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## GRINDING TEMPERATURE MODEL SIMPLIFICATION FOR THE OPERATION INFORMATION SUPPORT SYSTEM

**Annotation.** Grinding temperature mathematic models need for the designing, monitoring and diagnosing the grinding operation to boost the operation throughput without burns of the surface to be ground. This is fully relevant, for example, for CNC gear grinding machines. Once the problem of mentioned mathematic models development is solved, it becomes possible to develop appropriate computer subsystems to optimize and control the grinding operation on CNC machines at the stages of both production and its preparation. The urgency of solving this problem is confirmed by the large number of relevant publications, most of them are connected with Jaeger moving heat source. At the same time, the idea of replacing the fast-moving source with the time of action of the corresponding unmoving one, formulated for the first time by Jaeger, has not yet found a proper practical application. This article justifiably shows that the proximity of the results of calculating the maximum grinding temperature and the depth of its penetration by the two- and one-dimensional solutions practically takes place when the fast-moving heat source is characterized by the Peclet number which is more than 4. For this interval of the Peclet number change, a simplified formula for grinding temperature was first obtained for determining the temperature on the surface and on the depth of the surface layer. Then this simplified formula was investigated by comparing it with the well-known analytical solution of the one-dimensional differential equation of heat conduction for various values of the Peclet number. It is shown that in the range of the Peclet number from 4 to 20, which is the case for most modern grinding operations (flat, round, profile, and others), the difference in determining the grinding temperature by exact and approximate solutions does not exceed 11%. At the same time, the simplified solution obtained in the paper has an important new quality. The mathematical formula that describes this solution makes it possible to express explicitly the penetration depth of any given fixed temperature. If this fixed temperature leads to structural-phase transformations in the surface layer of the workpiece, then it becomes possible to determine the defective layer depth during grinding. In turn, the grinding stock for the grinding operation should be greater than the mentioned defective layer depth. New information on the state of the grinding system can be the basis for monitoring and diagnosing of the operation, as well as for designing this operation at the stage of production preparation. This, in turn, is a technological prerequisite for the development of appropriate computer subsystems that can be integrated into the CNC system of modern grinding machines.

**Keywords:** Grinding temperature; heating stage; Peclet number; thermal models; dimensionless temperature

### Introduction

The problem of the grinding productivity increasing can often be solved by adapting the grinding system elements to higher productivity, taking into account the temperature in the contact zone, i.e. between the grinding wheel and the workpiece [1-2]. To do this, an appropriate methodology has to be created. The analysis and study of thermal phenomena during grinding is part of this methodology which concerns the development of embedded subsystems for designing, monitoring and process diagnosing of the profile grinding operation on CNC machines. The development of such subsystems is an example of the information and communication technologies strategy implementation in the framework of the fourth industrial revolution in discrete and process manufacturing named “Industry 4.0”.

The temperature in the grinding zone is one of the main factors limiting the performance of grinding operation [1-3]. To optimize a grinding process

and boost its productivity, it is necessary to have true information about the grinding temperature which can be obtained by the methods observed in the literature. There are many works which are devoted to the study of thermal phenomena in grinding. In terms of applied research methods the analyzed literary references can be divided into the following groups: theoretical methods [4-7]; the theoretical ones with experimental verification [8-10]; theoretical ones with computer simulation of the temperature field [11-13]; computer simulation methods with experimental testing [13-18]; the only computer simulation ones [19-21].

### 1. Theoretical Background

#### 1.1. Two-dimensional Solution

One of the grinding heat criterion indicators is the Peclet number  $H = \frac{Vh}{2a}$ , which is determined by the velocity of the part being ground  $V$  (m/s), the half-width of the moving contact zone  $h$  (in m) and the thermal diffusivity  $a$  (in m<sup>2</sup>/s). For most grinding operations there is the condition of  $H \geq 4$  [10; 22-24] and this condition is the basis for determining

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the grinding temperature based on solving the one-dimensional differential equation of heat conduction [10; 22; 23; 25], since at  $H \geq 4$  the results of calculating the maximum grinding temperature by the two- and one-dimensional solutions (both for the workpiece surface and along the depth of the surface layer) differ by no more than 5-10 % [22; 23]. That is why the criterion  $H \geq 4$  may be used to choose the one-dimensional solution instead of two-dimensional one for determining the grinding temperature. In this regard, let's consider the two-dimensional solution in terms of determining the temperature and depth of the defective layer during grinding.

According to the work [24] a heat exchange scheme for surface grinding without forced cooling is represented as follows. A band heat source with the width of  $2h$  moves over the flat surface of a semi-infinite body along and in the positive direction of the  $z$ -axis and is infinitely long in the direction of the  $y$ -axis (Fig.1, a). The heat flux density  $q$  (in  $W/m^2$ ) over the entire surface of the moving contact is assumed uniform, i.e.  $q = \text{const}$ . The coordinate system is referenced to the moving heat source (Fig. 1, a).

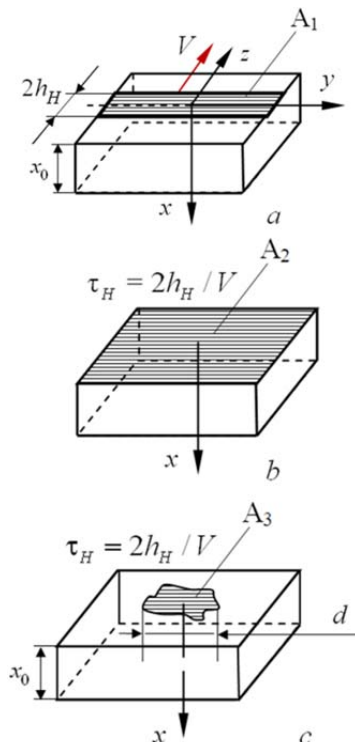


Fig.1. Moving zone  $A_1$  (a) and unmoving  $A_2$  (b) and  $A_3$  (c) zones in which there is the heat flux  $q$  at the stage of heating

The transition from the two-dimensional thermophysical scheme with a moving heat source (Fig.1, a) to the corresponding one-dimensional

schemes with unlimited (Fig. 1, b) and limited (Fig.1, c) unmoving flat sources is performed by replacing the velocity parameter of the heat source  $V$  (in m/s) by the time of its action.

The two-dimensional solution for determining the grinding temperature, as it is adopted in works [4; 22; 24; 25], has the following form

$$T_{2D}(X,Z,H) = \frac{2qa}{\pi\lambda V} \times \int_{Z-H}^{Z+H} \exp(-\xi) K_0\left(\sqrt{X^2 + \xi^2}\right) d\xi. \quad (1)$$

In the equation (1) the following designations are used:  $\lambda$  is the thermal conductivity in  $W/(m \cdot K)$  of the workpiece material;  $\xi = \frac{V(z-z')}{2a}$ ;  $V$  is the

heat source velocity, m/s;  $X, Z$  are dimensionless (i.e. relative) coordinates which correspond to dimensional coordinates  $x, z$  in m;  $H$  is the Peclet number or dimensionless heat source half-width which correspond to the dimensional half-width  $h$  (in m) (Fig. 1, a);  $K_0(s)$  stands for the zeroth-order modified Bessel function of the second kind. In equation (1) the following designations are used:

$$X = \frac{Vx}{2a}; Z = \frac{Vz}{2a}; H = \frac{Vh}{2a}.$$

Dividing both parts of equation (1) by the factor of  $2qa/\pi\lambda V$ , we obtain the following two-dimensional solution in a dimensionless form, e.g. in the interval of  $-20H \leq Z \leq +5H$ , i.e.

$$\Theta(X,Z,H) = \int_{Z-H}^{Z+H} \exp(-\xi) K_0\left(\sqrt{X^2 + \xi^2}\right) d\xi. \quad (2)$$

Let's introduce the notation

$$J(u) = \int_0^u \exp(-\xi) K_0\left(\sqrt{X^2 + \xi^2}\right) d\xi. \quad (3)$$

Using the property of a definite integral, expression (2) can be written as [24]

$$\int_{Z-H}^{Z+H} \exp(-\xi) K_0|\xi| d\xi = J(Z+H) - J(Z-H).$$

That is, the dimensionless temperature is equal to the difference between the values of the integral (2) found at its upper and lower limits. The surface temperature can be formally found from equation (2) at  $X = 0$ , i.e.

$$\Theta_{2D}|_{X=0} = \Theta(0,Z,H) = \int_{Z-H}^{Z+H} \exp(-\xi) K_0|\xi| d\xi. \quad (4)$$

Here  $|\xi| = \sqrt{\xi^2}$ .

In this case, the expression (3) will be

$$J(u)|_{X=0} = \int_0^u \exp(-\xi) K_0 |\xi| d\xi. \quad (5)$$

In turn

$$J(u)|_{X=0} = \int_0^u \exp(-\xi) K_0 |\xi| d\xi \approx \sqrt{2\pi|u|} - 1. \quad (6)$$

Thus, in the moving coordinate system (Fig. 1, a), the surface temperature under the heat source, i.e. at the stage of heating in the interval  $-H \leq Z \leq H$ , can be determined by the formula

$$\Theta_H(0, H, Z) = \sqrt{2\pi|H - Z|}, \text{ if } -H \leq Z \leq H. \quad (7)$$

In turn, behind the source, i.e. at the stage of cooling in the interval of  $Z < -H$ , according to the formula [24] we have

$$\Theta_C(0, H, Z) = \sqrt{2\pi} \times (\sqrt{H - Z} - \sqrt{|H + Z|}), \text{ if } Z < -H. \quad (8)$$

Below it will be shown that the formulas (7) and (8) completely coincide with the corresponding formulas for determining the dimensionless surface grinding temperature obtained from the one-dimensional

$$\begin{cases} \Theta_{cr}(\text{number}) \geq \Theta_{2D}(0, Z_0, H) = \int_{Z_0-H}^{Z_0+H} \exp(-\xi) K_0 |\xi| d\xi \\ \Theta_{cr}(\text{number}) = \Theta(X_{cr}, Z_X, H) = \int_{Z_X-H}^{Z_X+H} \exp(-\xi) K_0 (\sqrt{X_{cr}^2 + \xi^2}) d\xi, \end{cases} \quad (10)$$

where:  $Z_0$  and  $Z_X$  are dimensionless coordinates in the moving coordinate system (Fig. 1, a), on the surface,  $X = 0$ , and at depth of  $X = X_{cr}$ , respectively, in which the dimensionless temperature takes the maximum value.

These coordinates, fixed in each specific case, are located in the region of the heat source trailing edge (Fig. 1, a). For example, for an interval at least of  $5 \leq H \leq 20$ , the coordinate  $Z_0$  of the point of maximum temperature varies in the range of  $0.88H \leq |Z_0| \leq 0.96H$ , and at  $H = 0.5$  this coordinate is  $|Z_0| = 0.54$ . Here, the coordinate  $Z_0$  is taken modulo for convenience of explanation, since in the adopted coordinate system (Fig. 1, a) the value  $Z_0$  is negative, i.e.  $Z_0 \leq 0$ . When the inequality in the system (10) is satisfied, it is possible to determine the temperature rise coefficient

$$n = \frac{\Theta_{\max}(0, Z_0, H)}{\Theta_{cr}(\text{number})}.$$

Taking into account equations (2) and (4), we obtain a transcendental equation with one unknown  $X_{cr}(n)$ , i.e.

solution of the differential equation of heat conduction. If the temperature on the surface is equal to the critical value, then from equation (3) we obtain the condition (or criterion) of the heat damage appearance, i.e

$$\begin{aligned} \Theta_{cr}(\text{number}) &= \Theta_{\max}(0, Z_0, H) = \\ &= \int_{Z_0-H}^{Z_0+H} \exp(-\xi) K_0 |\xi| d\xi. \end{aligned} \quad (9)$$

Here  $\Theta_{\max}(0, Z_0, H)$  is the maximum surface temperature, depending on the coordinate  $Z_0$  for a given value  $H$ . If  $\Theta_{\max}(0, Z_0, H) = \Theta_{cr}(\text{number})$  then the critical grinding temperature penetration depth is equal to zero, i.e.  $X_{cr} = 0$ . If the current temperature according to equation (2) is greater than the critical value  $\Theta_{cr}(\text{number})$ , then the depth  $X_{cr}$  can be found taking into account equation (4). So we have

$$n = \frac{\int_{Z_0-H}^{Z_0+H} \exp(-\xi) K_0 |\xi| d\xi}{\int_{Z_X-H}^{Z_X+H} \exp(-\xi) K_0 (\sqrt{[X_{cr}(n)]^2 + \xi^2}) d\xi}.$$

In this case, the maximum temperature on the surface  $\Theta_{\max}(0, Z_0, H)$  is greater than the maximum (along the coordinate  $Z$ ) temperature at the depth  $X_{cr}(n)$  to which the temperature  $\Theta_{cr}(\text{number})$  penetrates i.e.

$$\begin{aligned} &\int_{Z_0-H}^{Z_0+H} \exp(-\xi) K_0 |\xi| d\xi = \\ &= n \int_{Z_X-H}^{Z_X+H} \exp(-\xi) K_0 (\sqrt{[X_{cr}(n)]^2 + \xi^2}) d\xi. \end{aligned} \quad (11)$$

From the equation (11) it is possible to find the dimensionless depth of the defective layer  $X_{cr}(n)$ , to which the critical temperature  $\Theta_{cr}(\text{number})$  penetrates, if the maximum surface grinding temperature  $\Theta_{\max}(0, Z_X, H)$  is  $n$  times greater than  $\Theta_{cr}(\text{number})$ .

### 1.2. Converting the Temperature Grinding Equation for the One-Dimensional Solution

Along with the dimensionless two-dimensional solution (2), the one-dimensional solution is also obtained under boundary conditions of the second kind. This solution for determining the maximum dimensionless temperature  $\Theta_H(X, H_H)$  with the notations mentioned above is [1; 26]

$$\Theta_H(X, H_H) = 2\pi\sqrt{H_H} \operatorname{ierfc} \frac{X}{2\sqrt{H_H}}. \quad (12)$$

The expression (12) can be represented as

$$\Theta_H(X, H_H) = 2\pi\sqrt{H} \times \left[ \frac{1}{\sqrt{\pi}} \exp\left(-\left(\frac{X}{2\sqrt{H_H}}\right)^2\right) - \frac{X}{2\sqrt{H_H}} \operatorname{erfc}\left(\frac{X}{2\sqrt{H_H}}\right) \right]. \quad (13)$$

We have shown in [22; 23; 27] that when determining the maximum grinding temperature both on the surface and depth of the surface layer using equations (1) and (13), the results differ by no more than 5-10 %, if the calculations are made in the interval of Peclet numbers change of  $H \geq 4$ .

From the expression (13) it can be seen that the maximum dimensionless grinding temperature on the surface ( $X = 0$ ) is determined by the expression

$$\Theta(0, H_H) = 2\sqrt{\pi H_H}.$$

Let's study the function

$$f = \frac{\Theta_H(X, H_H)}{2\pi\sqrt{H_H}} = \operatorname{ierfc} \frac{X}{2\sqrt{H_H}} \text{ contained in equation (12) using the MathCAD. With regard to the equation (13) we have}$$

tion (12) using the MathCAD. With regard to the equation (13) we have

$$f = \operatorname{ierfc} \frac{X}{2\sqrt{H_H}} = \frac{1}{\sqrt{\pi}} \exp\left(-\left(\frac{X}{2\sqrt{H_H}}\right)^2\right) - \frac{X}{2\sqrt{H_H}} \operatorname{erfc}\left(\frac{X}{2\sqrt{H_H}}\right). \quad (14)$$

Let us determine the intervals of change of both the dimensionless depth  $X$  and the argument

$\left(\frac{X}{2\sqrt{H_H}}\right)$  of the function (14). In determining the

intervals, we take into account that for most different grinding schemes, the Peclet number  $H$  varies within the interval of  $4 \leq H \leq 20$  [22; 27]. Therefore, in further studies it is necessary to study the interval of  $H$  change that will be not less than the one of  $4 \leq H \leq 20$ . Let us assume that the depth of the heated surface layer during grinding (dimensional coordinate  $x$ ) varies in the interval  $0 \leq x \leq 1 \cdot 10^{-3}$  m.

For a large number of grinding schemes, the velocity of the heat source moving (a part being ground velocity) varies in the range of  $0.1 \leq V \leq 20$  m/min. The coefficient of thermal diffusivity  $a$  for tool, carbon, alloyed and heavily alloyed steels varies in the range of  $4 \cdot 10^{-6} \leq a \leq 10 \cdot 10^{-6}$  m<sup>2</sup>/s [22; 27]. Taking into account the specified velocity intervals, we define the maximum and minimum values of the dimensionless parameters of the  $X$  and

$\left(\frac{X}{2\sqrt{H_H}}\right)$ , to wit:  $0 \leq X \leq 41.7$  and  $0 \leq \left(\frac{X}{2\sqrt{H_H}}\right) \leq$

96. Thus, we can construct a graph (Fig. 2, a) using equation (14).

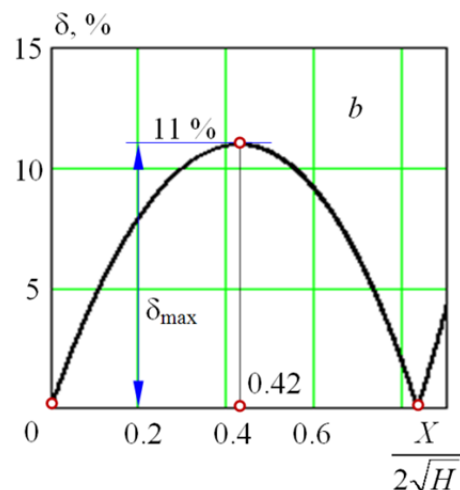
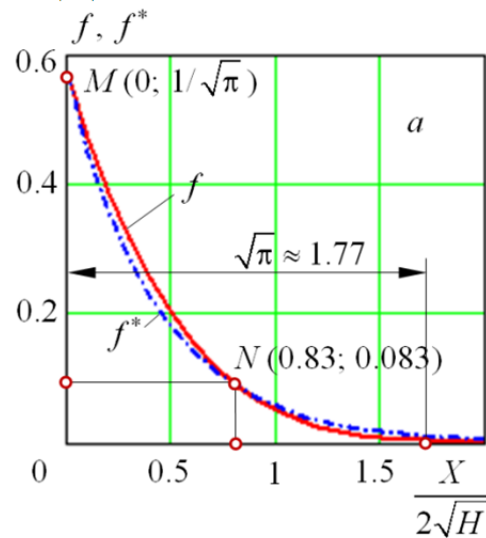


Fig. 2. Functions  $f = \operatorname{ierfc} \frac{X}{2\sqrt{H_H}}$  and

$$f^* = \frac{1}{\sqrt{\pi}} \exp\left(-\left(\frac{X}{2\sqrt{H_H}}\right)^2\right) \text{ (a) and the relative difference } \delta$$

between them in percentage (b) in relation to the function  $f$

It can be seen from the  $f$  curve (Fig. 2, a), that the theoretical interval  $0 < \left(\frac{X}{2\sqrt{H_H}}\right) < 96$  for changing the argument of the function being studied can in fact be replaced by interval of  $0 \leq \frac{X}{2\sqrt{H}} \leq \sqrt{\pi}$  (Fig. 2, a) or even by a narrower interval of  $0 \leq \frac{X}{2\sqrt{H}} \leq 1$  (Fig. 2, a).

Because the exponentially damped nature of the function  $f(X, H) = \text{ierfc} \frac{X}{2\sqrt{H}}$ , let's select the param-

$$\text{ierfc} \frac{0}{2\sqrt{H_H}} = \left( \frac{1}{\sqrt{\pi}} \exp \left( - \left( \frac{0}{2\sqrt{H_H}} \right)^2 \right) - \frac{0}{2\sqrt{H_H}} \text{erfc} \left( \frac{0}{2\sqrt{H_H}} \right) \right) = \frac{1}{\sqrt{\pi}} = \alpha.$$

Coefficient  $\beta$  can be found from the condition of the smallest mean square error between the functions  $f$  and  $f^*$ . It is found that  $\beta = 2.3026$ . At the same time it is known that  $\frac{1}{\lg e} = \ln 10 = 2.3026$ . That is why, we have (Fig. 2, a)

$$f^* = \frac{1}{\sqrt{\pi}} \exp \left( \frac{-X}{\lg e \cdot 2\sqrt{H_H}} \right). \quad (15)$$

Considering that

$$\exp \left( \frac{-X}{\lg e \cdot 2\sqrt{H_H}} \right) = 10^{-\left(\frac{X}{2\sqrt{H_H}}\right)}, \quad (16)$$

we finally get

$$\Theta_H^*(X, H) = 2\pi\sqrt{H} \frac{1}{\sqrt{\pi}} 10^{-\left(\frac{X}{2\sqrt{H}}\right)}, \quad 0 \leq H \leq H_H, \quad (18)$$

$$\Theta_C^*(X, H) = 2\sqrt{\pi} \left[ \sqrt{H} 10^{-\left(\frac{X}{2\sqrt{H}}\right)} - \sqrt{H - H_H} 10^{-\left(\frac{X}{2\sqrt{H-H_H}}\right)} \right], \quad H_H < H. \quad (19)$$

### Conclusion

1. The problem of the grinding productivity increasing can often be solved by adapting the grinding system elements to higher productivity, taking into account the temperature in the contact zone, i.e., between the grinding wheel and the workpiece. To do this, an appropriate methodology is created.

2. The analysis and study of thermal phenomena during grinding is part of this methodology which

eters of the exponent  $f^*(X, H_H) = \alpha \exp \left( \frac{-\beta X}{2\sqrt{H_H}} \right)$ , by means of which we can approximate the dependence  $f = \text{ierfc} \frac{X}{2\sqrt{H_H}}$ . To do this, it is necessary to define the coefficients in the equation  $f^* = \alpha \exp \left( \frac{-\beta \cdot X}{2\sqrt{H_H}} \right)$ .

The value of the  $\alpha$  coefficient is found from the condition  $X = 0$ . We get the equation

$$f(0, H_H) = \text{ierfc} \frac{0}{2\sqrt{H_H}} = f^*(0, H_H) = \alpha.$$

Taking into account the (14) we have

$$f^* = \frac{1}{\sqrt{\pi}} 10^{-\left(\frac{X}{2\sqrt{H}}\right)}. \quad (17)$$

Thus, the obtained dependence (17) can approximate the dependence  $f = \text{ierfc} \frac{X}{2\sqrt{H_H}}$  with a rela-

tive “error”  $\delta = \frac{|f - f^*|}{f} 100\%$ , which is no more than 10.92%, i.e.  $\delta_{\max} < 11\%$  (Fig. 2, b).

Multiplying the obtained dependence(17) by the same factor  $2\pi\sqrt{H_H}$ , we obtain the following two alternative equations for determining the dimensionless grinding temperature at the heating(with the index “H”) and cooling (with the index “C”) stages:

concerns the development of embedded subsystems for designing, monitoring and diagnosing of the grinding operation on modern CNC machines. The development of such subsystems is an example of the information and communication technologies strategy implementation in the framework of the fourth industrial revolution in discrete and process manufacturing.

3. An universal generalized indicator for the theoretical justification of the continuity and applicability of solutions of the Fourier differential equations of heat conduction is the Peclet number, which simultaneously characterizes the dimensionless time of the heat source during grinding, dimensionless half-width and dimensionless velocity of the moving heat source.

4. A simplified model for determining the grinding temperature is obtained, which differs from the corresponding analytical solutions of the two and one-dimensional differential equation of heat conduction by no more than 11 % for the Peclet number which is more than 4.

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## СПРОЩЕННЯ ТЕМПЕРАТУРНОЇ МОДЕЛІ ШЛІФУВАННЯ ДЛЯ СИСТЕМИ ІНФОРМАЦІЙНОГО ЗАБЕЗПЕЧЕННЯ ОПЕРАЦІЇ

*Анотація.* Математичні моделі для визначення температури шліфування необхідні при проектуванні, контролі і діагностиці операції шліфування для підвищення продуктивності цієї операції без припиків поверхні, що підлягає шліфуванню. Це повною мірою відноситься, наприклад, до зубошліфувальних верстатів з ЧПК. Як тільки проблема розробки зазначених математичних моделей вирішена, стає можливим розробити відповідні комп'ютерні підсистеми для оптимізації й регулювання операції шліфування на верстатах з ЧПК на етапах виробництва і його підготовки. Актуальність рішення цієї про-

блеми підтверджується значною кількістю відповідних публікацій, більшість із яких пов'язана із джерелом тепла, яке рухається, теорія якого розроблена Єгером. У той же час ідея заміни джерела, який швидко рухається, часом дії відповідного нерухливого джерела, уперше сформульована Єгером, ще не знайшла належного практичного застосування. У даній статті обґрунтовано наведено, що близькість результатів розрахунку максимальної температури шліфування і глибини її проникнення за двовимірним й одномірними рішеннями практично має місце, коли джерело тепла, яке швидко рухається, характеризується числом Пекле, що більше ніж 4. Для цього інтервалу зміни числа Пекле вперше була отримана спрощена формула для визначення температури шліфування на поверхні і на глибині поверхневого шару. Потім ця спрощена формула була досліджена шляхом її зіставлення з відомим аналітичним рішенням одномірного диференціального рівняння теплопровідності при різних значеннях числа Пекле. Показано, що в діапазоні числа Пекле від 4 до 20, що має місце для більшості сучасних операцій шліфування (плоского, круглого, профільного та інших), розходження у визначенні температури шліфування за точним і наближеним рішеннями не перевищує 11%. У той же час, отримане у статті, спрощене рішення має нову важливу якість. Математична формула, що описує це рішення, дозволяє виразити в явному виді глибину проникнення кожної наперед заданої фіксованої температури. Якщо ця фіксована температура призводить до структурно-фазових перетворень у поверхневому шарі оброблюваної заготовки, то з'являється можливість визначення глибини дефектного шару при шліфуванні. У свою чергу, припуск на операцію шліфування повинен бути більше зазначеної глибини дефектного шару. Нова інформація про стан технологічної системи може бути основою для моніторингу та технологічної діагностики операції, а також для проектування цієї операції на етапі підготовки виробництва. Це у свою чергу, є технологічною передумовою для розробки відповідних комп'ютерних підсистем, які можна інтегрувати в систему ЧПК сучасних шліфувальних верстатів.

**Ключові слова:** температура шліфування; температурні моделі; безрозмірна температура; рухливий джерело тепла; розподіл температури; форма теплового джерела; число Пекле

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## УПРОЩЕНИЕ ТЕМПЕРАТУРНОЙ МОДЕЛИ ШЛИФОВАНИЯ ДЛЯ СИСТЕМЫ ИНФОРМАЦИОННОГО ОБЕСПЕЧЕНИЯ ОПЕРАЦИИ

**Аннотация.** Математические модели для определения температуры шлифования необходимы при проектировании, контроле и диагностике операции шлифования для повышения производительности этой операции без прижогов поверхности, подлежащей шлифованию. Это в полной мере относится, например, к зубошлифовальным станкам с ЧПУ. Как только проблема разработки указанных математических моделей решена, становится возможным разработать соответствующие компьютерные подсистемы для оптимизации и регулирования операции шлифования на станках с ЧПУ на этапах производства и его подготовки. Актуальность решения этой проблемы подтверждается большим количеством соответствующих публикаций, большинство из которых связаны с движущимся источником тепла, теория которого разработана Егером. В то же время идея замены быстро движущегося источника временем действия соответствующего неподвижного источника, впервые сформулированная Егером, еще не нашла надлежащего практического применения. В данной статье обоснованно показано, что близость результатов расчета максимальной температуры шлифования и глубины ее проникновения по двумерному и одномерному решениям практически имеет место, когда быстро движущийся источник тепла характеризуется числом Пекле, которое больше чем 4. Для этого интервала изменения числа Пекле впервые была получена упрощенная формула для определения температуры шлифования на поверхности и на глубине поверхностного слоя. Затем эта упрощенная формула была исследована путём её сопоставления с известным аналитическим решением одномерного дифференциального уравнения теплопроводности при различных значениях числа Пекле. Показано, что в диапазоне числа Пекле от 4 до 20, который имеет место для большинства современных операций шлифования (плоского, круглого, профільного и других), различие в определении температуры шлифования по точному и приближенному решениям не превышает 11%. В то же время, полученное в статье, упрощенное решение обладает новым важным качеством. Математическая формула, которая описывает это решение, позволяет выразить в явном виде глубину проникновения любой наперед заданной фиксированной температуры. Если эта фиксированная температура приводит к структурно-фазовым



превращениям в поверхностном слое обрабатываемой заготовки, то появляется возможность определения глубины дефектного слоя при шлифовании. В свою очередь, припуск на операцию шлифования должен быть больше указанной глубины дефектного слоя. Новая информация о состоянии технологической системы может быть основой для мониторинга и технологической диагностики операции, а также для проектирования этой операции на этапе подготовки производства. Это в свою очередь, является технологической предпосылкой для разработки соответствующих компьютерных подсистем, которые можно интегрировать в систему ЧПУ современных шлифовальных станков.

**Ключевые слова:** температура шлифования; этап нагрева; число Пекле; температурные модели; безразмерная температура



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