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## IMPROVING THE DESIGNING METHOD OF THERMAL NETWORKS: SERIAL CONNECTION OF STREAMS

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### ABSTRACT

The paper presents an approach to the design of technical systems, the elements of which are interconnected and carry out an internal exchange of energy. The above analysis showed that for heat-exchange equipment when combining devices into systems, only iterative methods are currently used, a representative of which is Pinch analysis. A limitation of the iterative approach is the impossibility of obtaining an exact solution to such problems, which can only be achieved by analytical methods, which also make it possible to reveal some effects in systems that are practically unavailable for numerical solution. This indicates the absence of a rigorous proof of the existence of a solution and a problem in the construction of approximate solutions, due to the need to involve complementary hypotheses. The topological representation of the system modules allows us to consider the architecture as a network, which contributes to the analysis of the connections between the constituent elements and the identification of their mutual influence. Highlighted the typical connections of network elements such as serial, parallel, contour, which allows to unify the principles of building connections in the system. As an optimality criterion, the NTU parameter was chosen, which includes the heat exchange surface and is usually used when searching for a solution for heat exchangers of moving objects. An analytical solution to the problem of flow distribution and energy exchange efficiency in a system of two series-connected heat exchangers is obtained. His analysis showed that the formulation of the design problem based on the definition of matrix elements in relation to determinants allows not only to meet the requirements for the system, but also to determine the design parameters of its elements that satisfy their extreme characteristics.

**Keywords:** Design; Heat Exchangers; Network Architecture; Analytical Methods; Optimization; Efficiency

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### INTRODUCTION

The design is aimed at solving some optimization problem associated with ensuring the externality of a criterion that is significant for practice, for example, a minimum of the heat exchange surface or associated mass and size indicators, which is critical for moving objects. The problem of solving the problem for systems is that the system of conservation equations is incomplete and needs to be replenished. It seems natural to supplement the problem with analytical expressions about the proportionality of the potential difference to the amount of energy in the elements of the system, establishing the relationship between the final temperatures in the heat exchangers and the ratio of water numbers and NTU, introduced by Nusselt. The NTU parameter is the product of the heat exchange surface by the heat transfer coefficient, referred to the water number of the heating medium when searching for the optimal solution for heat exchangers. This approach leads to a system of transcendental equations for NTU and

the ratio of water numbers, the analytical solution of which is absent in the known literature. Attempts to solve the problem made it possible to establish the relationship between the efficiency of heat transfer, the ratio of water numbers and the NTU parameter, which made it possible to determine the minimum surface, but for one apparatus. Practical needs require a minimum sum of surfaces for two or more devices, which is an urgent design problem.

### LITERATURE REVIEW

It is shown in [1] that, on a global scale, one third of the energy consumed is in the industrial sector, and half is represented as heat, therefore, energy recovery can have a significant effect. Analysis of district heating networks indicates that the efficiency of heat transfer is influenced by almost all indicators: from the design and the nature of the movement of heat carriers to the topology of the heating network [2].

Achievement of cost-effective design is achieved by modeling the entire system and its components on the basis of library components on the assumption of an explicit formulation of the

structure of the network flow [3]. However, they are designed for individual devices and do not take into account their relationship. The nature of the movement of the coolant affects the efficiency, which can be seen from the comparison of counter-flow and direct-flow heat exchangers. The analysis is based on the use of a one-dimensional model of thermal conductivity and averaged over the entire volume of parameters: temperature head, conditional heat transfer coefficient, effective contact area of coolants [4]. The advantages of the counter-flow switching of flows are analyzed in [5], where the equivalence of the direct-flow and counter-flow schemes is shown only for very large and very small ratios of water numbers. However, the gain in NTU, a dimensionless parameter, which is considered as a factor of the size and, accordingly, the mass of the heat exchanger, belongs to the counterflow scheme. At the same time, the issue is considered in relation to a separate apparatus, and not a system in which there is a mutual influence of the components. This issue was considered in [6], where a variant of converting a direct-flow system to a counter-current one is presented.

Modeling systems with renewable energy sources with buffer storage of the coolant, accumulation and conversion of heat when changing operating modes [7], microelectronic heat exchange devices [8] with the interaction of elements requires system design. The problem is most clearly manifested in chemical-technological systems in gas-phase heterogeneous catalytic reactors, in which models of rectification technological schemes for product separation have been developed and implemented [9]. When calculating balances, the system of equations for generalized flows is supplemented with equations of functional relationships, which determine the component-wise composition of physical flows. Iterative procedures for optimization problems with probabilistic constraints have been developed, based on the partitioning of the domain, taking into account the uncertainty of the initial information [10]. At the same time, the incompleteness of the initial set of equations requires the introduction of additional hypotheses, as a rule, empirical ones, which affect the reliability of the simulation. In the system of heat exchangers, there are sequential, parallel, bypass and loop connections of equipment [11], in which the heat transfer ratios in the cascades are complex due to their interconnection, and the temperature distribution in the multistage cascade is not linear [12]. The Newton-Raphson method used to solve the system of equations uses an iterative method and it is assumed that there are sufficiently accurate values of all variables [13]. The principles of mathematical

modeling of heat transfer are most fully worked out in PINCH analysis, using the point at which the temperatures of the hot and cold streams are the closest [14]. For operating oil refineries, PINCH technologies make it possible to achieve a reduction in energy consumption by up to 30-50 % [16]. At the same time, the presented literature uses exclusively iterative methods for solving problems when combining devices into systems and does not pay attention to the development of analytical methods that allow obtaining an exact solution of such problems.

The need to change the concept to resolve the design contradiction is substantiated in [16]. Obviously, analytical methods make it possible to obtain not only an exact solution to the problem, but also to reveal some effects in systems that are practically inaccessible for a numerical solution. This indicates, firstly, the lack of a rigorous proof of the existence of a solution and, secondly, certain problems in the construction of approximate solutions, including due to problems with testing algorithms.

#### THE PURPOSE AND ARTICLE

The aim of the work is to obtain an analytical solution for the distribution of heat exchange efficiencies in a system of two series-connected heat exchangers.

To achieve the goal, the following tasks were set:

- 1) to develop an analytical model of the serial connection of two heating devices;
- 2) analyze the serial connection model.

#### ANALYTICAL MODEL OF SERIAL CONNECTION

In engineering and technological systems, a series connection of two devices is often found as subsystems. With a serial connection (Fig. 1), the entire process flow leaving the previous apparatus enters the next element completely, while the flow passes through each element of the circuit only once.

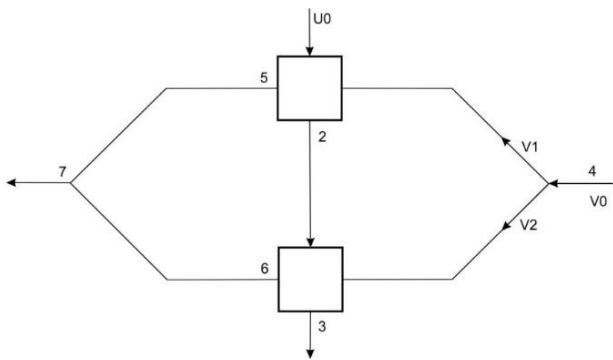
$$\begin{aligned}\alpha_1(T_0 - T_2) &= (T_5 - \Theta_0) \\ \alpha_2(T_2 - T_3) &= (T_6 - \Theta_0) , \\ \alpha_3(T_5 - T_{71}) &= (T_{72} - T_6)\end{aligned}$$

where:  $T_j$  – hot stream temperature;

$\Theta_0$  – temperature of the cold stream at the outlet;

$T_{kl}$  – temperature at the point of mixing of streams;

$\alpha_i$  – the ratio of the consumable heat capacities of the hot and cold streams.



**Fig. 1. Series connection of elements:**  
**UO - consumption heat capacity of a hot stream;**  
**VO - consumption heat capacity of a cold stream;**  
**V1 - consumption heat capacity of a cold stream branch**

Source: compiled by the authors

The incompleteness of the system of equations for the conservation of energy is obvious. In our opinion, the most fundamental replenishment of the conservation laws is their replenishment based on the hypothesis of proportionality between the amount of transferred (used) energy and the applied potential

$$\begin{aligned} (T_0 - T_2) &= \Phi_1(T_0 - \Theta_0) \\ (T_2 - T_3) &= \Phi_2(T_2 - \Theta_0), \\ (T_5 - T_{71}) &= \Phi_3(T_5 - T_6) \end{aligned}$$

where  $\Phi_n$  is the temperature efficiency of the intra-network energy exchange [6].

In matrix form, such a system of equations has the form

$$\begin{bmatrix} -\alpha_1 & 0 & 0 & -1 & 0 & 0 \\ \alpha_2 & -\alpha_2 & 0 & 0 & -1 & 0 \\ 0 & 0 & -\alpha_3 & \alpha_3 & 1 & -1 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 1 - \Phi_2 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 - \Phi_3 & \Phi_3 & 0 \end{bmatrix} \cdot \begin{bmatrix} T_2 \\ T_3 \\ T_{71} \\ T_5 \\ T_6 \\ T_{72} \end{bmatrix} = \begin{bmatrix} -\alpha_1 T_0 - \Theta_0 \\ -\Theta_0 \\ 0 \\ \Phi_1(T_0 - \Theta_0) - T_0 \\ -\Phi_2 \Theta_0 \\ 0 \end{bmatrix} \quad (1)$$

Let's introduce an independent variable

$$\Theta = \frac{T_0 - T_n}{T_0 - \Theta_0}, \quad (2)$$

$$\begin{bmatrix} \Theta_2 \\ \Theta_3 \\ \Theta_{71} \end{bmatrix} = \begin{bmatrix} \Phi_1 \\ \Phi_1 - \Phi_1 \Phi_2 + \Phi_2 \\ \alpha_1 \Phi_1 \Phi_3 - \alpha_1 \Phi_1 - \\ -\alpha_2 \Phi_2 \Phi_3 + \alpha_2 \Phi_1 \Phi_2 \Phi_3 + 1 \end{bmatrix},$$

$$\begin{bmatrix} \Theta_5 \\ \Theta_6 \\ \Theta_{72} \end{bmatrix} = \begin{bmatrix} 1 - \alpha_1 \Phi_1 \\ \alpha_2 \Phi_1 \Phi_2 - \alpha_2 \Phi_2 + 1 \\ \alpha_2 \Phi_1 \Phi_2 - \alpha_2 \Phi_2 - \\ -\alpha_1 \alpha_3 \Phi_1 \Phi_3 + \alpha_2 \alpha_3 \Phi_2 \Phi_3 - \\ -\alpha_2 \alpha_3 \Phi_1 \Phi_2 \Phi_3 + 1 \end{bmatrix}.$$

The requirement of equality of temperatures  $T_{71}$  and  $T_{72}$  allows expressing the ratio of flows in the mixing unit through the efficiency within the network energy exchange

$$\begin{aligned} T_{71} - T_{72} &= (\alpha_3 \Phi_3 + \Phi_3 - 1)(\alpha_1 \Phi_1 - \\ &- \alpha_2 \Phi_2 + \alpha_2 \Phi_1 \Phi_2)(T_0 - \Theta_0) \\ \alpha_3 &= \frac{1 - \Phi_3}{\Phi_3}. \end{aligned}$$

And this, in turn, will make it possible to close the task of determining flows, expressing them through the efficiency of the intra-network energy exchange

$$\begin{bmatrix} 1 & 1 \\ \Phi_3 & \Phi_3 - 1 \end{bmatrix} \cdot \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} V_0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} V_0(1 - \Phi_3) \\ V_0 \Phi_3 \end{bmatrix}.$$

And, therefore, determine the ratio of the fluxes  $\alpha$  in the conservation equations under the assumption of a unit heat capacity of the interacting fluxes

$$\begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} = \begin{bmatrix} \frac{U_0}{V_1} \\ \frac{U_0}{V_2} \\ \frac{V_1}{V_2} \end{bmatrix} = \begin{bmatrix} \frac{\alpha_0}{1 - \Phi_3} \\ \frac{\alpha_0}{\Phi_3} \\ \frac{1 - \Phi_3}{\Phi_3} \end{bmatrix},$$

$$\alpha_0 = \frac{U_0}{V_0}.$$

The most general requirement for a system can be formulated as the ratio of the amount of energy in it to the applied energy potential [6]

$$E = \frac{T_0 - T_3}{T_0 - \Theta_0}.$$

In essence, in such a setting, the formulated problem is the problem of determining the elements of the matrix by the value of the ratio of determinants, since

$$E = \Phi_1 - \Phi_1\Phi_2 + \Phi_2.$$

The generally accepted logic of solving the formulated problem leads to the replenishment of conservation laws with models of elements and iterative schemes for its solution. The disadvantages of this approach are well known [16].

#### ANALYSIS OF THE SERIAL CONNECTION MODEL

The key factor that determines both the operation of the designed system and its elements is the correspondence of the direction of the processes in the real system with their direction hypothetically chosen by the designer when setting its topology. It is well known that the direction of processes and their performance is determined by the second law of thermodynamics and, as a consequence, is associated with the uncertainty of the values of average energy measures both in the elements of the system and in its networks. In other words, the distribution of the energy potential dictates the conditions for the operability of the system and its elements through the uncertainty of the values of the average energy measures. The statement of the problem, the determination of the matrix elements that satisfy the requirement of the minimum uncertainty of the average energy measures, leads to the determination of the distribution of the efficiencies of the system elements in its topological representation in accordance with the requirements of the second law of thermodynamics. The continuity of the set of values of the elements of matrix (1) satisfying requirement (2) is obvious. However, the formulated requirements for the minimality of the uncertainty of the average energy measures and the construction based on the Shannon principle allow one to obtain for the subsystem shown in Fig. 1, the solution of the formulated problem as a finite subset of the values of the efficiencies of the inter-network and intra-network

energy exchange depending on  $E$  and  $\alpha$ . In addition, the extremeness of solutions (minimum uncertainty of average energy measures) ensures the maximum efficiency of energy transfer from the “hot” network to the “cold” network in its elements and the minimum energy dissipation in the mixing nodes.

Formally, the formulation of the problem on the value of the efficiency of the elements of the system ( $\Phi_n$ ) for the given efficiency of the system ( $E$ ) and the ratio of flows at the input to it ( $\alpha$ ) determines the finite set of functionals that depend both on the designer's choice of the elements themselves and their design at the next design stage based on the requirements systems to their integral characteristics (energy exchange efficiency).

The solutions obtained on the basis of the minimality of the uncertainty of the average energy measures determine three groups of roots for the efficiency of energy exchange depending on the efficiency of the system.

$$1 > E > 0$$

$$\Phi_3 = \frac{1}{2}; \quad \Phi_1 = \frac{E}{2};$$

$$\Phi_2 = \frac{E}{2 - E}$$

$$1 > E > 0$$

$$\Phi_3 = \frac{\Phi_2(\Phi_1 - 1)}{\Phi_1\Phi_2 - \Phi_2 - \Phi_1};$$

$$\Phi_1 = \Phi_2 = 1 - \sqrt{1 - E}$$

$$1 > E > 0,75$$

$$\Phi_3 = \frac{\Phi_2(\Phi_1 - 1)}{\Phi_1\Phi_2 - \Phi_2 - \Phi_1};$$

$$\Phi_1 = \frac{1 - \sqrt{4E - 3}}{2};$$

$$\Phi_2 = \frac{\sqrt{4E - 3} + 1}{2}$$

The presented roots not only satisfy the requirements for the subsystem, but, as noted above, in the presence of a model for a system element that relates efficiency to the parameters of its design, they allow us to find their values. So, for example, if the elements of the subsystem are counterflow heat exchangers, then their efficiency is determined by the function [6]

$$\Phi = \frac{1 - \exp[-NTU(1-z)]}{1 - z \cdot \exp[-NTU(1-z)]};$$

$$NTU = \frac{\ln\left(\frac{\Phi - 1}{z\Phi - 1}\right)}{z - 1}, \quad (4)$$

where  $z$  is the ratio of hot to cold flow at unit heat capacity.

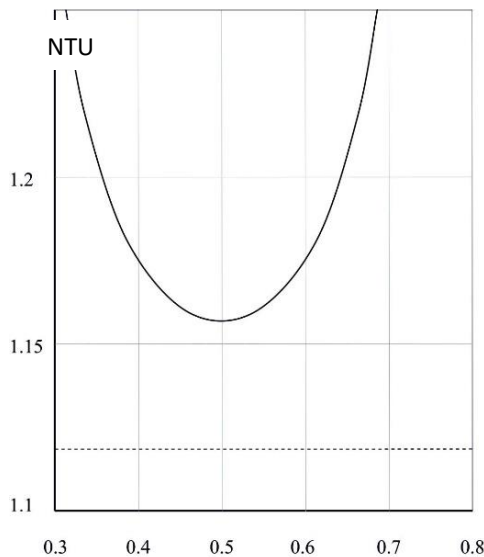
The NTU parameter depends on the heat exchange surface, and, therefore, other things being equal, is the mass and size characteristic of the subsystem, which in some circumstances can be extremely important if it is necessary to divide the "cold" stream into two. The simulation results for a similar situation are shown in Fig. 2.

The solution of the transcendental equation (4) with respect to NTU for a hot flow has the form

$$\frac{\ln\left(\frac{z\sqrt{1-E}}{z-\alpha+\alpha\sqrt{1-E}}\right)z}{\alpha-Z} +$$

$$+ \frac{\ln\left(\frac{z\sqrt{1-E}-\sqrt{1-E}}{\alpha+z-\alpha\sqrt{1-E}-1}\right)(z-1)}{1-\alpha-z};$$

$$\frac{\ln\left(\frac{E-1}{E\alpha-1}\right)}{\alpha-1}$$



**Fig. 2. Comparison of the total value of  $N_z$  in a system of two heat exchangers with the value of this value in a system consisting of one apparatus of the same type at  $\alpha = 0.5$  and  $E = 0.6$ , depending on the distribution of flows on the branches of the networks**

Source: compiled by the authors

It can be seen from the graphs that for the first two groups of roots, deviation from the extreme when the "cold" flow is distributed over the apparatus can lead to a significant increase in the heat exchange surface, and, consequently, in the mass and size characteristics of the subsystem. If we take as the "ideal" design, consisting of one heat exchanger, then, as can be seen from Fig. 2, the surface surplus (NTU) in the system of two devices, designed on the basis of the new design logic, does not exceed 10 %. It should be noted that with an increase in the requirements for system efficiency ( $E > 0.75$ ), the tendency to exceed the surface area (NTU) in a system of two units above the surface required by one unit remains and reaches 40% regardless of the choice of the root group.

In some cases, it becomes necessary to divide into two streams: "cold" and "hot". The system of equations for the subsystem shown in Fig. 1, taking into account the change in the direction of heat fluxes, will be written in the form

$$\alpha_1(T_0 - T_5) = (T_2 - \Theta_0)$$

$$\alpha_2(T_0 - T_6) = (T_3 - T_2) ,$$

$$\alpha_3(T_5 - T_{71}) = (T_{72} - T_6)$$

$$(T_0 - T_5) = \Phi_1(T_0 - \Theta_0)$$

$$(T_0 - T_6) = \Phi_2(T_0 - T_2) .$$

$$(T_5 - T_{71}) = \Phi_3(T_5 - T_6)$$

Or in matrix notation

$$\begin{bmatrix} -\alpha_1 & 0 & 0 & -1 & 0 & 0 \\ 0 & -\alpha_2 & 0 & 1 & -1 & 0 \\ \alpha_3 & 1 & -\alpha_3 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & \Phi_2 & 0 & 0 \\ 1 - \Phi_3 & \Phi_3 & -1 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} T_5 \\ T_6 \\ T_{71} \\ T_2 \\ T_3 \\ T_{72} \end{bmatrix} =$$

$$\begin{bmatrix} -\alpha_1 T_0 - \Theta_0 \\ -\alpha_2 T_0 \\ 0 \\ \Phi_1(T_0 - \Theta_0) - T_0 \\ -T_0 \\ 0 \end{bmatrix}$$

The substitution (2) introduced above after transformation makes it possible to write the solution of the system of equations in the form

$$\begin{bmatrix} \Theta_2 \\ \Theta_3 \\ \Theta_{72} \end{bmatrix} = \begin{bmatrix} 1 - \alpha_1 \Phi_1 \\ (\alpha_1 \Phi_1 - 1)(\alpha_2 \Phi_2 - 1) \\ \Phi_2 - \alpha_1 \Phi_1 \Phi_2 + \alpha_3 \Phi_1 \Phi_3 - \\ - \alpha_3 \Phi_2 \Phi_3 + \alpha_1 \alpha_3 \Phi_1 \Phi_2 \Phi_3 \end{bmatrix}$$

$$\begin{bmatrix} \Theta_2 \\ \Theta_6 \\ \Theta_{71} \end{bmatrix} = \begin{bmatrix} \Phi_1 \\ - \Phi_2 (\alpha_1 \Phi_1 - 1) \\ \Phi_2 \Phi_3 - \Phi_1 \Phi_3 + \Phi_1 - \alpha_1 \Phi_1 \Phi_2 \Phi_3 \end{bmatrix}$$

The requirement of equality of temperatures  $T_{71}$  and  $T_{72}$  allows expressing the ratio of flows in the mixing unit through the efficiency within the network energy exchange

$$\alpha_3 = \frac{1 - \Phi_3}{\Phi_3} \cdot \frac{(\alpha_3 \Phi_3 + \Phi_3 - 1)(\Phi_1 - \Phi_2 + \alpha_1 \Phi_1 \Phi_2)}{\Phi_3}$$

And to express the ratio of flows in the elements of the interconnection of energy through the efficiency within the interconnection of energy

$$\begin{aligned} U_0 - (U_1 + U_2) &= 0 \\ U_1 \Phi_3 - U_2(1 - \Phi_3) &= 0 \end{aligned}$$

$$\begin{bmatrix} U_1 \\ U_2 \end{bmatrix} = \begin{bmatrix} U_0(1 - \Phi_3) \\ U_0 \Phi_3 \end{bmatrix},$$

$$\begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} = \begin{bmatrix} \frac{U_1}{V_0} \\ \frac{U_2}{V_0} \\ \frac{U_1}{U_2} \end{bmatrix} = \begin{bmatrix} \alpha_0(1 - \Phi_3) \\ \alpha_0 \Phi_3 \\ \frac{1 - \Phi_3}{\Phi_3} \end{bmatrix}.$$

In this case, the efficiency of the system is determined by the ratio

$$\begin{aligned} E = \frac{T_0 - T_{71}}{T_0 - \Theta_0} &= \Phi_2 \Phi_3 - \Phi_1 \Phi_3 + \\ &+ \Phi_1 - \alpha_1 \Phi_1 \Phi_2 \Phi_3 \end{aligned}$$

and the group of roots for the efficiencies of the elements of the system, which ensures its extreme efficiency based on the minimum uncertainty of the average energy measures, is written in the form

$$\Phi_3 = \frac{1}{2}; \quad \Phi_1 = \Phi_2 = \frac{2(1 - \sqrt{1 - E\alpha_0})}{\alpha_0}.$$

As noted above, the presented roots not only satisfy the requirement for the subsystem, but, as noted above, in the presence of a model for a system element connecting the efficiency with the parameters of its design, find their values. So, for example, if the elements of the subsystem are counter flow heat exchangers, then their efficiency is determined by function (4), and its solution with respect to NTU has the form

$$\begin{aligned} & \left[ \frac{z \cdot \ln \left( \frac{\alpha + 2\sqrt{1 - E\alpha} - 2}{\alpha - 2\alpha z + 2\alpha z \sqrt{1 - E\alpha}} \right)}{\alpha z - 1} + \right. \\ & \left. \frac{\ln \left( \frac{\alpha + 2\sqrt{1 - E\alpha} - 2}{\alpha(2z - 2z\sqrt{1 - E\alpha} + 2\sqrt{2 - E\alpha} - 1)} \right) \cdot (z - 1)}{\alpha z - \alpha + 1} \right] \\ & \left( \frac{\ln \frac{E - 1}{E\alpha - 1}}{\alpha - 1} \right) \end{aligned}$$

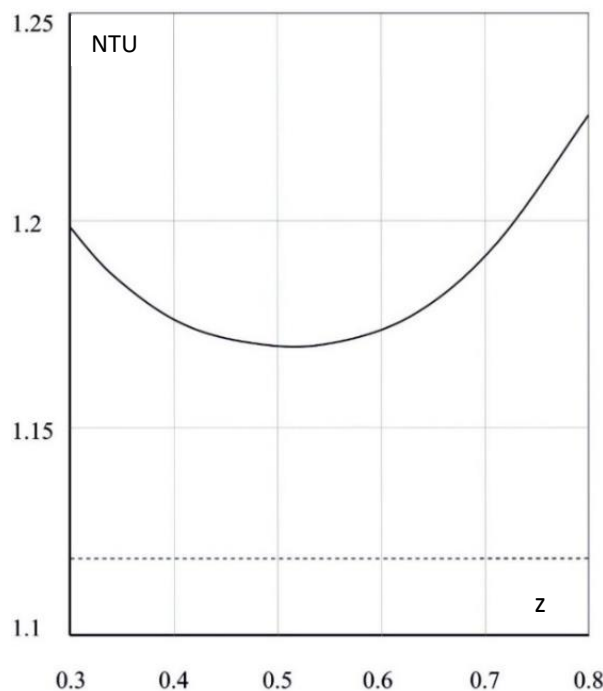


Fig. 3. Comparison of the total value of NTU in a system of two heat exchangers with the value of this value in a system consisting of one apparatus of the same type at  $\alpha = 0.5$ ;  $E = 0.6$ , depending on the distribution of flows on the branches of the networks

Source: compiled by the authors

### DISCUSSION OF THE ANALYSIS RESULTS

The problem of the so-called optimal design can be formulated, for example, on the basis of the NTU sum of squares, while the problem of flows

(water numbers) should supplement the problem of optimal design with a system of equations based on Kirchhoff's laws. We analyzed the consequences of this approach in the review [16].

The proposed approach is based on the statement that the amount of energy in an element is proportional to the energy potential at the entrance to it, i.e. efficiency of energy exchange.

The developed logic for solving such problems allows us to determine the extreme distribution of the efficiency of devices in the system. In this case, the task of designing a system is reduced to a finite set of optimization problems for one apparatus with extreme values of their efficiency.

An extreme distribution of efficiencies in the system leads to a distribution of potential providing a minimum NTU value.

Exact solutions of problems can serve as a test when constructing various iterative approaches to solving such problems, for example, when designing heat exchanger systems in technological systems of chemical-technological systems.

If we accept as the “ideal” design, consisting of one heat exchanger, then the surplus surface (NTU) in the system of two devices, designed on the basis of the new design logic, as in the case of the

separation of the “cold” flow, does not exceed 10%. At the same time, as in the case of a “cold” flow, with an increase in the requirements for system efficiency ( $> 0.75$ ), the tendency to exceed the surface (NTU) in a system of two devices above the surface required by one device remains and reaches 40 %. This confirms the effectiveness of the new approach associated with the analytical representation of the connection of heat exchangers in comparison with the iterative design approach, in which it is impossible to determine the global extremum, to solve inverse problems.

## CONCLUSION

1. An analytical solution to the problem of flow distribution and energy exchange efficiency in a system of two series-connected heat exchangers has been obtained.

2. As is obvious from the above results, the formulation of the design problem based on the determination of the matrix elements in relation to the determinants, allows not only to meet the requirements for the system, but also to determine the design parameters of its elements that satisfy their extreme characteristics.

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## **ВДОСКОНАЛЕННЯ МЕТОДУ ПРОЕКТУВАННЯ ТЕПЛОВИХ МЕРЕЖ: ПСЛІДОВНЕ З’ЄДНАННЯ ПОТОКІВ**

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### **АНОТАЦІЯ**

В роботі представлено підхід до проектування технічних систем, елементи яких взаємопов’язані і здійснюють внутрішній обмін енергією. Проведений аналіз показав, що для теплообмінної апаратури при об’єднанні апаратів в системи у теперішній час використовуються виключно ітераційні методи, представником яких являється Pinch-аналіз. Обмеженням ітераційного підходу є неможливість одержання точного рішення таких задач, що може бути досягнуто тільки аналітичними методами, які дозволяють виявити деякі ефекти в системах, що практично недосяжно при чисельному рішенні. Це вказує на відсутність строгого доказу наявності існування рішення і проблеми при побудові приблизних рішень внаслідок необхідності залучення доповнюючих гіпотез. Топологічне представлення модулів системи дозволяє розглядати архітектуру у вигляді мережі, що сприяє аналізу зв’язків між складовими елементами і виявленню їх взаємовпливу. Виділені типові з’єднання елементів мережі такі як послідовне, паралельне, контурне, що дозволяє уніфікувати принципи побудови зв’язків в системі. У якості критерію оптимальності обрано параметр NTU, який включає поверхню теплообміну і звичайно використовується при пошуку рішення для теплообмінних апаратів рухомих об’єктів. Одержано аналітичне рішення задачі про розподіл потоків і ефективність теплообміну в системі з двох послідовно з’єднаних апаратів. Його аналіз показав, що постановка задачі проектування на основі визначення елементів матриці по відношенню детермінантів, дозволяє не тільки



задовільнити вимоги до системи, але і визначити параметри конструкції її елементів, які задовільняють їх екстремальним характеристикам.

**Ключові слова:** Проектування; теплообмінні апарати; мережева архітектура; аналітичні методи; оптимізація; ефективність

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