8]–[9]. It is used *n*–fold differentiation of a target signal on parameter–amplitude *a* of test actions to separate the responses of the nonlinear dynamical system on partial components $\hat{y}_n(t)$ [9].

$ax(t)$ type test signal is sent to input of the system, where $x(t)$ – any function; $|a|\leq 1$ – scale factor for *n*–th order partial component allocation $\hat{y}_n(t)$ from the measured response of nonlinear dynamical system $y[ax(t)]$. In such case it is necessary to find *n*–th private derivative of the response on amplitude *a* at $a=0$

$$\hat{y}_n(t)=\underbrace{\int_0^t \dots \int_0^t}_{n\ times} w_n(\tau_1,\dots,\tau_n)\prod_{l=1}^{n} x(t-\tau_l)d\tau_l = \frac{1}{n!}\left.\frac{\partial^n y[a\,x(t)]}{\partial a^n}\right|_{a=0}. \quad (1)$$

Partial components of responses $\hat{y}_n(t)$ can be calculated by using the test actions and procedure (1).

Diagonal and subdiagonal sections of Volterra kernel are defined on basis of calculated responses.

Formulas for numerical differentiation using central differences for equidistant knots $y_r = y[a_r x(t)] = y[rhx(t)],\ r = -r_1, -r_1+1, \ldots, r_2$ with step of computational mesh on amplitude $h=\Delta a$ [9] are received. Volterra kernel of the first order is defined as the first derivative at $r_1 = r_2 = 1$ or $r_1 = r_2 = 2$ accordingly

$$y_0' = \frac{1}{2h}(-y_{-1} + y_1),$$
$$y_0' = \frac{1}{12h}(y_{-2} - 8y_{-1} + 8y_1 - y_2), \qquad (2)$$
$$y_0' = \frac{1}{60h}(-y_{-3} + 9y_{-2} - 45y_{-1} + 45y_1 - 9y_2 + y_3).$$

Volterra kernel of the second order is defined as the second derivative at $r_1 = r_2 = 1$ or $r_1 = r_2 = 2$, accordingly

$$y_0'' = \frac{1}{h^2}(y_{-1} - 2y_0 + y_1),$$
$$y_0'' = \frac{1}{12h^2}(-y_{-2} + 16y_{-1} - 30y_0 + 16y_1 - y_2), \qquad (3)$$
$$y_0'' = \frac{1}{180h^2}(2y_{-3} - 27y_{-2} + 270y_{-1} - $$
$$- 490y_0 + 270y_1 - 27y_2 + 2y_3).$$

Volterra kernel of the third order is defined as the third derivative at $r_1 = r_2 = 2$ or $r_1 = r_2 = 3$, accordingly

$$y_0''' = \frac{1}{2h^3}(-y_{-2} + 2y_{-1} - 2y_1 + y_2),$$
$$y_0''' = \frac{1}{8h^3}(y_{-3} - 8y_{-2} + 13y_{-1} - 13y_1 + 8y_2 - y_3) \quad , \qquad (4)$$

where n – the order of transfer function being estimated; A_l, ω_l and φ_l – amplitude, frequency and phase of l–th harmonics accordingly. It is supposed in research, that all amplitudes of A_l are equal and phases φ_l are equal to zero.

The amplitudes of the test signals $a_i^{(k)}$ and the corresponding coefficients $c_i^{(k)}$ for responses are shown in table 1, where:

k – order of the estimated Volterra kernel;

i – number of the experiment (i=1, 2, …, N); N - quantity of interpolation knots, i.e. quantity of identification experiments.

Thus, the test signal can be written in complex form:

$$x(t) = A\sum_{l=1}^{n}\cos(\omega_l t) = \frac{A}{2}\sum_{l=1}^{n}\left(e^{j\omega_l t} + e^{-j\omega_l t}\right). \qquad (5)$$

Then the n–th partial component in the response of system can be represented in the following form:

$$y_n(t) = \frac{A^n}{2^{n-1}}\sum_{m=0}^{\left[\frac{n}{2}\right]} C_n^m \sum_{k_1}^{n}\ldots$$
$$\ldots\sum_{k_n}^{n}\left|W_n\left(-j\omega_{k_1},\ldots,-j\omega_{k_m}, j\omega_{k_{m+1}},\ldots j\omega_{k_n}\right)\right| \times \quad , \qquad (6)$$
$$\times\cos\left(\left(-\sum_{l=0}^{m}\omega_{k_l} + \sum_{l=m+1}^{n}\omega_{k_l}\right)t + \arg W_n\left(-j\omega_{k_1},\ldots,-j\omega_{k_m}, j\omega_{k_{m+1}},\ldots, j\omega_{k_n}\right)\right)$$

where $\left[\frac{n}{2}\right]$ means function for extraction of an integer part of number.

The component with summary frequency $\omega_1+\ldots+\omega_n$ is selected from the response to test signal (2):

$$A^n\left|W_n(j\omega_1,\ldots,j\omega_n)\right|\cos[(\omega_1+\ldots+\omega_n)t + \arg W_n(j\omega_1,\ldots,j\omega_n)].$$

In [10] it is defined that during determination of multidimensional transfer functions of nonlinear systems it is necessary to consider the imposed constraints on choice of the test polyharmonic signal frequencies. It provides an inequality of combination frequencies in output signal harmonics: $\omega_1 \neq 0$, $\omega_2 \neq 0$ and $\omega_1 \neq \omega_2$ for the second order identification procedure, and $\omega_1 \neq 0$, $\omega_2 \neq 0$, $\omega_3 \neq 0$, $\omega_1 \neq \omega_2$, $\omega_1 \neq \omega_3$, $\omega_2 \neq \omega_3$, $2\omega_1 \neq \omega_2+\omega_3$, $2\omega_2 \neq \omega_1+\omega_3$, $2\omega_3 \neq \omega_1+\omega_2$, $2\omega_1 \neq \omega_2-\omega_3$, $2\omega_2 \neq \omega_1-\omega_3$, $2\omega_3 \neq \omega_1-\omega_2$, $2\omega_1 \neq -\omega_2+\omega_3$, $2\omega_2 \neq -\omega_1+\omega_3$ и $2\omega_3 \neq -\omega_1+\omega_2$ for the third order identification procedure.

Given method was fully tested on a nonlinear test object (fig. 1) described by Riccati equation:

$$\frac{dy(t)}{dt} + \alpha y(t) + \beta y^2(t) = u(t). \qquad (7)$$

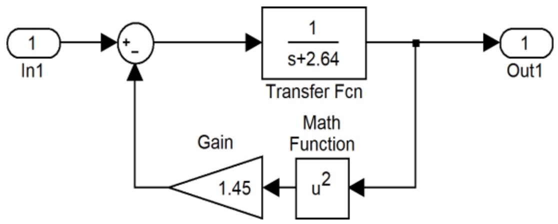

Figure 1. Simulink–model of the test object

Analytical expressions of AFC and PFC for the first, second and third order model were received:

$$|W_1(j\omega)| = \frac{1}{\sqrt{\alpha^2+\omega^2}}, \quad \arg W_1(j\omega) = -\mathrm{arctg}\,\frac{\omega}{\alpha}; \qquad (8)$$

$$|W_2(j\omega_1, j\omega_2)| = \frac{\beta}{\sqrt{(\alpha^2+\omega_1^2)(\alpha^2+\omega_2^2)[\alpha^2+(\omega_1+\omega_2)^2]}}, \qquad (9)$$

$$\arg W_2(j\omega_1, j\omega_2) = -\mathrm{arctg}\,\frac{(2\alpha^2-\omega_1\omega_2)(\omega_1+\omega_2)}{\alpha(\alpha^2-\omega_1\omega_2)-\alpha(\omega_1+\omega_2)^2}; \qquad (10)$$

$$|W_3(j\omega_1, j\omega_2, j\omega_3)| = \sqrt{[\mathrm{Re}(W_3(j\omega_1, j\omega_2, j\omega_3))]^2 + [\mathrm{Im}(W_3(j\omega_1, j\omega_2, j\omega_3))]^2} = \frac{2\beta^2}{3}\frac{1}{\sqrt{[\alpha^2+(\omega_1+\omega_2+\omega_3)^2](\alpha^2+\omega_1^2)(\alpha^2+\omega_2^2)(\alpha^2+\omega_3^2)}}\times$$
$$\times\frac{\sqrt{[3\alpha^2-(\omega_1+\omega_3)(\omega_1+\omega_2)-(\omega_1+\omega_2)](\omega_2+\omega_3)-(\omega_1+\omega_3)(\omega_2+\omega_3)]^2+16\alpha^2(\omega_1+\omega_2+\omega_3)^2}}{\sqrt{[\alpha^2+(\omega_1+\omega_2)^2][\alpha^2+(\omega_1+\omega_3)^2][\alpha^2+(\omega_2+\omega_3)^2]}} \quad (11)$$

$$\arg W_3(j\omega_1, j\omega_2, j\omega_3) = \mathrm{arctg}\frac{\mathrm{Im}W_3(j\omega_1, j\omega_2, j\omega_3)}{\mathrm{Re}W_3(j\omega_1, j\omega_2, j\omega_3)} = -\mathrm{arctg}\frac{DA-CB}{AB+CD}, \quad (12)$$

where $A = 3\alpha^2 - 3\omega_1\omega_2 - 3\omega_2\omega_3 - 3\omega_1\omega_3 - \omega_1^2 - \omega_2^2 - \omega_3^2$; $C = 4\alpha(\omega_1+\omega_2+\omega_3)$; $B = uw - vx$; $D = vw + ux$;

$$u = \alpha^3 - \alpha\omega_1\omega_3 - \alpha\omega_2\omega_3 - \alpha(\omega_1+\omega_2+\omega_3);\ w = (\alpha^2 - \omega_1\omega_2 - \omega_2\omega_3)(\alpha^2 - \omega_1\omega_2 - \omega_1\omega_3) - \alpha^2(\omega_1+\omega_2+\omega_3)^2;$$
$$v = (\omega_1+\omega_2+\omega_3)(2\alpha^2 - \omega_1\omega_3 - \omega_2\omega_3);\ x = \alpha(\omega_1+\omega_2+\omega_3)(2\alpha^2 - 2\omega_1\omega_2 - \omega_1\omega_3 - \omega_2\omega_3).$$

TABLE 1. AMPLITUDES AND CORRESPONDING COEFFICIENTS OF THE INTERPOLATION METHOD

Kernel order, k	Experiments quantity, N	$a_1^{(k)}$	$a_2^{(k)}$	$a_3^{(k)}$	$a_4^{(k)}$	$a_5^{(k)}$	$a_6^{(k)}$	$c_1^{(k)}$	$c_2^{(k)}$	$c_3^{(k)}$	$c_4^{(k)}$	$c_5^{(k)}$	$c_6^{(k)}$
1	2	-1	1					-0,5	0,5				
	4	-1	-0,5	0,5	1			0,0833	-0,6667	0,6667	-0,0833		
	6	-1	-0.67	-0,33	0,33	0,67	1	-0,0167	0,15	-0,75	0,75	-0,15	0,0167
2	2	-1	1					1	1				
	4	-1	-0,5	0,5	1			-0,0833	1,3333	1,3333	-0,0833		
	6	-1	-0.67	-0,33	0,33	0,67	1	0,0111	-0,15	1,5	1,5	-0,15	0,0111
3	4	-1	-0,5	0,5	1			-0,5	1	-1	0,5		
	6	-1	-0.67	-0,33	0,33	0,67	1	0,125	-1	1,625	-1,625	1	-0,125

III. THE TECHNIQUE OF TEST OBJECT IDENTIFICATION

The main purpose was to identify the multi frequency performances characterizing nonlinear and dynamical properties of nonlinear test object. Volterra model in the form of the second order polynomial is used. Thus, test object properties are characterized by transfer functions of $W_1(j\omega_1)$, $W_2(j\omega_1,j\omega_2)$, $W_3(j\omega_1,j\omega_2,j\omega_3)$ − by Fourier–images of weight functions $w_1(t)$, $w_2(t_1, t_2)$, $w_3(t_1, t_2, t_3)$.

Structure chart of identification procedure – determination of the 1–, 2– or 3–order AFC of CC is presented on fig. 2.

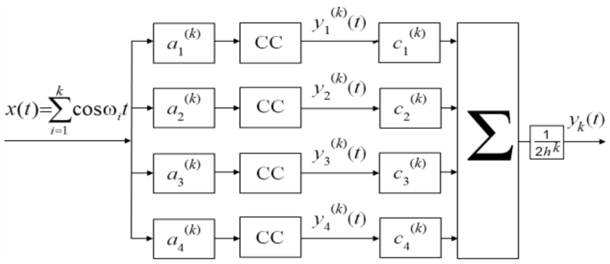

Figure 2. The structure chart of identification procedure using the first order Volterra model in frequency domain, number of experiments N=4

The weighted sum is formed from received signals – responses of each group (fig. 2). As a result the partial components of CC responses $y_1(t)$, $y_2(t)$ and $y_3(t)$ are got. For each partial component of response the Fourier transform (the FFT is used) is calculated, and from received spectrum only an informative harmonics (which amplitudes represent values of required characteristics of the first, second and third orders AFC) are taken.

The first order AFC $|W_1(j\omega_1)|$ and PFC $\arg W_1(j\omega_1)$, where ω_1=ω are received by extracting the harmonics with frequency f from the spectrum of the CC partial response $y_1(t)$ to the test signal $x(t)$=A/2(cos ωt).

The second order AFC $|W_2(j\omega_1,j\omega_2)|$ and PFC $\arg W_2(j\omega_1,j\omega_2)$, where ω_1=ω and ω_2=ω_1+Ω_1, were received by extracting the harmonics with summary frequency ω_1+ω_2 from the spectrum of the CC partial response $y_2(t)$ to the test signal $x(t)$=(A/2)(cos$\omega_1 t$+cos$\omega_2 t$).

The third order AFC $|W_3(j\omega_1,j\omega_2,j\omega_3)|$ and PFC $\arg W_3(j\omega_1,j\omega_2,j\omega_3)$, where ω_1=ω, ω_2=ω_1+Ω_1, ω_3=ω_2+Ω_2 were received by extracting the harmonics with summary frequency ω_1+ω_2+ω_3 from the spectrum of the CC partial response $y_2(t)$ to the test signal $x(t)$=(A/2)(cos$\omega_1 t$+cos$\omega_2 t$+cos$\omega_3 t$).

The results (first, second and third order AFC and PFC) which had been received after procedure of identification are represented in fig. 3–5 (number of experiments for the model N=4).

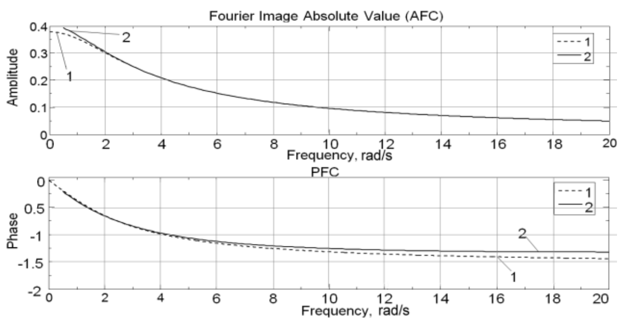

Figure 3. First order AFC and PFC of the test object: analytically calculated values (1), section estimation values with number of experiments for the model N=4 (2)

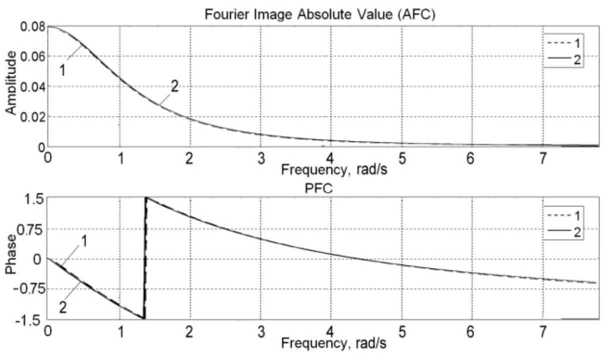

Figure 4. Second order AFC and PFC of the test object: analytically calculated values (1), subdiagonal cross–section values with number of experiments for the model N=4 (2), Ω_1=0,01 rad/s

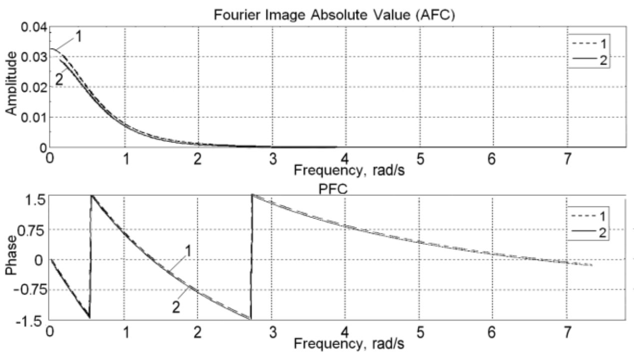

Figure 5. Third order AFC and PFC of the test object: analytically calculated values (1), subdiagonal cross-section values with number of experiments for the model N=6 (2), Ω_1=0,01 rad/s, Ω_2=0,1 rad/s

The surfaces shown on fig. 6–9 are built from subdiagonal cross–sections which were received separately. Ω_1 was used as growing parameter of identification with different value for each cross–section in second order characteristics. Fixed value of Ω_2 and growing value of Ω_1 were used as parameters of identification to obtain different value for each cross–section in third order characteristics.

The second order surfaces for AFC and PFC had been received after procedure of the test object identification and are shown in fig. 6–7 (number of experiments for the model N=4).

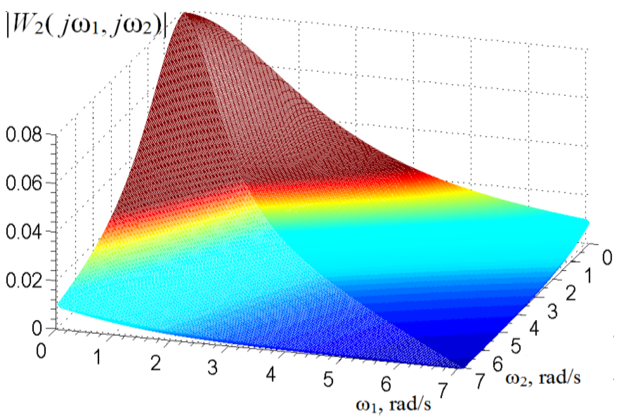

Figure 6. Surface of the test object AFC built of the second order subdiagonal cross–sections received for N=4, Ω_1=0,01 rad/s

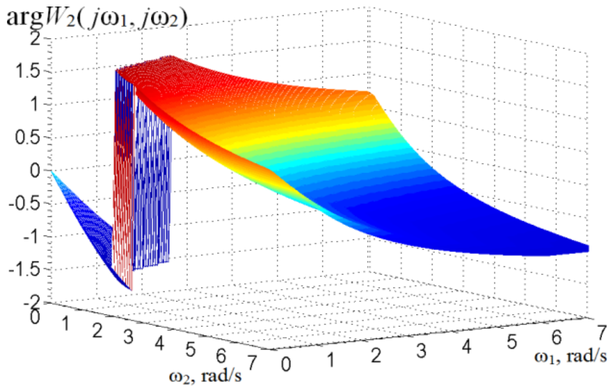

Figure 7. Surface of the test object PFC built of the second order subdiagonal cross–sections received for N=4, Ω_1=0,01 rad/s

The third order surfaces for AFC and PFC had been received after procedure of the test object identification and are presented in fig. 8–9 (number of experiments for the model N=6).

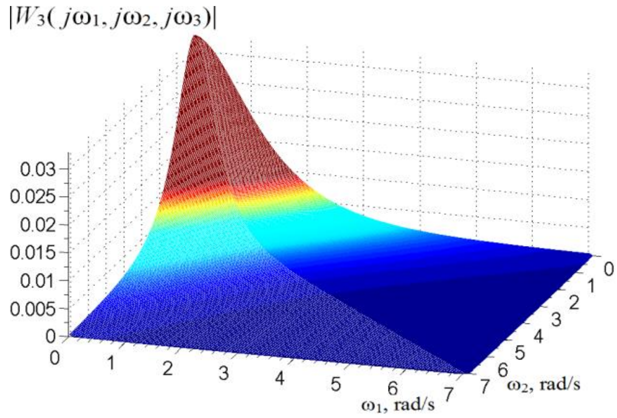

Figure 8. Surface of the test object AFC built of the third order subdiagonal cross–sections received for N=6, Ω_1=0,01 rad/s, Ω_2=0,1 rad/s

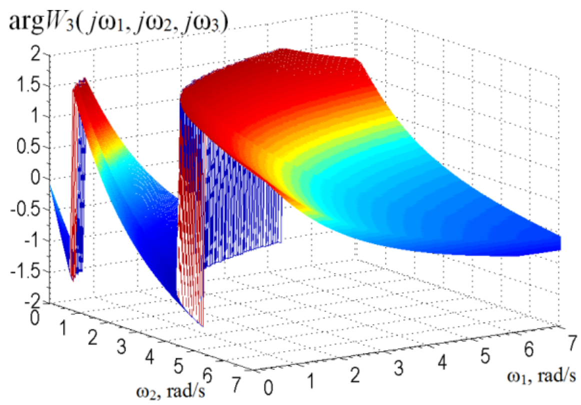

Figure 9. Surface of the test object PFC built of the third order subdiagonal cross-sections received for N=6, Ω_1=0,01 rad/s, Ω_2=0,1 rad/s

Numerical values of identification accuracy using interpolation method for the test object are represented in table 2.

TABLE 2. NUMERICAL VALUES OF IDENTIFICATION ACCURACY USING INTERPOLATION METHOD

Kernel order, k	Experiments quantity / approximation order, N	AFC relative error, %	PFC relative error, %
1	2	2.1359	2.5420
	4	0.3468	2.0618
	6	0.2957	1.9311
2	2	30.2842	76.8221
	4	2.0452	3.7603
	6	89.2099	5.9438
3	4	10.981	1.628
	6	10.7642	1.5522

IV. HARDWARE-SOFTWARE TOOLKIT AND TECHNIQUE OF RADIOFREQUENCY CC IDENTIFICATION

Experimental research of the Ultra High Frequency range CC were done. The main purpose was the identification of multifrequency performances that characterize nonlinear and dynamical properties of the CC. Volterra model in the form of the second order polynomial is used. Thus physical CC properties are characterized by transfer functions of $W_1(j2\pi f)$ and $W_2(j2\pi f_1,j2\pi f_2)$ − by the Fourier–images of weighting functions $w_1(t)$ and $w_2(t_1, t_2)$.

Implementation of identification method on the IBM PC computer basis has been carried out using the developed software in Matlab software. The software allows automating the process of the test signals forming with the given parameters (amplitudes and frequencies). Also this software allows transmitting and receiving signals through an output and input section of PC soundcard, to produce segmentation of a file with the responses to the fragments, corresponding to the CC responses being researched on test polyharmonic effects with different amplitudes.

In experimental research two identical S.P.RADIO A/S, RT2048VHF VHF–radio stations (the range of operational frequencies is 154,4−163,75 MHz) and IBM PC with Creative SBLive! soundcards were used. Sequentially AFC of the first and second orders were defined. The method of identification with number of experiments N=4 was applied.

General scheme of a hardware–software complex of the CC identification, based on the data of input–output type experiment was studied in [6].

The CC received responses $y[a_i x(t)]$ to the test signals $a_i x(t)$, compose a group of the signals, which amount is equal to the used number of experiments N (N=4), shown in fig.10.

In each following group the signals frequency increases by magnitude of chosen step. A cross–correlation was used to define the beginning of each received response. Information about the form of the test signals given in [7] were used.

In described experiment with use of sound card the maximum allowed amplitude was A=0,25V (defined experimentally). The range of frequencies was defined by the sound card pass band (20…20000 Hz), and frequencies of the test signals has been chosen from this range, taking into account restrictions specified above. Such parameters were chosen for the experiment: start frequency f_s =125 Hz; final frequency f_e =3125 Hz; a frequency change step Δf=125 Hz; to define AFC of the second order determination, an offset on frequency $F_1=f_2-f_1$ was increasingly growing from 201 to 3401 Hz with step 100 Hz.

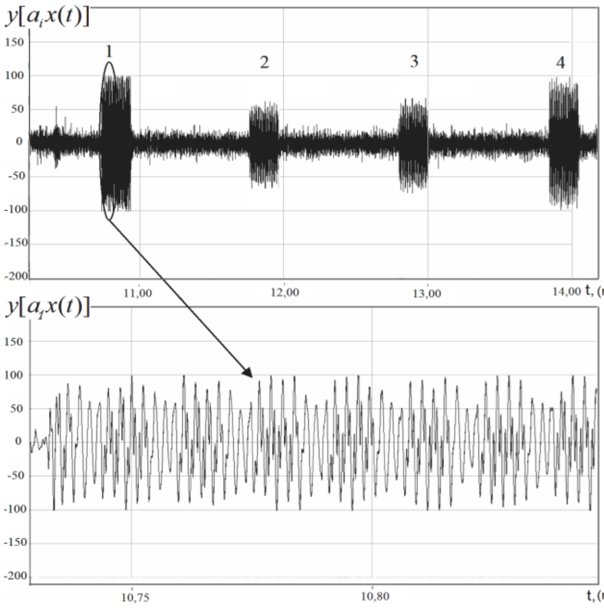

Figure 10. The group of signals received from CC with amplitudes: -1 (1); 1 (2); -0,644 (3); 0,644 (4); N=4

The weighed sum is formed from received signals – responses of each group (fig. 2). As a result we get partial component s of response of the CC $y_1(t)$ and $y_2(t)$. For each partial component of response a Fourier transform (the FFT is used) is calculated, and from received spectra only an informative harmonics (which amplitudes represents values of required characteristics of the first and second orders AFC) are taken.

The first order AFC $|W_1(j2\pi f)|$ is received by extracting the harmonics with frequency f from the spectrum of the partial response of the CC $y_1(t)$ to the test signal $x(t)=A/2(\cos 2\pi ft)$.

The second order AFC $|W_2(j2\pi f, j2\pi(f+F_1))|$, where $f_1=f$ and $f_2=f+F_1$, was received by extracting the harmonics with summary frequency f_1+f_2 from the spectrum of the partial response of the CC $y_2(t)$ to the test signal $x(t)=(A/2)(\cos 2\pi f_1 t+\cos 2\pi f_2 t)$.

The wavelet noise–suppression was used to smooth the output data of the experiment [11]. The results received after digital data processing of the data of experiments (wavelet “Coiflet” de–noising) for the first and second order AFC are presented in fig. 11–13.

The surface shown in fig. 13 was built from subdiagonal cross–sections that have been received separately. A growing parameter of identification Δf with different value for each section was used.

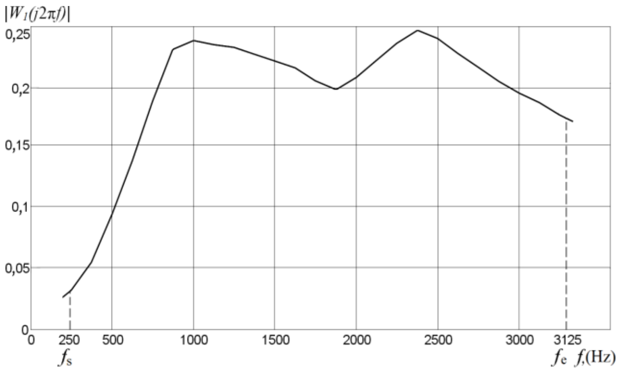

Figure 11. AFC of the first order after wavelet "Coiflet" second level denoising

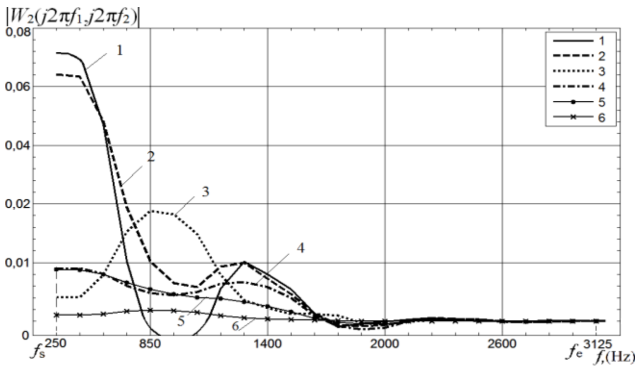

Figure 12. Subdiagonal cross–sections of AFCs of the second order after wavelet "Coiflet" second level de–noising at different frequencies: 201 (1), 401 (2), 601 (3), 801 (4), 1001 (5), 1401 (6) Hz

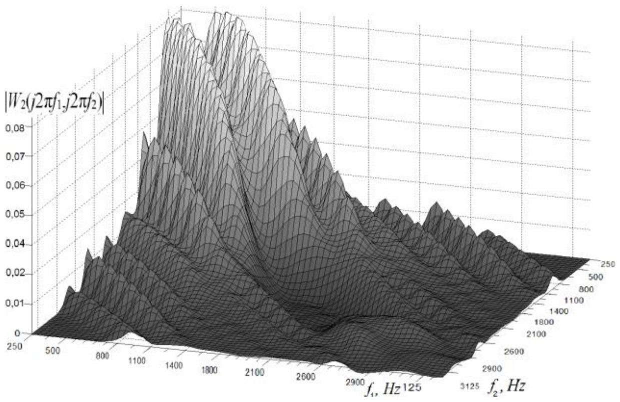

Figure 13. Surface built of AFC cross–sections of the second order after wavelet "Coiflet" 3rd level de–noising

Conclusions

The method based on Volterra model using polyharmonic test signals for identification nonlinear dynamical systems is analyzed. The method based on composition of linear responses combination on test signals with different amplitudes were used to differentiate the responses of object for partial components. New values of test signals amplitudes were defined and model were validated using the test object. Excellent accuracy level for received model is achieved as in linear model so in nonlinear ones. Given values are greatly raising the accuracy of identification in compare to amplitudes and coefficients written in [12]-[13]. The identification accuracy of nonlinear part for the test object has grown almost at 10 times while the standard deviation in best cases is no more than 5%.

Interpolation method of identification using the hardware methodology written in [13] is applied for constructing of informational Volterra model as an APC of the first and second order for UHF band radio channel.

Received results had confirmed significant nonlinearity of the test object characteristics that leads to distortions of signals in different type radio devices.

In the further researches it is necessary to study the third order frequency characteristics of the real CC for ability to decrease level of high order nonlinear distortions in telecommunication systems.

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