DOI: https://doi.org/10.15276/aait.02.2021.5
UDC 004 + 621.316.7

DISCRETE APPROXIMATION OF CONTINUOUS OBJECTS WITH MATLAB

Olga I. Tolochko[1)]
ORCID: https://orcid.org/0000-0002-6871-0653; tolochko.ola@gmail.com. Scopus ID: 57192559239
Stefan Palis[2)]
ORCID: https://orcid.org/0000-0002-1941-2289; stefan.palis@ovgu.de. Scopus ID: 24766914900
Oleksii O. Burmelov[1)]
ORCID: https://orcid.org/0000-0003-2057-4717; oleksii.burmelov@gmail.com. Scopus ID: 57203539371
Danylo V. Kaluhin[1)]
ORCID: https://orcid.org/0000-0003-1293-4479; danylo.kaluhin@gmail.com. Scopus ID: 57210207513
[1)] National Technical University of Ukraine “KPI”. 37, Peremogy Av. Kyiv, 03056, Ukraine
[2)] Otto von Guericke Universität Magdeburg. 2, Universitätsplatz. Magdeburg, 39106, Germany

ABSTRACT

This work is dedicated to the study of various discrete approximation methods for continuous links, which is the obligatory step in the digital control systems synthesis for continuous dynamic objects and the guidelines development for performing these operations using the MATLAB programming system. The paper investigates such sampling methods as pulse-, step-, and linearly invariant Z-transformations, substitution methods based on the usage of numerical integration various methods and the zero-pole correspondence method. The paper presents examples of using numerical and symbolic instruments of the MATLAB to perform these operations, offers an m-function improved version for continuous systems discretization by the zero-pole correspondence method, which allows this method to approach as step-invariant as linearly invariant Z-transformations; programs for continuous objects discrete approximation in symbolic form have been developed, which allows to perform comparative analysis of sampling methods and systems synthesized with their help and to study quantization period influence on sampling accuracy by analytical methods. A comparison of discrete transfer functions obtained by different methods and the corresponding reactions in time to different signals is performed. Using of the developed programs it is determined that the pulse-invariant Z-transformation can be used only when the input of a continuous object receives pulse signals, and the linear-invariant transformation should be used for intermittent signals at the input. The paper also presents an algorithm for applying the Tustin method, which corresponds to the replacement of analogue integration by numerical integration using trapezoidal method. It is shown that the Tustin method is the most suitable for sampling of first-order regulators with output signal limitation. The article also considers the zero-pole correspondence method and shows that it has the highest accuracy among the rough methods of discrete approximation. Based on the performed research, recommendations for the use of these methods in the synthesis of control systems for continuous dynamic objects are given.

Keywords: Discrete Approximation; Continuous Objects; Z-transformation; Transfer Function; Zeros; Poles; Sample Time; Transients

***For citation*: Tolochko O. I., Palis Stefan, Burmelov O. O., Kaluhin D. V. Discrete Approximation of Continuous Objects with MATLAB. *Applied Aspects of Information Technology.* 2021; Vol.4 No.2: 178–191. DOI: https://doi.org/10.15276/aait.02.2021.5**

INTRODUCTION

Usually, in most electromechanical automatic control systems, the transients in the control object are continuous. However, due to the known advantages of digital correction devices over analog ones, digital controllers, filters, state observers, identification devices, etc. are often used to control continuous objects.

Design of discrete control devices for continuous dynamic objects is performed in one of the following ways:

- design continuous control devices based on a continuous controlled object, and then convert them into a discrete form (the method of continuous models);
- build discrete models of the controlled object and based on them design digital control devices.

Therefore, in both cases it is necessary to be able to find discrete approximations of analog transfer functions, which determines the relevance of the research topic.

The difference is that the first approach involves the choice of quantization tact after the synthesis, and the second – before the synthesis [1].

The problem can be solved in the following ways:

1) by means of Z-transformations [2, 3];

2) by replacing the analog integration operator 1/s with one of the digital integration operators [4, 5];

3) by replacing the zeros and poles on the s-plane with the corresponding zeros and poles on the Z-plane [5, 6].

Generally, the problem has no exact solution, due to the fact that during sampling of the input signal information about its value between the quantization nodes gets lost. Therefore, the output of the discrete model cannot depend on these values, while the response of the continuous system depends on all values of the input signal.

But still there are situations in which a discrete model can be accurate in understanding that the meaning of the process $u(t)$ on the discrete interval $(k-1)T \le t \le kT$) is uniquely determined by the sequence $[u(0), u(T), ..., u((k-1)T)]$, where T is Sample Time.

This is the case for pulse systems with amplitude-pulse first type modulation and for digital control systems, if the input process is formed by a computer. In the latter case, the discrete input process is converted into a continuous one by means of extrapolators. For such cases, an "exact" solution is possible with Z-transformation methods usage.

Therefore, sampling methods using Z-transformations can be called ***accurate*** meaning that they ensure match of continuous and discrete output signals at moments of the time, that are multiple of the interruption period, at a certain type of input signal.

In this regard, there are **pulse-invariant, step-invariant and linear-invariant** Z-transformation, which provides an exact match of the reactions of the original continuous and discretized systems to the Dirac delta function, the Heaviside unit function and the linear function, respectively.

Methods of the second group sampling of are called **substitution**. These methods are essentially **approximate (inaccurate),** because they involve the replacement of continuous integrators in detailed structures with digital integrators, which are designed using a priori approximate methods of numerical integration.

Approximate methods include the method of correspondence of zeros-poles. This is caused by the fact that the transfer functions obtained with Z-transformations methods include not only the so-called "system zeros", associated with unambiguous dependencies with zeros of the analog object, but also "sampling zeros", which can only be determined approximately.

The process of sampling by analytical methods for continuous high-order objects requires a lot of time and attention. For example, the exact methods usage is associated with the representation of the original analog transfer function as the sum of elementary transfer functions for which *Z*-transformations can be found from existing tables.

Therefore, it is appropriate to simplify the conversion process by using modern software.

One of the most convenient packages for solving this problem is the MATLAB package [7, 8] with the distribution of Control Toolbox [9, 10], designed to perform analysis and synthesis of linear dynamical systems. Many of these operations can be performed not only in numerical form, but also in analytical form, using the application for symbolic programming – Extended Symbolic Toolbox, which covers the capabilities of one of the best symbolic mathematics packages – Maple5 [11, 12].

Using the listed above software requires certain experience and sometimes its improvement.

LITERATURE REVIEW

Discrete approximation issues of continuous systems are considered in many textbooks on the automatic control theory [14, 15]. These sources provide tables of elementary analogue units transformation and outline the method of obtaining transformations for more complex systems. Moreover, Z-transformation with zero-order extrapolation, i.e. step-invariant transformations, are considered in the most detail. But quite often the signal at the input of the sampled link is not stepwise, but changes quite smoothly by any arbitrary law, and when using DC converters or frequency converters, the signals are pulsed. In this case, the best results are given by linear-invariant and pulse-invariant transformation, respectively [4]. Of the substitution methods, quite rightly the most attention is paid to the Tustin method, which corresponds to the replacement of analogue integration by numerical integration using the trapezoidal method. But sometimes there is a need to use other substitution methods, in particular the method of zeros-poles correspondence, which ensures the discrete characteristic polynomial identity with polynomials obtained by Z-transforms.

This method in most sources is either not considered at all or considered very superficially.

Sources [17, 18] are devoted to the MATLAB package tools application to perform a discrete approximation of the continuous systems mathematical description. In sources [19, 20] described in detail the sampling process using MATLAB with practical examples.

Quite often it is advisable to perform transformations in analytical form, which can be significantly accelerated by using software for symbolic mathematics [10]. Insufficient attention has also been paid to this issue.

It could be proposed many examples in various fields of the usage of discrete models formation methods: electric drive and automation [21, 22], energetic [23, 24], robotics [25] and others.

THE PURPOSE OF THE ARTICLE

The aim of this work is to study different discrete approximation methods and their relationship and guidelines development for the use of sampling MATLAB tools.

MAIN PART. PROBLEM FORMULATION

Suppose we have a continuous dynamic system with an input signal $u(t)$ and the output signal $y(t)$, described in the region of the Laplace variable s by transfer function (TF) in polynomial form (***Transfer Function Polynomial***):

$$W(s)=\frac{H_m(s)}{G_n(s)}=\frac{\beta_m s^m+\beta_{m-1}s^{m-1}+...+\beta_1 s+\beta_0}{s^n+\alpha_{n-1}s^{n-1}+...+\alpha_1 s+\alpha_0}. \quad (1)$$

In this TF denominator $G_n(s)$ is rationed by the coefficient at the highest degree of the Laplace operator.

Using the polynomials expansion in the numerator and denominator of the transfer function (1) into factors, we obtain the transfer function in Zero-Pole-Cain form:

$$W(s)=K\frac{(s-z_1)(s-z_2)...(s-z_m)}{(s-p_1)(s-p_2)...(s-p_n)}, \quad (2)$$

where: $\mathbf{Z}=[z_1,z_2,...,z_m]$ – vector of zeros (Zeros); $\mathbf{P}=[p_1,p_2,...,p_n]$ – vector of poles (Poles); $K=\beta_m$.

In state space such an object is described by matrix equations:

$$s\mathbf{X}(s)=\mathbf{A}\mathbf{X}(s)+\mathbf{B}u(s), \quad (3)$$

$$y(s)=\mathbf{C}\mathbf{X}(s)+Du(s), \quad (4)$$

where: $\mathbf{X}=[x_1, x_2, ..., x_n]^T$ – state variables vector; $\mathbf{A}$ – state matrix of size $n\times n$; $\mathbf{B}$ – input vector-column of size $n\times 1$; $\mathbf{C}$ – output line vector of size $1\times n$; D – bypass coefficient.

The problem is to determine equivalent discrete transfer function (DFT) for a given sample time period T:

$$W(z)=K_d\frac{(z-z_{d1})(z-z_{d2})...(z-z_{dm})}{(z-p_{d1})(z-p_{d2})...(z-p_{dn})}=$$

$$=\frac{H_{m_d}(z)}{G_n(z)}=\frac{\beta^*_{m_d}z^{m_d}+\beta^*_{m_d-1}z^{m_d-1}+...+\beta^*_1 z+\beta^*_0}{z^n+\alpha^*_{n-1}z^{n-1}+...+\alpha^*_1 z+\alpha^*_0}, \quad (5)$$

or equivalent equations in the state space:

$$z\mathbf{X}(z)=\mathbf{A}_d\mathbf{X}(z)+\mathbf{B}_d u[z], \quad (6)$$

$$y(z)=\mathbf{C}_d\mathbf{X}(z)+D_d u(z), \quad (7)$$

that is, in the construction of a continuous object discrete model. Equivalence in this case means the reactions equality of a continuous system and its discrete model to some input action. Most often, the equality of reactions means that $y[k]=y(t_k)$, when: $u[k]=u(t_k)$, where $t_k=kT$, k – quantization step number.

The relationship between equations in the state space and transfer functions is determined by the expressions:

$$W(s)=\mathbf{C}(s\mathbf{I}-\mathbf{A})^{-1}\mathbf{B}+D=\mathbf{C}\frac{Adj(s\mathbf{I}-\mathbf{A})}{det(s\mathbf{I}-\mathbf{A})}\mathbf{B}+D, \quad (8)$$

$$W(z)=\mathbf{C}_d(z\mathbf{I}-\mathbf{A}_d)^{-1}\mathbf{B}_d+D_d, \quad (9)$$

where: $\mathbf{I}$ – single diagonal matrix in size $n\times n$; Adj($\mathbf{X}$) – union matrix (matrix composed of algebraic additions to the matrix-argument); det($\mathbf{X}$) – is the determinant of the matrix.

The aim of the study is to determine the correspondences between the parameters of continuous objects mathematical description and their discrete approximations using different methods and to develop recommendations for their application.

Z-TRANSFORMATION RESEARCH

It is known that the pulse-invariant transformation is carried out by the formula:

$$W_{imp}(z)=T\cdot Z\{W(s)\}, \quad (10)$$

step-invariant – by the formula

$$W_{zoh}(z)=\frac{z-1}{z}\cdot Z\left\{\frac{W(s)}{s}\right\}, \quad (11)$$

and linear-invariant – by the formula

ISSN 2617-4316 (Print)
ISSN 2663-7723 (Online)

$$W_{foh}(z) = \frac{(z-1)^2}{Tz} \cdot Z\left\{\frac{W(s)}{s^2}\right\}. \qquad (12)$$

Also are known equations that relate parameters of the corresponding equations in the state space [4], [15]. For example, for a step-invariant transformation are valid next equations:

$$\mathbf{A}_d = e^{\mathbf{A}T}, \qquad \mathbf{B}_d = \left(\int_0^T e^{\mathbf{A}\tau} d\tau\right)\mathbf{B} = \mathbf{A}^{-1}(\mathbf{A}_d - \mathbf{I})\mathbf{B},$$

$$\mathbf{C}_d = \mathbf{C}, \quad D_d = D. \qquad (13)$$

These formulas are used by the m-function c2d (continues to discrete), which is accessed in the format:

SysD = c2d (SysC, T, method),

where SysC – is the original continuous object created, for example, by using the functions tf (Transfer Function Polynomial), zpk (Zero-Pole-Gain) або ss (State Space); SysD – is a discrete object obtained from a given continuous by using one of the methods determined by the method parameter, Z-transformation at a given quantization period T. To perform pulse-invariant transformation, the last string parameter must be 'imp' (Impulse), for step-invariant – 'zoh' (Zero Order Hold), and for linear-invariant – 'foh' (First Order Hold).

As a continuous object, lets select the object with the TF:

$$W(s) = \frac{4s(2s+1)}{24s^3 + 10s^2 + 6s + 1} = 0{,}3333\ \frac{s(s+0{,}5)}{(s+0{,}2)(s^2+0{,}22s+0{,}21)}, \qquad (14)$$

with three poles and two zeros:

$$\mathbf{P} = [-0{,}11 \pm 0{,}44i,\ -0{,}20], \quad \mathbf{Z} = [0,\ -0{,}5]. \qquad (15)$$

Discrete transfer functions obtained from TF (14) by the method considered *Z*-transformation at $T = 2$, have the form:

$$W_{imp}(z) = \frac{0{,}67z^3 - 0{,}66z^2 + 0{,}06z}{z^3 - 1{,}69z^2 + 1{,}33z - 0{,}43} = 0{,}67 \cdot \frac{z(z-0{,}89)(z-0{,}11)}{(z-0{,}67)(z^2 - 1{,}02z + 0{,}65)}; \qquad (16)$$

$$W_{zoh}(z) = \frac{0{,}61z^2 - 0{,}82z + 0{,}21}{z^3 - 1{,}69z^2 + 1{,}33z - 0{,}43} = 0{,}61 \cdot \frac{(z-1)(z-0{,}34)}{(z-0{,}67)(z^2 - 1{,}02z + 0{,}65)}; \qquad (17)$$

$$W_{foh}(z) = \frac{0{,}32z^3 - 0{,}11z^2 - 0{,}34z + 0{,}12}{z^3 - 1{,}69z^2 + 1{,}33z - 0{,}43} = 0{,}32\ \frac{(z-1)(z-0{,}37)(z+1{,}03)}{(z-0{,}67)(z^2 - 1{,}02z + 0{,}65)}. \qquad (18)$$

The quantization period is purposely chosen to be large in order to more clearly observe the behaviour of discrete signals.

Impulse response $w(t)$ of the original continuous system with the transfer function (14) and discrete systems with the transfer functions (16)-(18) are shown in Fig. 1.

Fig. 2 shows transients $h(t)$ of the same objects in Fig. 3 – their response to a signal that increases linearly from 0 to 1, and then fixed at the achieved level.

Graphs analysis shows that the digital model impulse response value, synthesized by the pulse-invariant Z-transform, as expected, at each discrete interval matches with continuous object impulse response values at the beginning of the interval. But the step response of this model differs significantly from the discretized continuous link step response at any time, including the initial and steady values.

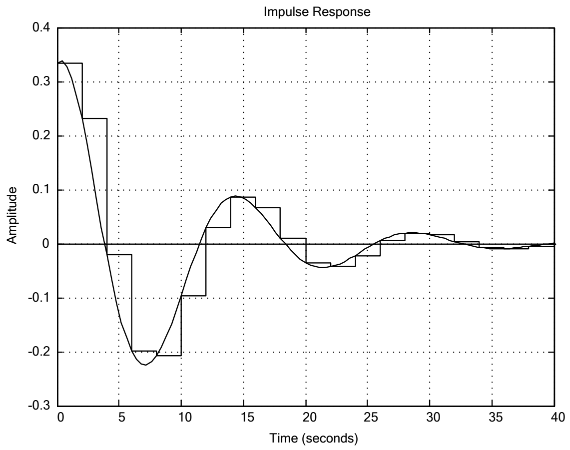
Accordingly, its response to a linear input signal has an error that increases with time. Therefore, this type of discrete approximation can be used only when there is confidence that the signal at the input will always have a form close to ideal pulses.

Of course, the reactions deviation of the discrete and continuous links to the same input signal decreases with decreasing quantization period. This applies to all studies performed in this paper.

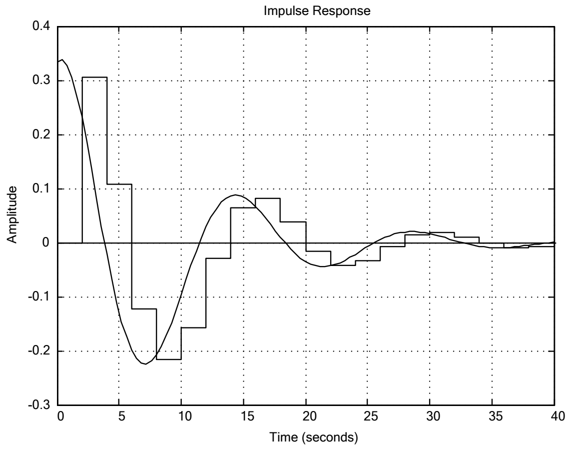
When using a Z-transformation with zero-order extrapolation, the best result is achieved by gradually changing the input signal. The reaction of such a discrete model to other types of input signals is characterized by a one quantization period delay.

When using Z-transformation with first-order extrapolation, the best result is achieved by linearly changing the input signal, but quite satisfactory results are obtained with other types of input signals: the response to the pulse is distorted only in the first two quantization steps, and the jump response matches with the continuous object transition function approximately at the quantization periods midpoints.

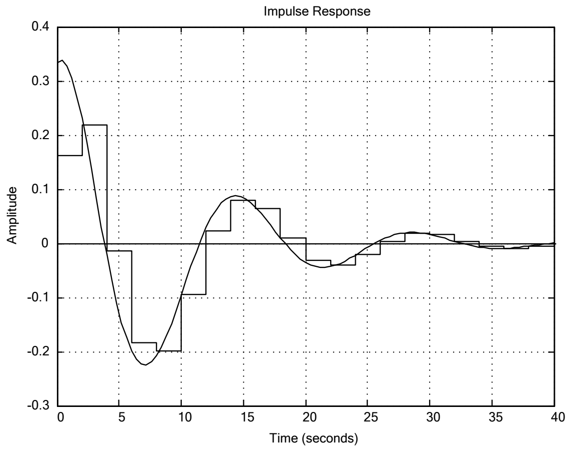
Similar studies performed for a sinusoidal input signal also confirm the advantages and universalism of continuous objects discrete approximation by the Z-transformation method with first-order extrapolation.



a



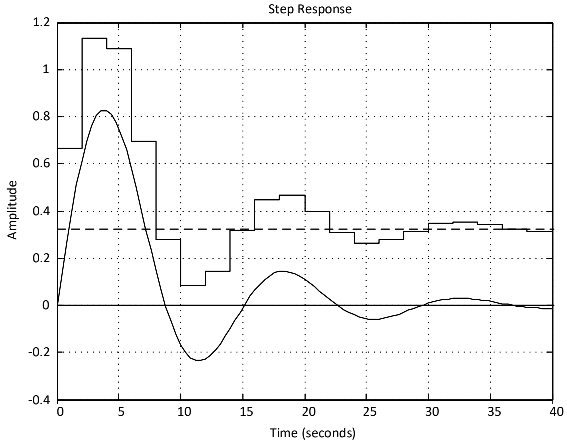
b



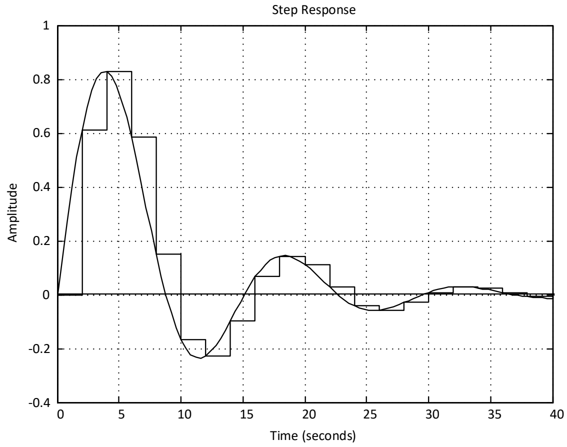
c

Fig. 1. **Impulse response of a continuous object and its “accurate” digital models**
a – pulse-invariant transformation; b – step-invariant transformation; c – linear-invariant transformation

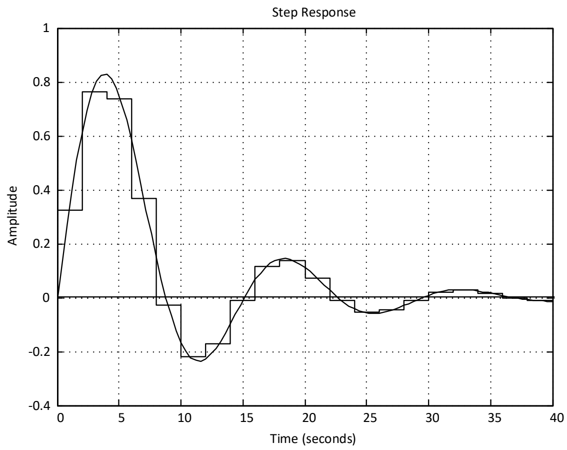
***Source:* Compiled by the authors**



a



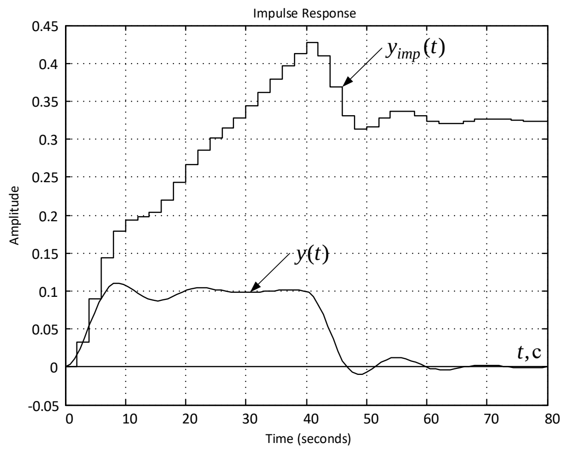
b



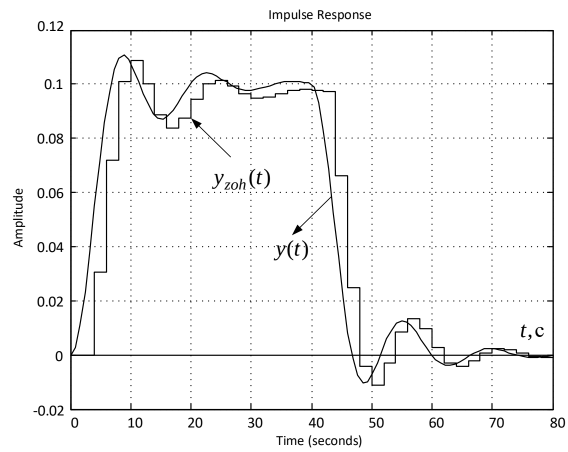
c

Fig. 2. **Step response of a continuous object and its “accurate” digital models using different transformation methods**
a – pulse-invariant transformation; b – step-invariant transformation; c – linear-invariant transformation
***Source:* Compiled by the authors**

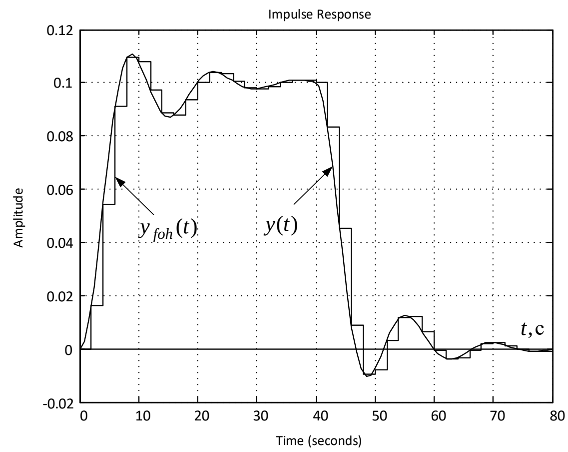
ISSN 2617-4316 (Print)
ISSN 2663-7723 (Online)



a



b



c

Fig. 3. **Reactions of a continuous object and its "accurate" digital models to a linear input signal using different transformation methods a – pulse-invariant transformation; b – step invariant transformation; c – linear-invariant transformation**

***Source:* Compiled by the authors**

Comparing the transfer functions of discrete models, it is seen that they all have the same denominators and different numerators.

To analyze the features of the obtained discrete approximations (16) - (18) for a continuous object (14) lets determine the discrete poles and the so-called discrete system zeros for the selected quantization period, corresponding to the zeros and poles of the continuous object (15):

$$\mathbf{P}_d = \exp(T\mathbf{P}) = [0,51 \pm 0,62i;\ 0,67], \qquad (19)$$

$$\mathbf{Z}_d = \exp(T\mathbf{Z}) = [1;\ 0,37]. \qquad (20)$$

It can be shown that all transfer functions obtained by Z-transform methods have discrete poles associated with the corresponding continuous poles by relations (19), i.e. they all have the same characteristic polynomials:

$$G_{imp}(z) = G_{zoh}(z) = G_{foh}(z) = \prod_{i=1}^{n}(z - p_{di}). \qquad (21)$$

Their order matches with the order of the continuous object:

$$n_d = n. \qquad (22)$$

The situation with zeros of discrete models is different. Most books on the theory of automatic control do not provide any information on their definition. From the references given in this work this question is considered only in [26], where it is noted that discrete transfer functions with *FOH* extrapolation always have a numerator order equal to the denominator order, regardless of the converted continuous object numerator order:

$$m_{foh} = n = n_d, \qquad (23)$$

and discrete models with ZOH extrapolation have a numerator order of one less, except when in a continuous object $m = n$, i.e.

$$m_{zoh} = \begin{cases} n_d & \text{при} \quad m = n, \\ n_d - 1 & \text{при} \quad m \le n-1. \end{cases} \qquad (24)$$

From the considered example it is seen that the relation (7.30) is characteristic also for impulse-invariant approximation:

$$m_{imp} = n = n_d. \qquad (25)$$

The zeros number excess in a continuous object and its discrete approximations is due to the fact that the digital models zeros of continuous links consist of system zeros and quantization zeros [1]. The difference between them is that when $T \to 0$ the first approaches to the discrete transformation of continuous object zeros:

$$\lim_{T\to 0} z_{dsi} = \exp(Tz_i),\quad i=1,2,...,m, \qquad (26)$$

and the second - to –1:

$$\lim_{T\to 0} z_{dkj} = -1,\qquad j=m+1,\ m+2,\ ...,\ m_d. \qquad (27)$$

The quantization zeros in general case can even be non-minimal-phase, i.e. have an amplitude greater than 1.

The zeros analysis of the above discrete models shows the following:

1) step- and linear-invariant digital models have close to each other system zeros $\mathbf{Z}_{ds\,zoh} = [1;\ 0,34]$, $\mathbf{Z}_{ds\,foh} = [1;\ 0,37]$, and the system zeros of the linear-invariant model are almost not differ from the zeros obtained by formula (7.27);

2) the linear-invariant digital model has zero quantization in its transfer function $\mathbf{Z}_{dk\,foh} = -1,04$, close to -1, which increases the order of the numerator and provides forcing of transients;

3) when using pulse-invariant sampling system zeros $\mathbf{Z}_{ds\,imp} = [0,88;\ 0,11]$ significantly different from zeros (7.27), and zero quantization has zero value: $\mathbf{Z}_{dk\,foh} = 0$, which causes a significant difference between this model and the other two, including the difference in state values of the transition functions.

To analyze the DFT parameters change during changing the quantization tact and the continuous system parameters, it is desirable to perform the transformation analytically. To solve this problem, we can use the MATLAB Extended Symbolic Toolbox, which includes functions that perform direct (laplace) and inverse (ilaplace) Laplace transform, as well as direct (ztrans) and inverse (iztrans) Z-transformation. With these and some auxiliary symbolic functions, you can find analytical expressions for continuous systems discrete models and on the contrary. For example, a program that finds a discrete model of an aperiodic link:

$$W(s) = \frac{1}{Ts+1}, \qquad (28)$$

with first-order extrapolation and quantization period Ts, may look like this:

```
syms t s z T Ts n% Description of symbolic variables
w=1/(T*s+1) % Continuous TF
h=ilaplace(w/s^2) % Incont. step response
hd=subs(h,t,n*Ts) % Discrete transition function
wd=(z-1)^2/(Ts*z)*ztrans(hd); % Discrete TF
wd1=subs(wd,exp(-Ts/T),pd); % Substitution
  % Simplification of the obtained analytical expression:
[num,den]=numden(wd1); % Decomposition of DTF on
                       %numerator and denominator
wd1=num/den;  % DTF formation
disp('Wd foh(z)='), pretty(wd1).
```

When executing this program we get:

```
w =
1/(T*s + 1)
h =
t - T + T*exp(-t/T)
hd =
Ts*n - T + T*exp(-(Ts*n)/T)
Wd foh (z)=
         T - T pd - Ts pd - T z + Ts z + T pd z
       - ------------------------------------------------ .
                    Ts (pd - z)
```

RESEARCH OF SUBSTITUTIONAL METHODS OF DISCRETIZATION

Substitution methods are identical to replacing the analog integration operator 1/s with one of the digital integration operators.

Most of the methods of digital integration is to divide a figure, the area of which should be divided into several simple figures, calculate their areas and sum them up. Most often, the integration interval is divided into several equal parts. Tach of them performs local interpolation of the subintegral function by power polynomials. Depending on the power of the polynomial n various methods of numerical integration are created: at $n=0$ – the method of rectangles, at $n=1$ – trapezoidal method, at $n=2$ - the method of parabolic trapezoids, which is usually called the Simpson method. This series can be continued, but by discretizing continuous dynamic links with substitution methods, it is limited to replacing analog integrators on digital, synthesized methods of rectangles and trapezoids.

The Simulink application of the MATLAB package uses 3 types of discrete integrators that perform numerical integration via ***Forward Euler method, Backward Euler method and Trapeziodal method*** and have the following TF:

$$W_{FE}(z) = \frac{y_{FE}(z)}{u(z)} = \frac{T}{z-1}, \qquad (29)$$

$$W_{BE}(z) = \frac{y_{BE}(z)}{u(z)} = zW_{FE}(z), \qquad (30)$$

ISSN 2617-4316 (Print)
ISSN 2663-7723 (Online)

$$W_{tr}(z)=\frac{y_{tr}(z)}{u(z)}=\frac{T}{2}\cdot\frac{z+1}{z-1}=\frac{W_{BE}(z)+W_{FE}(z)}{2}. \quad (31)$$

The following substitution formulas are received from formulas (29) - (31):

$$s=\frac{z-1}{T}, \quad (32)$$

$$s=\frac{z-1}{Tz}, \quad (33)$$

$$s=\frac{2}{T}\cdot\frac{z-1}{z+1}. \quad (34)$$

The substitution usage (32) is often called the **Euler method,** substitutions (33) – **the modified Euler method, and substitutions (34) – the bilinear transformation or the Tustin method.**

From the idea behind these methods, it follows that the substitution (32) matches the Z-transformation using a zero-order hold (ZOH), and the substitution (34) matches the Z-transformation using a first-order hold (FOH) at the discrete approximation of the integrator. Unfortunately, due to the increasing complexity of the object structure, the matching and even the desire for it are practically lost.

It should be noted **that the Euler transformation does not guarantee the stability of the digital controller with a stable continuous [4, 15]**.

Only one of the substitution transformations, namely the Tustin transformation can be performed with the m-function *c2d*:

```
SysD = c2d (SysC, T, 'tustin'),
```

But one can create his own m-function to perform any of the substitution transformations using both symbolic mathematics and numerical operations:

```
function SysD = c2d_podst (SysC, Ts, method)
 [numC, denC] = tfdata (SysC);
 numC = numC {1}, denC = denC {1};
 syms tsz T
 if length (numC) == 1, Ha = numC;
 else Ha = poly2sym (numC, s); end
 w = poly2sym (numC, s) / poly2sym (denC, s);
 switch method
 case 'Euler', wd = subs (w, s, (z-1) / T);
 case 'EulerM', wd = subs (w, s, (z-1) / T / z);
 case 'Tust', wd = subs (w, s, 2 / T * (z-1) / (z + 1));
 end
 wd = expand (wd); wd = collect (wd, z); wd = simplify (wd);
 [num, den] = numden (wd);
 num = collect (num, z); den = collect (den, z);
 T = Ts;
 numc = eval (num); denc = eval (den);
 if findstr (char (numc), 'z') == [], H = numc;
 else H = sym2poly (numc); end
 G = sym2poly (denc); % H = sym2poly (numc);
 H = H / G (1); G = G / G (1);
SysD = tf (H, G, T);
```

When discretizing the aperiodic link (28) at $T=0,1$ s with a quantization period $T_s=0,02$ s we get the following digital objects:

$$W_{FE}(z)=\frac{0,2}{z-0,8},\ W_{tr}(z)=\frac{0,091(z+1)}{z-0,82}.$$

Comparison of them with discrete transfer functions of the same link, obtained by methods of Z-transformations

$$W_{zoh}(z)=\frac{0,18}{z-0,82},\ W_{foh}(z)=\frac{0,094(z+0,94)}{z-0,82},$$

shows that the Euler transformation tends to approach a step-invariant Z-transformation, and the Tustin transformation tends to approach a linear-invariant Z-transformation.

More detailed studies give the following results:

1) zeros and poles of all substitution transformations differ from zeros and poles of Z-transformations;

2) only the Tustin transformation complements the transfer function with approximate values of discretization zeros $z_d=-1$; both of Euler's transformations add zero to the discrete transfer function instead of zero discretization: $z_d=0$;

3) euler transformation forms a discrete system, which can become unstable at large periods of quantization, despite the stability of the corresponding continuous object; therefore, it can be used only for continuous systems described by low-order differential equations, and for small periods of interruption (compared to the object's own time constants);

4) the modified Euler transformation is characterized by much lower discretization accuracy than the Tustin transformation.

From the above formulations it follows that the best of the permutation transformations is the Tustin transformation, which in its parameters and properties is close to the Z-transformation with first-order extrapolation. This conclusion matched with the generally accepted method of approximate discrete approximation of continuous objects, which provides a compromise between simplicity and accuracy of discretization.

INVESTIGATION OF DISCRETISATION BY THE METHOD OF CONFORMITY OF ZERO-POLES

This method displays continuous poles p_i and zeros z_i in discrete, based on the relationship between the Laplace operator and the discrete operator:

$$p_{di} = e^{Tp_i}, \; z_{di} = e^{Tz_i}. \qquad (35)$$

Let us show that these relations (35) usage is not sufficient for qualitative discretization in understanding the similarity of the dynamic and static properties of a continuous object and its digital model.

Let us assume that a continuous object has 3 valid poles: $\boldsymbol{P} = [\alpha_1 \alpha_2 \alpha_3]$ and no zeros:

$$W(s) = \frac{\beta_0}{(s-\alpha_1)(s-\alpha_2)(s-\alpha_3)}. \qquad (36)$$

Then its discrete model synthesized using formulas (37) will be as follows:

$$W_{zp}(z) = \frac{\beta_0}{(z-e^{\alpha_1 T})(z-e^{\alpha_2 T})(z-e^{\alpha_3 T})}. \qquad (37)$$

Let us assume that the output signal $y(t)$ of continuous object (36) is a reaction to a single stepped effect. The following initial conditions are valid for the output signal:

$$y(0)=0, \quad y'(0)=0, \quad y''(0)=0.$$

Let $y(kT)$ – the reaction of the digital model (36) to a single step function. As $W_{zp}(z)$ have a number of poles that exceeds the number of zeros by 3, the signal at the output of the digital object will appear only on the third tact.

Taking everything into consideration, we formulate the following conclusion: ***if the number of poles of the discrete link exceeds the number of zeros by*** r***, the reaction of this object to the input signal will start with*** r ***-th quantization tact.***

In addition, the transfer functions (36) and (37) provide different fixed values of the transition functions.

In order to eliminate these shortcomings, several zeros should be added to the transfer function of the discrete model and adjust its gain.

But the method of determining additional zeros and coefficients K_d transfer function (37) is practically not mentioned in the literature.

For adequate discrete approximation of the studied method, we will use the experience of transformation methods analysis described above and the known provisions of the automatic control theory.

From the above considerations comes the following method of a continuous dynamic object discretization by the method of correspondence of zeros and poles:

1) find the poles p_i (i = 1,2, …, n) of a continuous system by solving the characteristic equation

$$G_n(s) = s^n + \alpha_{n-1}s^{n-1} + ... + \alpha_1 s + \alpha_0 = 0, \qquad (38)$$

or by searching for eigenvalues of the state matrix

$$\mathbf{P} = \text{eig}(s\mathbf{I} - \mathbf{A}); \qquad (39)$$

2) find the zeros z_i (i = 1,2,…, m) of a continuous system by solving the equation

$$H_m(s) = \beta_m s^m + \beta_{m-1}s^{m-1} + ... + \beta_1 s + \beta_0 = 0; \quad (40)$$

3) calculate the corresponding discrete poles and discrete system zeros according to formula (35):

$$p_{di} = \exp(Tp_i), \; i = 1, 2, ..., n, \qquad (41)$$

$$z_{dsi} = \exp(Tz_i), \; i = 1, 2, ..., m; \qquad (42)$$

4) complement the discrete transfer function with quantization zeros

$$z_{dkj} = -1, \qquad (43)$$

so that the order of the numerator adjusted in this way m_d was one less than the order of the denominator:

$$m_d = n-1, \; j = m+1, m+2, ..., n-1, \qquad (44)$$

or equal to it:

$$m_d = n, \; j = m+1, m+2, ..., n-1, n; \qquad (45)$$

5) select from the vectors of analog zeros and poles the neutral ones (those that are 0 for a continuous system, and for a discrete – 1) and count their number μ and ν accordingly);

6) calculate the transmission ratio in the steady state of the continuous link according to the formula

$$k = K\prod_{i=1}^{n_1}(-p_{1i}) \Big/ \prod_{j=1}^{m_1}(-z_{1j}), \qquad (46)$$

where $K = \beta_m$, $m_1 = m - \mu$, $n_1 = n - \nu$ – number of non-zero analog zeros and poles, respectively;

7) calculate the coefficient K_d discrete object by the formula

$$K_d = kT^{\nu-\mu} \cdot \prod_{i=1}^{n_{d1}}(1-p_{d1i}) \Big/ \prod_{j=1}^{m_{d1}}(1-z_{d1j}), \quad (47)$$

where $p_{d1i}, z_{d1j}, m_{d1}, n_{d1} = n_1$ – non-single discrete zeros and poles and their quantity, respectively.

The transfer functions obtained by discretization of a continuous object (14) according to the above method have the form: at $m_d = n-1$

ISSN 2617-4316 (Print)
ISSN 2663-7723 (Online)

$$W_{zp0}(z)=\frac{0,66z^2-0,9z+0,24}{z^3-1,69z^2+1,33z-0,43}=$$

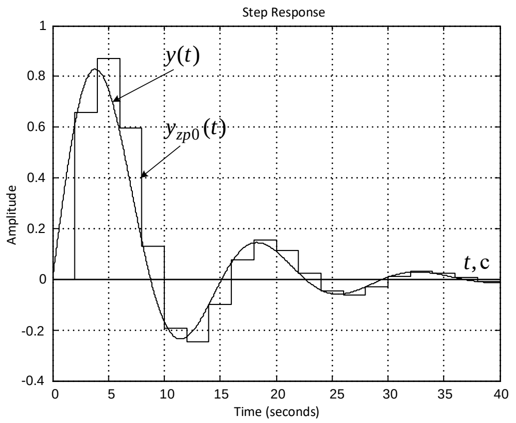
$$=\frac{0,66(z-1)(z-0,37)}{(z-0.67)(z^2-1,02z+0,65)}, \qquad (48)$$

and at $m_d=n$

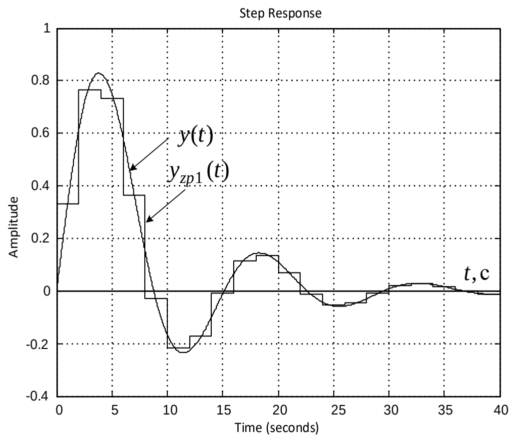
$$W_{zp1}(z)=\frac{0,33z^3\text{-}0,12z^2-0,34z+0,12}{z^3-1,69z^2+1,33z-0,43}=$$

$$=\frac{0,3291(z-1)(z-0,3679)(z+1)}{(z-0,6681)(z^2-1,024z+0,6505)}\ . \qquad (49)$$

They are very similar to the transfer functions obtained by Z-transformation methods with extrapolation of zero (24) and first (25) orders: $W_{zp0}(z)\approx W_{zoh}(z)$ and $W_{zp1}(z)\approx W_{foh}(z)$.



a



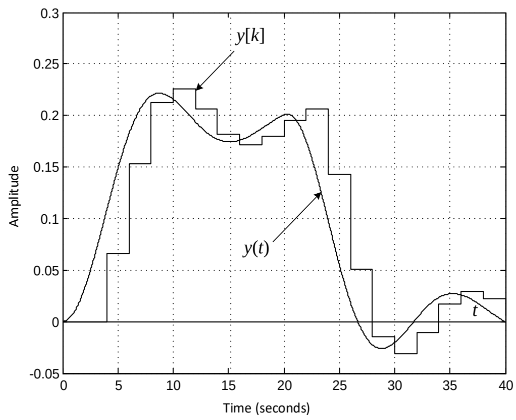
b

Fig. 4. **Step response of a continuous system and its discrete analogues, determined by the method of zero of poles matching at $m_d=n-1$ (a) and $m_d=n$ (b)**

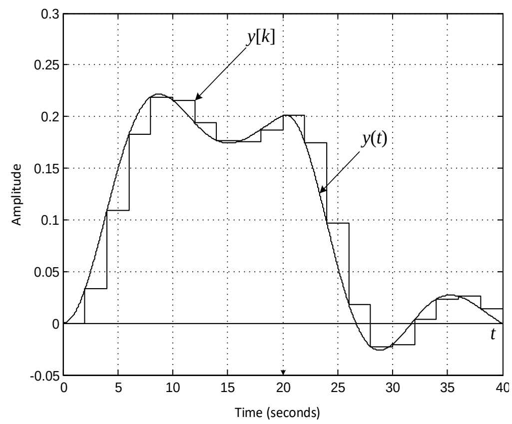
***Source:* Compiled by the authors**

In order to demonstrate the results of the two proposed above methods of matching zeros and poles of continuous and discrete systems the step response of the studied continuous system and its discrete approximations found by the above method is shown in Fig. 4, and reactions of the same links to the linear input signal with restriction are shown in Fig. 5. The graphs confirm that even with such an inflated quantization period, the obtained step response approach the step responses shown in Fig. 2b and Fig. 2c.

It is necessary to emphasize the fact that discrete approximation by a method of zeros of poles matching at $m_d=n$ for this example has a significantly smaller deviation from the approximation of the method of Z-transformation with first-order extrapolation than using the Tustin method, which is often recommended for use as an approximate discretization method.



a



b

Fig. 5. **The response of a continuous system and its discrete analogues, determined by the method of zero of poles matching at $m_d=n-1$ (a) and $m_d=n$ (b) on the linear input signal**

***Source:* Compiled by the authors**

This is confirmed by the graphs of the corresponding deviations shown in Fig. 6.

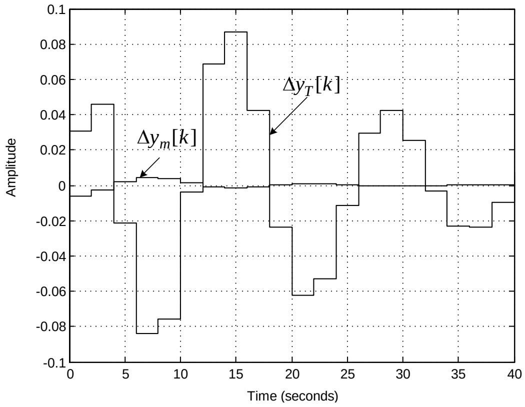


Fig. 6. **Deviation of step response of discrete systems created by Tustin methods and matching of zero-poles from the step response of the system formed by the method of Z-transformation with FOH**

***Source:* Compiled by the authors**

The obtained results give grounds for revision of the generally accepted recommendation on approximate discrete approximation of continuous systems. Another advantage of the matching method is that discrete systems obtained in this way do not need to be tested for stability.

Testing of the *c2d* function showed that it performs approximation via matched method with the number of zeros, identical to the step-invariant discretization.

Approximation of the matched method to the FOH method by increasing the order of the numerator of the transfer function to the order of the denominator by introducing another zero discretization, as proposed above, is not provided in this function.

For software implementation of the continuous object discretization method via the matching method proposed in this work, we can recommend a function, an abbreviated version of which (without diagnosing the input parameters) has the form:

```
function sysd = c2d_matched (sys, Ts, method)
 [Zero, Pole, Gain, ts] = zpkdata (sys);
 z = Zero {1}; p = Pole {1};
 zd = exp (z * Ts)
 pd = exp (p * Ts)
 if method == 'zoh',
 zd = [zd; -ones (length (pd) -length (zd) -1,1)];
 else zd = [zd; -ones (length (pd) -length (zd), 1)];
 end
 p1 = p; z1 = z;
 iz = find (p == 0); p1 (from) = [];
 izz = find (z == 0); z1 (izz) = [];
 pd1 = pd; zd1 = zd; pd1 (from) = []; zd1 (izz) = [];
 k = Gain (1) * real (prod (-z1) / prod (-p1))
 if isempty (izz), izz = 0; end
 if isempty (from), from = 0; end
 Kd = k * real (prod (1-pd1) / prod (1-zd1)) *…
 Ts ^ (iz-izz);
 sysd = zpk (zd, pd, Kd, Ts);
```

CONCLUSIONS

The paper investigates exact (in a sense) and approximate methods of continuous objects discrete approximation, gives examples of using MATLAB numerical and symbolic tools to perform these operations, offers an improved m-function version for continuous systems discretization method of zero-pole correspondence, developed programs for discrete approximation of continuous objects in symbolic form, which allows to perform a comparative analysis of sampling methods and systems synthesized with their help by analytical methods.

On the basis of the performed researches it is possible to reach the following conclusions:

1) pulse-invariant Z-transform can be used only when the input of a continuous object receives pulse signals, for example, from valve converters, for example, from DC converters or from frequency converters); in other cases, the usage of such a transformation may lead to unacceptable errors;

2) if the signals at the input of the continuous link change to a greater extent abruptly, i.e. have gaps of the first kind, then you should use a step-invariant Z-transformation, but the use of linear-invariant transformation is also possible;

3) if the signals at the input of the continuous link have an arbitrary continuous character, then the best sampling method is linear-invariant Z-transformation, but with sufficiently small quantization periods it can and even should be replaced by step-invariant transformation, which is simpler at the stage of synthesis and at the stage of implementation;

4) of the approximate methods in the last two cases, the Tustin method and the zero-pole correspondence method can be used;

5) the method of zero-pole correspondence has the greatest accuracy among approximate methods, which, depending on the number of zero-poles, is almost equal to step- or linear-invariant Z-transforms, but for high-order objects it is easier because it does not involve previously continuous transfer function decomposition for the elementary functions sum; it is sensible to use it for discrete approximation of the regulation objects mathematical

ISSN 2617-4316 (Print)
ISSN 2663-7723 (Online)

description, for example, electric motors, multimass electromechanical systems, etc .;

6) tustin's method is the most suitable for first-order regulators sampling with output signal limitation.

REFERENCES

1. Goodwin, G. Grebe, S. & Salgado, M. "Design of Control Systems" (in Russian). *Publ. Binom. Knowledge Laboratory*. 2004. 911 p.
2. Kotsegub, P. Kh. & Kotsegub, Kh. P. "Theory of Pulsed Electromechanical systems" (in Russian), Donetsk National Technical University. Donetsk: Ukraine. 2004. 262 p.
3. Kalechman, M. "Practical MATLAB Applications for Engineers". *CRC Press Taylor & Francis Group*. New-York: USA. 2007. 708 p.
4. Andrievsky, B. R. & Fradkov, A. L. "Selected Chapters of the Theory of Automatic Control with Examples in the MATLAB Language" (in Russian). *Nauka Publications*. 2000. 475 p.
5. Ishmatov, Z. Sh., Kazakov, E. G. & Mezeushev, D. V. "Modern Control Theory: Laboratory Practice". UPI (Ural Polytechnic Institute). Yekaterinburg: Russian Federation, 2006. 50 p.
6. Eihab B. M. Bashier. "Practical Numerical and Scientific Computing with MATLAB and Python". *CRC Press Taylor & Francis Group*. New-York: USA. 2020. 349 p.
7. "MATLAB. The Language of Technical Computing". [Software documentation] / MATLAB – Active link: https://uk.mathworks.com/help/matlab/.
8. Huei-Huang, L. "Programming and Engineering Computing with MATLAB". *SDC Publications*. 2020. 520 p.
9. "MATLAB. The Language of Technical Computing". [Software documentation] / MATLAB – Active link: https://uk.mathworks.com/help/control/index.html.
10. Dingyu, X. & Yang Quan C. "Modeling, Analysis and Design of Control Systems in MATLAB and Simulink". *Publ. World Scientific*. 2015. 580 p.
11. "MATLAB. The Language of Technical Computing". [Software documentation] / MATLAB – Active link: https://uk.mathworks.com/help/symbolic/.
12. Desmond, J., Higham, Nicholas J. Higham "MATLAB Guide". *Third Edition – SIAM-Society for Industrial and Applied Mathematics*. 2018. 502 p.
13. Popovich, M. G. & Kovalchuk, O. V "Theory of Automatic control" (in Russian). *Publ. Libid*, Kiev: Ukraine. 2007. 656 p.
14. Miroshnik, I. V. "Automatic Control theory. Linear Systems"(in Russian). SPb.-Peter: Russian Federation. 2005. 336 p.
15. Phillips, C. A., Nagle, H. & Chakrabortty. A. "Digital Control System Analysis & Design". [4th Edition]. *Publ. Pearson*. 1995. 528 p.
16. Peresada, S. M., Kovbasa, S. M. & Zaichenko, Yu. M. "Dynamic Performances of the Shunt Active Power Filter Control System". *Applied Aspects of Information Technology*. *Publ. Nauka i Tekhnika*. 2021; Vol.4 No.1: 47–56. DOI: https://doi.org/10.15276/aait.01.2021.4.
17. Ogata, K. "Discrete-Time Control Systems". [2nd Edition]. *Pearson*. 1995. 745 p.
18. Ogata K. "Solving Control Engineering Problems with MATLAB (MATLAB Curriculum Series)". *Englewood Cliffs. NJ: Prentice Hall*. 2014. 527 p.
19. Tolochko, O. I. "Analysis and Synthesis of Electro-Mechanical Systems for the Support of the State" (in Russian). Donetsk: Ukraine. *Nord-Press*. 2004. 298 p.
20. Tolochko. O. I. & Rozkaryaka, P.I. "Analysis of the Shortfalls in the up-to-date Discrete Systems in the MATLAB Package" (in Russian). *Journal Elektrotekhnika and Elektroenergetika*. Zaporizhzhya: Ukraine. ZNTU (Zaporizhzhya National Technical University). 2006; No.1: 10–14.
21. Tolochko, O. I., Kaluhin, D. V., Palis, Stefan & Oshurko, S. V. "Field Weakening Control for Induction Motors Based on Copper and Iron Losses Minimization". *Applied Aspects of Information Technology*. *Publ. Nauka i Tekhnika*. 2020; Vol.3 No.2: 44–57. DOI: https://doi.org/10.15276/aait.02.2020.3.

22. Sadovoi, O. V., Klyuyev, O. V., Sokhina, Yu. V. & Filin, I. V. “Information System of Minimization Consumption Reactive Power in Asynchronous Electric Drive with Vector Control”. *Applied Aspects of Information Technology*. *Publ. Nauka i Tekhnika*. 2020; Vol.3 No.2: 74–84. DOI: https://doi.org/10.15276/aait.02.2020.5.

23. Tolochko, O. I. & Rozkaryaka, P. I. “A Digital Setting Device for Optimal Control on Positional Electric Drives” (in Russian). Proceedings of Donetsk National Technical University. Donetsk: Ukraine. 2010; No.1: 97–109.

24. Sinchuk, O. N., Kozakevych, I. A. & Vornikov, D. N. “Control System of Wind Generator Based onSwitched Reluctance Motor”. *Applied Aspects of Information Technology. Publ. Nauka i Tekhnika*. 2019; Vol.2 No.3: 230–242. DOI: https://doi.org/10.15276/aait.03.2019.5.

25. Arsirii, O. O., Yadrova, M. V., Kondratyev,S. B. & Stelmakh, D. E. “Development of the Intelligent Softwareand Hardware Subsystem Forcapturing an Object by Robotmanipulator”. *Herald of Advanced Information Technology*. *Publ. Nauka i Tekhnika*. 2020; Vol.3 No.2: 42–51. DOI: https://doi.org/10.15276/hait.02.2020.4.

26. Starostin, S. S. & Tolochko, O. I. “The Value of Parameters in Digital Controllers of Electric Drives by Design Methods of Continuous Systems” (in Russian). *Collection of Scientific works of DSTU (Dnipro State Technical University)*. Dneprodzerzhinsk: Ukraine. 2007. p. 179–182.

Conflicts of Interest: the authors declare no conflict of interest

Received 17.12.2020
Received after revision 12.03.2021
Accepted 16.03. 2021

DOI: https://doi.org/10.15276/aait.02.2021.5

УДК 004 + 621.316.7

ДИСКРЕТНА АПРОКСИМАЦІЯ НЕПЕРЕРВНИХ ДИНАМІЧНИХ ОБ‘ЄКТІВ В СЕРЕДОВИЩІ ПАКЕТУ MATLAB

Ольга Іванівна Толочко[1)]

ORCID: https://orcid.org/0000-0002-6871-0653; tolochko.ola@gmail.com. Scopus ID: 57192559239

Стефан Паліс[2)]

ORCID: https://orcid.org/0000-0002-1941-2289; stefan.palis@ovgu.de. Scopus ID: 24766914900

Олексій Олегович Бурмельов[1)]

ORCID: https://orcid.org/0000-0003-2057-4717; oleksii.burmelov@gmail.com. Scopus ID: 57203539371

Данило Владиславович Калугін[1)]

ORCID: https://orcid.org/0000-0003-1293-4479; dadylo.kaluhin@gmail.com. Scopus ID: 57210207513

[1)] Національний технічний університет України «Київський політехнічний інститут імені Ігоря Сікорського», пр.Перемоги, 37. Київ, 03056, Україна

[2)] Магдебурзький університет імені Отто фон Геріке, Університетська площа, 2. Магдебург, 39106, Німеччина

АНОТАЦІЯ

Дана робота присвячена дослідженню різних методів дискретної апроксимації неперервних ланок, що є обов‘язковим етапом синтезу цифрових систем керування неперервними динамічними об‘єктами та розробці методичних рекомендацій щодо виконання цих операцій за допомогою інструментів системи програмування MATLAB. В роботі досліджені такі методи дискретизації як імпульсно-, ступінчасто- та лінійно інваріантні Z-перетворення, підстановчі методи основані на застосуванні різних методів числового інтегрування та метод відповідності нулів-полюсів. У роботі наведено приклади використання для здійснення цих операцій числових та символьних інструментів пакету MATLAB, запропоновано удосконалений варіант m-функції для дискретизації неперервних систем методом відповідності нулів полюсів, що дозволяє даному методу наближуватися як до ступінчасто-інваріантного, так і до лінійно-інваріантного Z-перетворень; розроблено програми для дискретної апроксимації неперервних об‘єктів у символьному вигляді, що дозволяє виконувати порівняльний аналіз методів дискретизації і синтезованих за їх допомогою систем та досліджувати вплив періоду квантування на точність дискретизації аналітичними методами. Виконано порівняння між собою дискретних передавальних функцій, отриманих різними методами, та відповідних реакцій у часі на різні сигнали. За допомогою розроблених програм визначено, що імпульсно-інваріантне Z-перетворення можна використовувати тільки у тому випадку, коли на вхід неперервного об'єкту надходять імпульсні сигнали, а лінійно-інваріантне перетворення доцільно використовувати при стрибкоподібних сигналах на вході. Також в роботі наведено алгоритм застосування методу Тастіна, що відповідає заміні аналогового інтегрування чисельним інтегруванням методом трапецій. Показано, що метод Тастіна є найбільш придатним для дискретизації регуляторів першого порядку з обмеженням вихідного сигналу. В статті також розглянуто метод відповідності нулів-полюсів та показано, що він має найбільшу точність серед приближних методів дискретної апроксимації. На основі виконаних досліджень наведено рекомендації щодо використання цих методів при синтезі систем керування неперервними динамічними об'єктами.

Ключові слова: Дискретна апроксимація; неперервний об'єкт; Z-перетворення; передавальні функції; нулі; полюси; період квантування; перехідні процеси

ABOUT THE AUTHORS

Olga Ivanivna Tolochko, Dr. Sci. (Eng), Professor of Electrical Engineering and Automation Systems Department, National Technical University of Ukraine "KPI", 37, Peremogy Av. Kyiv, 03056, Ukraine

ORCID: https://orcid.org/0000-0002-6871-0653; tolochko.ola@gmail.com

Research field: Energy-Efficient Control of AC and DC Electric Drives; Optimal Control of Positional Electric Drive Systems; Damping of Torque Load and Metal Structures of Crane Mechanisms Vibrations by Electric Drive Means

Ольга Іванівна Толочко, доктор технічних наук, професор кафедри Автоматизації електромеханічних систем та електроприводу. Національний технічний університет України «Київський політехнічний інститут імені Ігоря Сікорського», пр. Перемоги, 37. Київ, 03056, Україна

Stefan Palis, Doctor-Engineer, Junior-professor, Professor of Institute of Automation Engineering (OvGU) and Institute of Electrical Power Engineering (MPEI), Otto von Guericke Universität Magdeburg, 2, Universitätsplatz. Magdeburg, 39106, Germany.

ORCID: https://orcid.org/0000-0002-1941-2289; stefan.palis@ovgu.de

Research field: Nonlinear Control Theory; Control of Distributed Parameter Systems; Analysis and Optimization of Electrical Energy Systems

Стефан Паліс, доктор-інженер, професор факультету Електротехніки та інформаційних технологій університету і кафедри Електроенергетичних систем Інституту електроенергетики. Магдебурзький університет імені Отто фон Геріке. Університетська площа, 2. Магдебург, 39106, Німеччина

Oleksii Olegovich Burmelov, PhD Student of Electrical Engineering and Automation Systems Department. National Technical University of Ukraine "KPI", 37, Peremogy Av. Kyiv, 03056, Ukraine

ORCID: https://orcid.org/0000-0003-2057-4717; oleksii.burmelov@gmail.com

Research field: Electric Drive; Permanent Magnet Synchronous Motor; Speed Vector Control; Optimal Control; Loss Minimization; Energy Efficiency

Олексій Олегович Бурмельов, аспірант кафедри Автоматизації електромеханічних систем та електроприводу. Національний технічний університет України «Київський політехнічний інститут імені Ігоря Сікорського», пр. Перемоги, 37. Київ, 03056, Україна

Danylo Vladyslavovych Kaluhin, PhD Student of Electrical Engineering and Automation Systems Department, National Technical University of Ukraine "KPI", 37, Peremogy Av. Kyiv, 03056, Ukraine

ORCID: https://orcid.org/ 0000-0003-1293-4479; danylo.kaluhin@gmail.com

Research field: Electric Drive; Induction Motor; Speed Vector Control; Optimal Control; Loss Minimization; Energy Efficiency

Данило Владиславович Калугін, аспірант кафедри Автоматизації електромеханічних систем та електроприводу. Національний технічний університет України «Київський політехнічний інститут імені Ігоря Сікорського», пр. Перемоги, 37. Київ, 03056, Україна