

# Multidimensional Hierarchical Model of Behavioral Testing of Distributed Information Systems

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## I. INTRODUCTION

Perspective distributed information systems (DIS), rapidly expanding their scope of application, are characterized by a sharp complication of the problems solved with their help, with a significant increase in the criticality of their application, growing intelligence with the use of fuzzy, evolutionary and neural methods, and the speed bordering on the real time of the functioning of objects of the domain. Component autonomy and mobility, as well as the growth of the degree and structure of their interactions, both internal - intercomponent and external - with external objects of the domain and the global network, are becoming more and more significant, developing features of modern DIS. These properties of DIS acquire the character of general and indicate that they obtained the properties of multi-agent and dynamic systems that promptly form special distributed and shared structures of tasks, resources and processes in the environments of the accommodation infrastructure. It should be noted that a significantly higher level of dynamic, situational communications, coordination and cooperation in such systems exacerbates the risks of access, uncertainty, functional disability, failures and errors, incorrect and malicious actions. As a rule, these risks are reduced or even eliminated by a set of security measures and information protection systems, for example, by means of authorization/authentication, digital signature, encryption, attributes and access rights/trust, multilevel screening, subject-logical virtualization. However, a higher level of reliability of DIS functioning is provided by additional means of their formal check and diagnosis. Thus, analysis, design, maintenance of DIS, often NP-complex, with the use of many complex technologies, with a sharp increase in the degree of efficiency of both their construction and verification, in particular, based on behavioral online and offline testing and diagnosis, used special FPGA testing.

For the analysis of component cooperations and DIS, in general, commonly used methods are deterministic, probabilistic, fuzzy, evolutionary check, diagnosis and testing of their structural, functional and informational properties and mechanisms, characterized by acceptable values of the reliability of work and resource costs. At the same time, the significantly increased dynamism, uncertainty, intelligence and diversity of situational component DIS cooperations, both in the distribution within the fuzzy boundaries and in the tasks to be solved, impose ever more stringent time requirements for the existing methods of check, diagnosis and testing up to real of time. This limits the use of most of the known methods to systems of medium complexity.

Solving these problems uses hardware methods, decomposition and paralleling of the presented check and diagnostic analysis, and can also be based on the development of formal models, expansion of the class of analyzed properties and mechanisms of DIS, in particular, additional research of testing of multidimensional, hierarchical, heterogeneous and multi-purpose behavioral models on base of systems of hierarchical Petri nets (PN) - for situational spatial-temporal cooperations of components and processes into DIS.

The relevance of the work is due to the need to develop the existing methods of decomposition, behavioral testing of DIS with the features of situational dynamic multidimensional, hierarchical spatial-temporal, multiagent cooperations with the accumulation of knowledge of check behavior, in particular, the near (frequent, adjacent near) and distant (episodic, indirectly remote) circle, represented by systems of hierarchical PN. The proposed technology of behavioral testing of DIS increases the efficiency of the formation of verified, secure, dynamic, spatial-temporal, distributed systems of tasks, resources and processes for DIS. The result is a reduction in the time of analysis, testing and recovery of DIS, close to real time.

## II. BUILDING THE MULTIDIMENSIONAL HIERARCHICAL MODELS OF BEHAVIOURAL TESTING OF DIS

A formal model of behavioral control determines the conditions for its implementation, taking into account which one or another method of behavioral control is built. The DIS models built, for example, on the basis of PN, simple and extended, in particular hierarchical, have great flexibility, power, and expressiveness. Representing many asynchronous parallel processes using the chip

mechanism, simple and extended Petri nets allow you to implicitly and naturally implement spatial, time-parallel decomposition of the DIS model, hierarchical PN additionally provide the possibility of its temporal, time-consistent decomposition. The extended PN  $S(f)$  can have a fairly general form, in particular:

$$S(f)=(P, T, X, Y, In, Pb, Ep, Et, F, S, M_0, L, K), \quad (1)$$

where  $P, T$  are the sets of positions and transitions,  $X, Y$  are the sets of input and output signals, respectively, for variable conditions, events, actions and functions, arising and placed in positions and transitions;  $In \subset N$  In transitions;  $Pb \subset [0; 1] \subset D$  is the set of probability coefficients in the range  $[0; 1]$ ;  $Ep \subset N$  - a set of integer energy costs for the formation of conditions, events and the execution of functions for positions from  $P$ ;  $Et \subset N$  is the set of integer energy inputs for performing actions and functions for transitions from  $T$ ;  $F: (P \times X \times In \times Pb \times Ep \rightarrow) \cup (T \times Y \times In \times Pb \times Et \rightarrow P)$  - extended conditional incidence relation of transition positions;  $S: (P \rightarrow X \times In \times Pb \times Ep) \cup (T \rightarrow Y \times In \times Pb \times Et)$  - extended correspondence of values of variable conditions, events, actions, functions, time intervals, probability coefficients to positions and transitions;  $M_0: P \rightarrow N$  - initial marking,  $(M: P \rightarrow N$  - function of current marking);  $L: (T \times Y \times In \times Pb \times Et \rightarrow \{0, 1\})$  - transition predicate;  $K: ((P \times X \times In \times Pb \times Ep) \rightarrow (P \times X \times In \times Pb \times Ep)) \cup ((T \times Y \times In \times Pb \times Et) \rightarrow (T \times Y \times In \times Pb \times Et))$  is the function of modifying the values of variable conditions, events, actions, functions, time intervals, probability coefficients for positions and transitions.

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Due to their asynchronous-event nature and token mechanism (parallel processes), multicomponent spatial decomposition of the PN can be performed for any subgraph of the PN graph, provided that the properties of the bipartite structure are preserved. Such a decomposition can formally be based on a two-component partition of the sets of positions and transitions, taking into account their incidence of the form:

$$\begin{aligned} S'(f) &= (P', T', X', Y', In', Pb', Ep', Et', F', S', M_0', L', K'), \\ S''(f) &= (P'', T'', X'', Y'', In'', Pb'', Ep'', Et'', F'', S'', M_0'', L'', K''), \end{aligned} \quad (2)$$

where  $P' \cup P'' = U$  и  $P' \cap P'' = \emptyset$ ,  $T' \cup T'' = U$  и  $T' \cap T'' = \emptyset$ ,  $X', X'' \subset X$ ,  $Y', Y'' \subset Y$ ,  $In', In'' \subset In$ ,  $Pb', Pb'' \subset Pb$ ,  $Ep', Ep'' \subset Ep$ ,  $Et', Et'' \subset Et$ ,  $M_0', M_0'' \subset M_0$ ,  $F'$  and  $F''$  - narrowing the ratio  $F$ ,  $S'$  и  $S''$  - narrowing the correspondence  $S$ ,  $L'$  and  $L''$  - narrowing the predicate function  $L$ ,  $K'$  and  $K''$  - narrowing the function  $K$ .

As a result of decomposition, a spatial ( $S$  - Space) model is formed, defined by the set of all Petri subnets  $S(f)_h^S \in S(f)^{S\wedge} = \cup_{h \in H} S(f)_h^S$  of the form  $S'(f)$ ,  $S''(f)$  for the original PN  $S(f)$ , the structure of their connections in the composition, both the connections between the Petri subnets themselves, and the connections of the PSN with the external inputs and outputs of the composition — the inputs and outputs of the original PN  $S(f)$ . Additionally, functional-alphabetic relationships can be distinguished for inputs and outputs of PSN in accordance with the structure of their connections.

The resulting network decomposition model  $nS$  from  $\forall S(f)_h^S \in S(f)^{S\wedge}$ , which represents the spatial components of the DIS and their connections, has the form:

$$nS = (X, Y, S(f)^{S\wedge}, \alpha^\wedge), \quad (3)$$

where  $X$  is the input alphabet on the boundary  $nS$ ;  $Y$  is the output alphabet on the boundary  $nS$ ;  $S(f)^\wedge$  is the set of component Petri subnets  $\forall S(f)_h \in S(f)^\wedge$ ;  $\alpha^\wedge$  is the set of functional-alphabetic correspondences (connections) between Petri subnets from  $S(f)^\wedge$  to  $nS$ .

In  $nS$ , the operations of composition of functional PSN are used, which provide for parallel work, taking into account the markup function  $M$ . These are operations: serial connection  $(S(f)_k \equiv S(f)_m)$ , when the output positions  $S(f)_k$  are the input positions for  $S(f)_m$ ; parallel to the connection  $S(f)_k \times S(f)_m$ , when  $S(f)_k$  and  $S(f)_m$  have common input positions; feedback connections  $(S(f)_k \equiv S(f)_m)$  when the output positions  $S(f)_k$  are input positions for  $S(f)_m$  and at the same time some output positions  $S(f)_m$  are input positions for  $S(f)_k$ .

A multi-level hierarchical extended PN can be represented on the basis of a two-level hierarchical extended PN  $2iS$  of the form:

$$2iS = (S(f), \cup_{i \in I} S(f)_i^p, \cup_{j \in J} S(f)_j^t, Sg_{iS}), \quad (4)$$

where  $S(f)$  is the highest PN from the upper level of the hierarchy;  $S(f)^p = \cup_{i \in I} S(f)_i^p$  is the set of PSN of the lower hierarchy level that replace (with synchronization, translation) macro positions from  $P' = \cup_{i \in I} p_i'$ , where  $P' \subset P$ , for the PN  $S(f)$  the upper level through the substitution of hierarchical correspondences  $\chi^p, \mu^p$ ;  $S(f)^t = \cup_{j \in J} S(f)_j^t$  is the set of PSN s of the lower level of the hierarchy, replacing the macro transitions from  $T' = \cup_{j \in J} t_j'$ , where  $T' \subset T$ , for the PN  $S(f)$  of the upper level through substitution of hierarchical correspondences  $\nu, Sg_{iS} = \{\chi^{\rightarrow p}, \chi^{p \rightarrow}, \nu^{\rightarrow}, \nu^{\rightarrow}\}$  is the signature of the hierarchical correspondences themselves, in which  $\chi^{\rightarrow p}$  is the partial correspondence of the substitution of inputs to split macro positions from  $P' = \cup_{i \in I} p_i'$  for the PN  $t S(f)$  of the upper level at the entrances to new initial positions from the set  $\cup_{i \in I} P_{S(f)_i}$  for PSN  $S(f)^p = \cup_{i \in I} S(f)_i^p$  of the lower level;  $\chi^{p \rightarrow}$  is the partial correspondence between the substitution of outputs from split macro positions from  $P' = \cup_{i \in I} p_i'$  for the top-level PN  $S(f)$  to the outputs from new end positions from the set  $\cup_{i \in I} P_{S(f)_i}$  for PSN  $S(f)^p = \cup_{i \in I} S(f)_i^p$  lower level;  $\nu^{\rightarrow}$  is a partial correspondence between the substitution of inputs to split macro transitions from  $T' = \cup_{j \in J} t_j'$  for the upper level PN  $S(f)$  to the inputs of new initial transitions from the set  $\cup_{j \in J} T_{S(f)_j}$  for PSN  $S(f)^t = \cup_{j \in J} S(f)_j^t$  of the lower level,  $\nu^{\rightarrow}$  is the partial correspondence of the substitution of the outputs from the split macro transitions from  $T' = \cup_{j \in J} t_j'$  for the PN  $S(f)$  of the upper level to the outputs of the new final transitions from the set  $\cup_{j \in J} T_{S(f)_j}$  for PSN  $S(f)^t = \cup_{j \in J} S(f)_j^t$  of the lower level.

As a result of decomposition, a temporary ( $T$  - Temporal) model is formed, determined by the set of all PSN  $\forall S(f)_h^T \in S(f)^{T\wedge} = \cup_{h \in H} S(f)_h^T$  included in it, in particular, of the form  $S(f)_i^p \in S(f)^p$  or  $S(f)_j^t \in S(f)^t$ , by the structure of their correspondence-substitutions instead of the positions and transitions of PSN from  $S(f)^{T\wedge}$ , including the original PN  $S(f)$ , that is, the structure of auto-substitutions in the temporary composition. Additionally, functional-alphabetic relations can be distinguished for alphabets of conditions, events, actions, functions, in particular, inputs and outputs in accordance with their substitutions.

Obtained on the basis of the two-level model  $2iS$  as a result of temporary hierarchical decomposition, the multi-level model  $iS$  from  $\forall S(f)_h^T \in S(f)^{T\wedge}$ , which represents the temporary components of the DIS and their relationships, has the form:

$$iS = (S(f)^{T\wedge}, Sg_{iS}), \quad (5)$$

where  $S(f)^{T\wedge} = \cup_{h \in H} S(f)_h^T$  is the set of all PSN included in it, both with replaced positions / transitions and their substituting,  $Sg_{iS} = \{\chi^{\rightarrow p}, \chi^{p \rightarrow}, \nu^{\rightarrow x}, \nu^{x \rightarrow}\}$  is the signature of the hierarchical correspondences themselves, in which  $\chi^{\rightarrow p}$  is the partial correspondence of the entries in the split macro positions from  $P' = \cup_{i \in I} p_i'$  for some PSN  $S(f)_i^T \in S(f)^{T\wedge}$  to the inputs of new initial positions from the set  $\cup_{h \in H} P_{S(f)_h}$  for PSN  $S(f)_h^{pT} \in S(f)^{pT\wedge} \subseteq S(f)^{T\wedge}$ ;  $\chi^{p \rightarrow}$  - partial correspondence of substituting exits from split macro positions from  $P' = \cup_{i \in I} p_i'$  for the PSN  $S(f)_i^T \in S(f)^{T\wedge}$  to exits from new end positions from the set  $\cup_{h \in H} P_{S(f)_h}$  for PSN  $S(f)_h^{pT} \in S(f)^{pT\wedge} \subseteq S(f)^{T\wedge}$ ;  $\nu^{\rightarrow x}$  is the partial correspondence between the substitution of inputs to split macro transitions from  $T' = \cup_{j \in J} t_j'$  for some PSN  $S(f)_j^T \in S(f)^{T\wedge}$  to the inputs of new initial transitions from the set  $\cup_{h \in H} T_{S(f)_h}$  for PSN  $S(f)_h^{tT} \in S(f)^{tT\wedge} \subseteq S(f)^{T\wedge}$ ;  $\nu^{x \rightarrow}$  is the partial correspondence of substituting outputs from split macro transitions from  $T' = \cup_{j \in J} t_j'$  for some PSN  $S(f)_j^T \in S(f)^{T\wedge}$  to the exits from new finite transitions from the set  $\cup_{h \in H} T_{S(f)_h}$  for PSN  $S(f)_h^{tT} \in S(f)^{tT\wedge} \subseteq S(f)^{T\wedge}$ . The hierarchy of PSN  $iS$ , formed by correspondences from  $Sg_{iS}$  on the set  $S(f)^{T\wedge}$ , can formally have a network structure and even include feedbacks, which should be justified by the object load of the model.

Let  $S(f)^\wedge = S(f)^S \wedge S(f)^{T\wedge}$ . As a result, the network hierarchical model obtained as a result of the combined spatial-temporal decomposition has the following form:

$$niS = (X, Y, S(f)^\wedge, \alpha^\wedge, Sg_{iS}), \quad (6)$$

The choice of a spatial-temporal decomposition of the initial PN model  $S(f)$  for the DIS depending on its dimension, overall structural complexity, specific subject load and tuning suggests the existence of a multitude of solutions with a corresponding system of criteria that performs target structuring. The presented multidimensional spatial-temporal and hierarchical representations of PN models defined by the correspondences  $\alpha^\wedge$  and  $Sg_{iS}$  on the joint set  $S(f)^\wedge$  can give a general approach to the construction of such a set of solutions and decomposition criteria, to the choice of its variant.

From the formal point of view, the set of entities and relations of PN from  $S(f)^\wedge$ , including their positions  $P = \cup_{h \in H} p_h$ , transitions  $T = \cup_{h \in H} t_h$ , chips  $M = \cup_{h \in H} m_h$ , the conditions  $C = \cup_{h \in H} c_h$ , events  $E = \cup_{h \in H} e_h$  (as systems of conditions), actions  $A = \cup_{h \in H} a_h$  (as systems of functions) and functions  $F = \cup_{h \in H} f_h$ , including inputs  $X = \cup_{h \in H} x_h$  and outputs  $Y = \cup_{h \in H} y_h$ , where  $X \subseteq C \cup E$  and  $Y \subseteq A \cup F$ , based on partitions or coverings, can be assigned, including multiple, to different spatial-temporal subsystems of the simulated DIS, that is, different submodels of PN from  $S(f)^\wedge$ . Moreover, these coverings and partitions determine the nature of their interactions - strong connectedness, connectedness, weak connectedness, unconnectedness. In the case of only partitions, the spaces are not connected - not interacting or mutually independent.

In the general case, for relations  $R = R_{c-e} \cup R_{e-a} \cup R_{a-f}$  between entities (Quintessence) from the sets  $Q = C \cup E \cup A \cup F \cup X \cup Y$  of PSN  $S(f)^\wedge$  "event-conditions"  $R_{c-e}$ , "action-events"  $R_{e-a}$ , "action-functions"  $R_{a-f}$  is assigned to the  $n-n$  type. It is possible to accept each submodel  $S(f)_h \in S(f)^\wedge$ , whether it is the above network spatial PSN  $S(f)_h^S \in S(f)^S$  or a temporary PSN substituting for a position or transition  $S(f)_h^{pT} \in S(f)^{pT\wedge}$  or  $S(f)_h^{tT} \in S(f)^{tT\wedge}$  of the corresponding DIS subsystem, as existing in a separate autonomous space-time or dimension. In this case, it is possible to speak of a multidimensional DIS model, and, in it, time decompositions  $S(f)_h^{pT} \in S(f)^{pT\wedge}$  or  $S(f)_h^{tT} \in S(f)^{tT\wedge}$  also form the corresponding hierarchies of temporal measurements similar to formal point of view of spatial dimensions.

In such a multidimensional model, on the one hand, the obtained formal, simple, and multiple submodel projections of the form  $S(f)_h^S$  or  $S(f)_h^{pT}$ ,  $S(f)_h^{tT}$ , decomposing the PN  $S(f)$ , in the form  $nS$ ,  $iS$ ,  $niS$  allow you to highlight simple and complex submodels of the corresponding subsystems of DIS. On the other hand, the inverse formal composition of submodels from  $S(f)_h^S$  or  $S(f)_h^{pT}$ ,  $S(f)_h^{tT}$  of the DIS subsystems, presented as simple and complex projections, as a result forms the general compositional model  $nS$ ,  $iS$ ,  $niS$  for  $S(f)$  with the degree of their interaction, determined by coverings and partitions of entities from  $Q$  and relations from  $R$  combined into composition of submodels.

Entities from  $Q$  and relations from  $R$  can be divided into sets with elements of the same multiplicity and form a hierarchy of static interactions of the submodels  $Hi(S(f)^\wedge)^{static}$ , in which the ranks are determined by the multiplicity, as well as the correspondence relations  $\alpha^\wedge$ ,  $Sg_{iS}$ . The dynamic interaction of submodels, determined by the statistics of activation of entities from  $Q$  and relations from  $R$  during the functioning of submodels from  $S(f)^\wedge$  and the model  $S(f)$ , can be obtained directly in the process of modeling the behavior of the DIS as a whole. The hierarchy  $Hi(S(f)^\wedge)$ , additionally weighted by the dynamic statistics on the links, allows one to determine the structured quantitative measure / metric  $Met(S(f)^\wedge)$  for each submodel  $S(f)_h \in S(f)^\wedge$  of the form  $Met(S(f)_h^\wedge) = (Met(S(f)_h^\wedge)^{static}, Met(S(f)_h^\wedge)^{dynamic})$ , consisting of static and dynamic components.

The metric  $Met(S(f)^\wedge)$   $Met(S(f)^\wedge)$  admits ranking in the behavioral interaction of submodels from  $S(f)^\wedge$   $S(f)^\wedge$  in the general complex model  $nS$ ,  $iS$ ,  $niS$   $nS$ ,  $iS$ ,  $niS$ . The metric of behavioral interaction of the  $Met(S(f)^\wedge)$   $Met(S(f)^\wedge)$  submodels, in turn, allows us to determine

the so-called “near, middle, and far circles” of the interaction — often, infrequently, graph-space-time structures of its components (submodels from  $S(f)^{\wedge}S(f)^{\wedge}$ ) - Chains, Trees, Hammocks, Strongly Coupled Components SCC.

### III. MODELS OF BEHAVIORAL TESTING OF DIS

The use of extended PN in the organization of behavioral online testing of DIS involves the recognition of characteristic fragments of the behavior of a reference PN in the working behavior of the checked PN, as a model of the project or the implementation of the DIS.

The class of checked properties  $Pr$  of the reference PN  $S(f)$ , for which the deviations of the tested SP  $S(f)^{\wedge}$  are determined and the control model is determined, includes deviations of the incidence correspondences  $F^{\wedge}$  and  $S^{\wedge}$  from the reference correspondences  $F$  and  $S$  with the restriction  $|P^{\wedge}| \leq |P|$  and  $|T^{\wedge}| \leq |T|$ . The error class of the PN  $S(f)^{\wedge}$  is represented by the static part - its correspondences  $F^{\wedge}$  and  $S^{\wedge}$ , and the dynamic part - by its marking functions  $M^{\wedge}$ , predicates  $L^{\wedge}$ , modification of variables  $K^{\wedge}$ .

The model of behavioral testing  $cS$  has the form:

$$cS = (W^{\wedge}, Pr, Ci, Cp, Cf, Sg_{cS}), \quad (7)$$

where  $W^{\wedge}$  is the set of words of external (not structured by recognized positions and transitions) behaviour, which extends the incidence relation  $F$ , understood as the reachability relation on the unified set  $P \cup T$ ;  $Pr$  is the checked properties based on the total incidence  $F$ ;  $PrU = \{PrX \cup PrY\}$  is the checked properties based on the particular  $S$  included in  $F$ ;  $Ci$  - the identifying properties (identifiers of positions or transitions), for some  $ci_{jk} \in Ci$  defined as two of the form  $ci_{jkp} = (p_{jk}, W_{jk})$ ,  $W_{jk} = \bigcup_{jki \in 1k} W_{jki} \subset W_j$  identifiers of positions or  $ci_{jkt} = (t_{jkt}, W_{jk})$ ,  $W_{jk} = \bigcup_{jki \in 1k} W_{jki} \subset W_j$  identifiers of transitions for the reference  $S(f)$  are uniquely incident to the corresponding positions of the  $p_{jk}$  and transitions of the  $t_{jk}$ , on the set of relations  $\{\xi, \neg\xi, \varepsilon, \psi\}$  of compatibility, incompatibility, uncertainty and precedence (quasi order) are valid, taking into account the incidence of positions and transitions;  $Cp$  - the checked primitives, based on properties  $Pr$  and identifiers  $Ci$ ;  $Cf$  - recognized checked fragments of behaviour of reference PN  $S(f)$ , included the primitives  $Cp$ ;  $Sg_{cS} = \{\alpha, \beta, \gamma\}$  - signature of operations:  $\alpha$ -identification of positions or transitions;  $\beta$ -sameness of positions or transitions;  $\gamma$ -determinism of the behaviour of unmarked positions or transitions.

The network spatial model of check  $cnS$  for interacting in a system PSN has the form of a six:

$$cnS = (CS, node^{\wedge}, R_{T^{-1}(node^{\wedge})}, Tr_{T(node^{\wedge})}, Sg_{cnS}, R_{(S(f)^{\wedge})}, Tr_{(S(f)^{\wedge})}), \quad (8)$$

where:  $CS = \bigcup_{h \in H} cS_h$  - a lot of control models for network spatial memory bandwidths of the previously given form;  $node^{\wedge}$  - the set of selected nodes (selected pairs of adjacent positions and transitions for the PSN  $S(f)^{\wedge}$ , in which there is a convergence-divergence of chip flows and selected entities from  $Q$  and  $R$  represent the behavior of implementation (control) and transportation (observation) inside and at the boundaries of the composition  $nS$ ;  $R_{T^{-1}(node^{\wedge})}$  is the set of sub-models of the implementation of behavior (in the form of minimized PSN) on each node  $\forall node_h \in node^{\wedge}$  of the composition  $nS$  - the system PN  $S(f)$  from its common inputs through the reverse (to common inputs) simple (without repetitions) graph network spatial structures from the PSN of the form  $S(f)^{\wedge}_{RT^{-1}(node^{\wedge})}$  with the nodes  $node_{RT^{-1}(node^{\wedge})} \subseteq node^{\wedge}$  selected in them;  $Tr_{T(node^{\wedge})}$  is the set of submodels of behavior transportation (in the form minimized CSP) from each of the nodes  $\forall node_h \in node^{\wedge}$  of the composition  $nS$  - system PSN  $S(f)$  to its common outputs through direct (common outputs) simple graph network spatial structures from the CSP of the form  $S(f)^{\wedge}_{TrT(node^{\wedge})} \subseteq S(f)^{\wedge}$  with nodes selected in them  $node_{TrT(node^{\wedge})} \subseteq node^{\wedge}$ ;  $Sg_{cnS} = \{\bullet^{Y''}, \bullet^{X''}, \times^{Y''}, \times^{X''}, *^{Y''}, *^{X''}\}$  - the signature of the inter-component, between the PSN from  $S(f)^{\wedge}$  through the corresponding nodes from  $node^{\wedge}$  network direct and reverse operations compositions - serial  $\bullet^{Y''}, \bullet^{X''}$ , parallel  $\times^{Y''}, \times^{X''}$ , with feedback  $*^{Y''}, *^{X''}$  - for the behavior represented by constrictions (for control and observation) of the PSN;  $R_{(S(f)^{\wedge})}$  - the set of submodels of the implementation of behavior (in the form of minimized PSN) at the inputs (input nodes  $node^{in}_{(S(f)^{\wedge})} \subseteq node^{\wedge}$  of each of the PSN  $\forall S(f)_h \in S(f)^{\wedge}$  implemented through the nodes  $\forall R_{T^{-1}(S(f)_h)} \subseteq R_{T^{-1}(node^{\wedge})}$ ; corresponding to  $S(f)_h$ ;  $Tr_{(S(f)^{\wedge})}$  - sets of submodels of behavior transportation (in the form of minimized PSN) from the outputs (output nodes  $node^{out}_{(S(f)_h)} \subseteq node^{\wedge}$  of each PSN  $\forall S(f)_h \in S(f)$  transported through the nodes  $\forall Tr_{T(S(f)_h)} \subseteq Tr_{T(node^{\wedge})}$  corresponding to  $S(f)_h$ .

The network check model  $cnS$  accepts, as input, component check models  $CS$  and limits them to the conditions of realizability (controllability) and transportability (observability). In the model  $cnS$ , in addition to the set of component  $CS$  check models for all component PSN  $\forall S(f)_h \in S(f)^{\wedge}$ , the construction of all proper implementation models from  $\forall R_{(S(f)_h)} \in R_{(S(f)^{\wedge})}$  and transportation  $\forall Tr_{(S(f)_h)} \in Tr_{(S(f)^{\wedge})}$  in the network composition  $nS$  and reuse of node behavior implemented by  $R_{T^{-1}(node^{\wedge})}$  and transported by  $Tr_{T(node^{\wedge})}$  in node  $node^{\wedge}$  composition  $nS$  is emphasized.

The set of output words determined on the basis of  $Tr_{T(node^{\wedge})}$  and transported by the network  $nS$ , respectively, from the outputs of nodes from  $node^{\wedge}$  and PSN from  $S(f)^{\wedge}$ , are based on a number of additional models, in particular, models of specially generated check graphs  $\forall G_{S(f)_h} \in G_{S(f)}$  for all  $\forall S(f)_h \in S(f)^{\wedge}$ . When solving online testing problems, the  $2iS$  hierarchy imposes conditions for inheriting check behavior in hierarchical transitions from  $2iS$  under replacement mappings for detailed PSN from  $\bigcup_{i \in I} S(f)_i^p, \bigcup_{j \in J} S(f)_j^t$  instead of the corresponding macropositions and macrotransitions of the system PN  $S(f)$ .

Thus, the organization of the 2-hierarchy of check primitives and, as a result, the behavioral working control performed when they are covered is possible provided that the verifiable properties and position identifiers in the replacement PSN from the  $\bigcup_{i \in I} S(f)_i^p, \bigcup_{j \in J} S(f)_j^t$  in the signature  $Sg_{iS} = \{\chi^{\rightarrow p}, \chi^{\rightarrow t}, v^{\rightarrow p}, v^{\rightarrow t}\}$  hierarchical mappings  $\chi^{\rightarrow p}, \chi^{\rightarrow t}$  of the positions and mappings  $v^{\rightarrow p}, v^{\rightarrow t}$  transitions from  $\bigcup_{i \in I} S(f)_i^p, \bigcup_{j \in J} S(f)_j^t$  of each two-level  $2iS$  hierarchy. This condition restricts the sets  $\bigcup_{i \in I} S(f)_i^p, \bigcup_{j \in J} S(f)_j^t$  of junior implementers of the memory bandwidth in the hierarchical maps from  $Sg_{iS}$  to be valid for storing checked properties and identifiers. In this case, the set of preserving hierarchical transitions forms five compatible hierarchies derived from the initial

hierarchy  $2iS = (S(f), \cup_{i \in I} S(f)_i^p, \cup_{j \in J} S(f)_j^t, Sg_{iS})$ , - the reference checked properties  $2iPr$ , position identifiers  $2iCi$ , control primitives  $2iCp$ , fragments of fixed recovered behavior  $2iCf$ , fragments of fixed unrecovered behavior  $2iW^f$ :

$$\begin{aligned} 2iPr &= (Pr, \cup_{i \in I} Pr_i^p, \cup_{j \in J} Pr_j^t, Sg_{iPr}), \\ 2iCi &= (Ci, \cup_{i \in I} Ci_i^p, \cup_{j \in J} Ci_j^t, Sg_{iCi}), \\ 2iCp &= (Cp, \cup_{i \in I} Cp_i^p, \cup_{j \in J} Cp_j^t, Sg_{iCp}), \\ 2iCf &= (Cf, \cup_{i \in I} Cf_i^p, \cup_{j \in J} Cf_j^t, Sg_{iCf}), \\ 2iW^f &= (W^f, \cup_{i \in I} W_i^{p^f}, \cup_{j \in J} W_j^{t^f}, Sg_{iW^f}), \end{aligned} \quad (9)$$

where  $Sg_{iPr} = Sg_{iS} \subseteq Sg_{iS}$ ,  $Sg_{iCi} = Sg_{iS} \subseteq Sg_{iS}$ ,  $Sg_{iCp} = Sg_{iS} \subseteq Sg_{iS}$ ,  $Sg_{iCf} = Sg_{iS} \subseteq Sg_{iS}$ ,  $Sg_{iW^f} = Sg_{iS} \subseteq Sg_{iS}$  is the set of subsets  $Sg_{iS}$ , that is, the map  $Sg_{iS} = \{Sg_{iS}, Sg_{iPr}, Sg_{iCi}, Sg_{iCp}, Sg_{iCf}, Sg_{iW^f}\}$  is a special covering of the  $Sg_{iS}$  map, including the  $Sg_{iS}$  map itself.

Then for defined two-level check hierarchy model of two-level check  $2iCIS$  has the form:

$$2iCIS = (cS, \cup_{i \in I} cS_i^p, \cup_{j \in J} cS_j^t, Sg_{iCIS}). \quad (10)$$

The set of retaining hierarchical transitions forms the top five compatible hierarchies derived from the initial hierarchy  $iS = (S(f)^{T\wedge}, Sg_{iS})$ ,  $iPr$  is the hierarchy for the reference checked properties  $Pr^{T\wedge} = \cup_{h \in H} Pr_h^T$ ,  $iCi$  is the hierarchy for the identifiers of the positions  $Ci^{T\wedge} = \cup_{h \in H} Ci_h^T$ ,  $iCp$  - hierarchy for control primitives  $Cp^{T\wedge} = \cup_{h \in H} Cp_h^T$ ,  $iCf$  - hierarchy for fragments of fixed restored behavior  $Cf^{T\wedge} = \cup_{h \in H} Cf_h^T$ ,  $iW^f$  - hierarchy for fragments of fixed unrecovered behavior  $W^{T\wedge} = \cup_{h \in H} W_h^f$ .

$$\begin{aligned} iPr &= (Pr^{T\wedge}, Sg_{iPr}), \quad iCi = (Ci^{T\wedge}, Sg_{iCi}), \\ iCp &= (Cp^{T\wedge}, Sg_{iCp}), \quad iCf = (Cf^{T\wedge}, Sg_{iCf}), \\ iW^f &= (W^{T\wedge}, Sg_{iW^f}), \end{aligned} \quad (11)$$

Obtained on the basis of the  $2iCIS$  two-level behavioral control model as a result of temporary hierarchical decomposition, the  $ciS$  multi-level behavioral control model for the  $iS$  hierarchy from  $\forall S(f)_h^T \in S(f)^{T\wedge}$ , which represents the corresponding time components  $cS^{T\wedge} = \cup_{h \in H} cS_h$  of the multilevel RIS behavioral control and their hierarchical relationships in the  $Sg_{iCIS}$  mapping signature is:

$$ciS = (cS^{T\wedge}, Sg_{ciS}). \quad (12)$$

where  $cS^{T\wedge} = \cup_{h \in H} cS_h$  is the set of all hierarchical submodels of behavioral control included in it for  $S(f)^{T\wedge} = \cup_{h \in H} S(f)_h^T$ , firstly, for submodels with replaceable positions/transitions and previous detailed properties, by identifiers, primitives, fragments, and secondly, for submodels that replace positions / transitions and contain new detailed properties, identifiers, primitives, fragments. The hierarchy of multilevel behavioral check  $ciS$ , as well as the hierarchy of PN  $iS$ , formed by matches from  $Sg_{iCIS}$  on the set  $cS^{T\wedge}$ , can formally have a network structure and include feedbacks, which should be justified by the subject load of the control model.

Thus, for the  $iS$  hierarchy, a hierarchical model of multilevel behavioral testing  $ciS$  is formed.

Based on the hierarchical network model  $niS$ , a hierarchical network model of behavioral check  $cniS$  is formed as follows:

$$cniS = (cS^{S\wedge}, cS^{T\wedge}, node^\wedge, R_T^{-1}(node^\wedge), Tr_{T(node^\wedge)}, Sg_{cniS}, R_{(S(f)^\wedge)}, Tr_{(S(f)^\wedge)}, Sg_{ciS}). \quad (13)$$

In the online testing, analysis of the hierarchical mappings  $\{\chi^{\rightarrow p}, \chi^{p \rightarrow}, v^{\rightarrow}, v^{\leftarrow}\}$  of the highest PN  $S(f)$  and any component PSN from  $S(f)^+ = S(f) \cup ((\cup_{i \in I} p_i) \cup (\cup_{j \in J} t_j)) \cup \cup_{i \in I} S(f)_i^p \cup \cup_{j \in J} S(f)_j^t$   $2iS$  hierarchy is performed for relations quasi-order  $\psi^+ = \psi \cup (\cup_{i \in I} \psi_i) \cup (\cup_{j \in J} \psi_j)$ , compatibility  $\xi^+ = \xi \cup (\cup_{i \in I} \xi_i) \cup (\cup_{j \in J} \xi_j)$ , representing the higher and lower levels of synchronization behavior of the hierarchical transition for  $S(f)^+$ .

#### IV. EVALUATION OF BEHAVIORAL TESTING MODELS OF DIS

Representation of the Petri network  $S(f)$ , where  $|P|=n_p$ ,  $|T|=n_t$ ,  $n = n_p + n_t$ ,  $|X|=m$ ,  $|Y|=L$ , in the memory of the monitoring system using list structures requires for the upper limit of the total number of conditional fields:

$$c_{iS}^{max} = (n_t(4n_p + 3L + 4) + n_p(3m + 2)) + (\sum_{i \in I} n_{ti}(4n_{pi} + 3L_i + 4) + n_{pi}(3m_i + 2)) + (\sum_{j \in J} n_{tj}(4n_{pj} + 3L_j + 4) + n_{pj}(3m_j + 2)). \quad (14)$$

The complexity of the check analysis of the PN  $S(f)$  is determined by the upper bound:

$$\begin{aligned} c_{ciS}^{max} &= n_t(4n_p + 3L + 4) + n_p(3m + 2) + 2n_t n_p (n_t - 1) + 2(2L m n_p n_t)^{n_t} - \\ &- 3) + (n_t - 1)(n_p n_t)! + (\sum_{i \in I} n_{ti}(4n_{pi} + 3L_i + 4) + n_{pi}(3m_i + 2) + 2n_{ti} n_{pi} (n_{ti} - 1) + 2(2L_i m_i n_{pi} n_{ti})^{n_{ti}} - 3) + \\ &+ (n_{ti} - 1)(n_{pi} n_{ti})! + (\sum_{j \in J} n_{tj} n_{ij} (4n_{pj} + 3L_j + 4) + n_{pj}(3m_j + 2) + 2n_{ij} n_{pj} (n_{ij} - 1) + 2(2L_j m_j n_{pj} n_{ij})^{n_{ij}} - 3) + \\ &+ (n_{ij} - 1)(n_{pj} n_{ij})!. \end{aligned} \quad (15)$$

The comparison of the work of online testing programs based on automata-deterministic and Petri-evolutionary methods for dynamic DIS of onboard automated control systems (BSAC) and terminal video surveillance (TVS) confirmed a) decrease in computational complexity of online testing; b) reducing timer time of the check with preservation of c) the length of the check and their d) completeness.

## V. CONCLUSIONS

The paper presents the results of the development of the method of behavioral online testing of distributed information systems based on a special model of behavioral control of extended Petri nets and characterized by the features of the “wave” evolutionary parallelism of the “background” control analysis. A special model of behavioral control of extended Petri nets is based on determining the compliance of the reference and verifiable extended Petri nets, representing respectively the reference and verifiable components of DIS. The use of the model made it possible to determine the basic conditions for constructing a check method applied both at the system level and at the component level.

Decomposition increases the flexibility of the organization of behavioral workers control by taking into account the features of DIS. The greatest reduction was achieved on the components of the DIS of special behavior, in particular, with partial definiteness of model functions.