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TAKING ACCOUNT OF THE SHIFT AND INERTIA OF ROTATION IN PROBLEMS OF DIAGNOSTICS OF THE SPECTRA OF CRITICAL FORCES MECHANICAL SYSTEMS

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Abstract

Diagnostics of stability of the cores constructions, elements of the carrier system of the ship and port facilities, reduces to the definition of critical forces, excess which causes a transition of the system from one equilibrium state to another. Such a transition often leads to the destruction of the constructions or other forms of accidents, and therefore it is extremely undesirable and for practice it is important to knowledge of a specific spectrum of critical forces and their corresponding forms of loss of stability.

Keywords: Boundary Element Method, Core System, Critical Force, Mechanics, Stability

1. INTRODUCTION

Elements of load-bearing structures of ships and port structures are loaded with significant external loads. These loads cause a state of compression both in all elements and in individual parts of structures. The danger of such a state lies in the interaction of the loss of stability and the transition of the structure from one equilibrium state to another. Note that in this case, external loads can be significantly less than the limiting ones. Therefore, for the safe operation of such structures, it is necessary to diagnose the spectrum of critical forces at which stability loss occurs. This diagnostics can be carried out when analyzing the stability models of elastic systems, where to increase the accuracy, the shift and inertia of rotation of rectilinear cores are additionally taken into account.

All loads on elastic systems can be conventionally divided into conservative and non-conservative ones. Conservative loads include so-called "dead" forces, when their line of action moves along with the construction only in parallel with the original direction. This can not be said about non-conservative forces. Systems with non-conservative forces are widely used in the life of modern society. Such systems include systems with internal energy sources, i.e. rockets, planes, space orbital stations, drill rigs and platforms, automobiles, ships, submarines, turbines, internal combustion engines, metal cutting machines, various cranes, instruments, etc.

If conservative stability problems can be solved by a static method, non-conservative problems are solved only by the dynamic method [1]. The main element of the dynamic method is the solution of the Cauchy problem for transverse oscillations of the core, taking into account the longitudinal force. In contrast to the statistical method, the critical force in the dynamic method is determined at the point where become equal (merging) two neighboring frequencies of the eigen oscillations.

For this purpose, the calculation of the entered the initial value reduces the force, and the frequencies (at least two) of the eigen oscillations are fixed. Further, the value of compression force increases and the frequency variation is monitored. The process continues until the two neighboring frequencies become equal with certain accuracy. The value of compression force in this case will be critical.

The necessity of applying the dynamic method greatly complicates the decision of non-conservative stability problems. This requires a very effective method for determining the frequencies of the eigen oscillations. Among other methods, in this respect, the IPE is highlighted. It allows to obtain the exact spectrum of frequencies (eliminates the shortcoming of the MCE), and in the transcendental frequency equation there are no break points of the 2nd kind (eliminating the lack of the method of displacement). Additional positive factors are the simple logic of the formation of a dynamic matrix of stability, the absence of operations of multiplication, circulation and addition of matrices, good stability of numerical operations in calculating a determinant and etc.

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2. ANALYSIS OF RECENT RESEARCH AND PUBLICATIONS

There are no publications on diagnostics of the spectrum of critical forces of non-conservative stability problems taking into account shear and inertia of rotation.

Equation (1) does not take into account the deformation of the shear and the inertia of the rotation under oscillations [1-5]:

$$\begin{bmatrix} EIv(x) \\ EI\varphi(x) \\ M(x) \\ Q(x) \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & -A_{13} & -A_{14} \\ A_{21} & A_{11} & -A_{23} & -A_{13} \\ -A_{31} & -A_{21} & A_{33} & A_{23} \\ -A_{41} & -A_{31} & A_{43} & A_{33} \end{bmatrix} \begin{bmatrix} EIv(0) \\ EI\varphi(0) \\ M(0) \\ Q(0) \end{bmatrix} + \int_0^x \begin{bmatrix} A_{14}(x-\xi) \\ A_{13}(x-\xi) \\ -A_{23}(x-\xi) \\ -A_{33}(x-\xi) \end{bmatrix} q_y(\xi)d\xi, \quad (1)$$

Therefore, they describe well enough the transverse oscillations of the core with a large length-to-cell ratio (ℓ / h > 10) and at low frequencies. However, for framework systems of foundations for heavy adjacent to the port or shipyard equipment and similar structures, when $\ell_{пв}/nh$ < 6, where n - number of tones of oscillations; h - characteristic cross-sectional dimension; $\ell_{пв}$ – is the length of the half-wave of the elastic line of the core, it is already necessary to take into account the shear and the inertia of rotation [3-6]. The problem of constructing more precise solutions for transverse oscillations of the core is also very relevant in the theory of stability in connection with the application of the dynamic method. The differential equation of the transverse oscillations of a straight-line core, taking into account the deformations of the shear and the inertia of rotation, was brought out by a prominent compatriot, a scientist prof. S.P. Tymoshenko [1]. His model is now proved as the most accurate and widely used in various problems of constructions mechanics [7-9]. For the application of the model of S.P. Tymoshenko in the problem of stability, needs to be supplemented its longitudinal force Fx. For this purpose, examined the rod, compressed guardian force F1 and the force of the F2, with a fixed line action (Fig. 1).

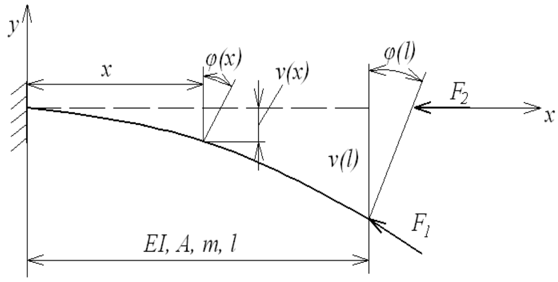

Fig. 1. The core, compressed guardian force and force with the fixed line of action

3. PURPOSE OF THE WORK

The purpose of the work is to diagnose the spectrum of critical forces based on the analysis of models of transverse vibrations of structural members based on the properties of Timoshenko beams, where it is possible to additionally take into account the shift and inertia of rotation. More precise mathematical models that allow you to specify the values of critical force, it was very important in practical terms for various engineering constructions and machines.

4. SUMMARY OF THE BASIC MATERIAL

We perform the diagnostics of the spectrum of critical forces using the numerical-analytical method of boundary elements [1]. This method allows you to create models of stability of rod, plate and shell structures, which are structural elements of port and ship structures.

Let us imagine non-conservative combinations of stability, when in one construction there is a combination of different variants of the behavior of compressive forces. As an example, let us consider the definition of the critical forces of the free frame in combination of guardian force with different variants of the behavior of compressive forces.

Suppose that a monitoring force is applied to the frame node 1 (Fig. 2), and in the node 2 is a force with a fixed action line. In this case, this will be a combination of non-conservative the problems of M. Beck and V.I Reuta.

Equations of equilibrium of node 2 will take the form:

$$M^{4-2}_{(\ell)} = M^{2-1}_{(0)} + F\left[v^{4-2}_{(\ell)} = -v^{1-3}_{(0)}\right];$$
$$Q^{4-2}_{(\ell)} = -N^{2-1}_{(0)} + F\left[\varphi^{4-2}_{(\ell)} = \varphi^{1-3}_{(0)}\right];$$
$$N^{4-2}_{(\ell)} = Q^{2-1}_{(0)}. \quad (2)$$

As a result, in the dynamic stability matrix A, two compensating elements A (13,16) = F / EI and A (14,7) = -F / EI are added, that is, there is a variable topology, due to 2 elements. By varying the parameter F of the rods 1-3 and 4-2 (in the matrix A, for the rod 4-2, we need to use the block of fundamental functions of equation (1)), we fix the changes of the frame frequencies. The graphs ωi = f (F) are shown in Fig. 3.

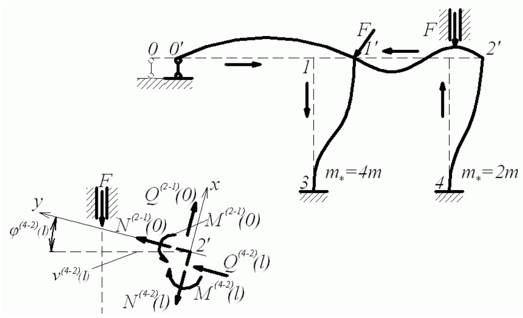

Fig. 2. Scheme of the frame

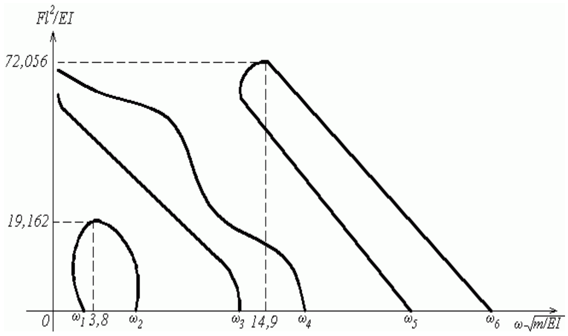

Fig. 3. The graphs ωi = f (F)

In this case, the frame first enters the flutter and F1 = 19,162EI / ℓ2 at $\omega = 3{,}8\sqrt{EI/m}$, and then the type of loss of stability takes place in Euler , then the flutter comes to the second time with F2 = 72,056EI / ℓ2, etc. It can be seen that two non-conservative forces significantly reduce the first critical force (F1 = 121,78EI / ℓ2), and F1 = 19,162EI / ℓ2 is only 2.5 times the first critical force at dead forces (FE1 = 7,66EI / ℓ2; FЭ2 = 27,35EI / ℓ2).

If, in the frame node 2, force is applied with a line of action passing through a fixed point, then its equilibrium equations will appear as:

$$M_{(\ell)}^{4-2} = M_{(0)}^{2-1}\ ;\ Q_{(\ell)}^{4-2} = -N_{(0)}^{2-1} + F\left[v_{(\ell)}^{4-2}/r + \varphi_{(\ell)}^{4-2}\right];$$

$$N_{(\ell)}^{4-2} = Q_{(0)}^{2-1}\ . \qquad (3)$$

Then the compensating elements A (13,16) = 0; A (14,16) = F / rEI. The remaining nonzero elements of the matrix A are unchanged. The study of the behavior of frequencies showed that they all tend to zero, each separately, i.e. Only the Euler type is the loss of stability.

Applying in the node 2 the "dead" force F, we obtain the equations of its equilibrium:

$$M_{(\ell)}^{4-2} = M_{(0)}^{2-1}\ ;\ Q_{(\ell)}^{4-2} = -N_{(0)}^{2-1} + F\varphi_{(\ell)}^{4-2}\ ;$$

$$N_{(\ell)}^{4-2} = Q_{(0)}^{2-1}\ . \qquad (4)$$

In comparison with the previous case, A (14,16) = 0. In this case, the frequencies of the eigen oscillations of the frame (each individually) tend to zero. Consequently, the combination in an elastic system of non-conservative and conservative forces, when the parameter F grows proportionally, does not lead to a flutter or divergence.

The MGE can solve even more complicated non-conservative stability problems, which are described by differential equations with variable coefficients. Such tasks are encountered in aircraft and rocket construction, when the variables are rigidity, core mass, or longitudinal compressive force. In this case, the core is sampled into separate parts, within the limits of which a correct differential equation with constant coefficients is considered, i.e. a system with distributed parameters is replaced by a set of systems with constant parameters. The following is an analysis of the behavior of the frequencies of the eigen oscillations of the sample system.

Application of the model S.P. Tymoshenko

More precise solutions of differential equations open up new possibilities when solving various problems, including stability problems. In the case of non-conservative stability problems of a rectilinear core, it can be noted that the problems of M. Beck and V.I. Reuta is sufficiently well investigated only on the basis of approximate solutions (1). The desire to clarify the existing results led to the appearance of works [1, 7, 8–11] where the model of S.P. Tymoshenko was used. In these papers only the problem of M. Beck was studied, and in an incomplete measure. In this connection, scientific and practical interest raises a more complete and detailed solution of non-conservative problems, which we will consider in a combined form (Fig. 1).

Consider the simultaneous action of forces F1, F2. The linear boundary conditions of this problem are quite simple:

$$EIv(0) = EI\varphi(0) = 0\ ;\ M(\ell) = F_2 v(\ell)\ ;$$

$$Q(\ell) = F_2\varphi(\ell)\ . \qquad (5)$$

For x = ℓ and given boundary conditions, equation (1) is reduced to the form (B = 0).

1	$-1 + a_4 A_{13}/EI$	$b_4 A_{14}/EI$	$-A_{13}$	$-A_{14}$	$EIv(\ell)$	
2	$a_4 A_{23}/EI$	$-1 + b_4 A_{24}/EI$	$-A_{23}$	$-A_{24}$	$EI\varphi(\ell)$	$= 0$
3	$\left[-F_2 + a_4(A_{33} - 1)\right]/EI$	$b_4 A_{34}/EI$	A_{33}	A_{34}	$M(0)$	
4	$a_4 A_{43}/EI$	$\left[-F_2 + b_4(A_{44} - 1)\right]/EI$	A_{43}	A_{44}	$Q(0)$	

For F2 = 0, the equation $\left|A_*\ (\omega, F_x)\right|$ = 0= 0 represents the task of M. Beck , while F1 = 0, the problem of V.I. Reut based on the model of S.P. Tymoshenko, i.e. Additionally, the shear, the inertia of the rotation and the deformed condition of the rod are taken into account. Determining the method of sequential selection of the roots of the equation and the coordinates of the points of the fusion of the first two frequencies, one can find the critical forces of various non-conservative problems stability. The results obtained are summarized in Table. 1.

Table 1. The critical forces of various non-conservative problems stability

Problems stability	The coordinates of the points of merging the first two bands	The relationship the height to the width of the cross section *h/b*; *A= b h* = 0,01 m²			
		1,0	2,0	3,0	4,0
M. Beck $F_1 = F$; $F_2 = 0$	$\frac{F_g\ell^2/EI}{\omega\ell^2\sqrt{m/EI}}$	20,57 / 11,35	20,33 / 11,05	20,25 / 10,099	20,21 / 10,09
	$\frac{F_z\ell^2/EI}{\omega\ell^2\sqrt{m/EI}}$	20,99 / 10,68	20,52 / 10,95	20,37 / 10,99	20,30 / 10,96
V.I.Reut $F_1 = 0$; $F_2 = F$	$\frac{F_g\ell^2/EI}{\omega\ell^2\sqrt{m/EI}}$	9,12 / 16,02	9,31 / 16,52	9,38 / 16,66	9,42 / 16,70
	$\frac{F_z\ell^2/EI}{\omega\ell^2\sqrt{m/EI}}$	19,34 / 12,02	19,72 / 11,42	19,85 / 11,34	19,91 / 11,22
Combined $F_1 = F$; $F_2 = F$; $F_g = 2F$	$\frac{F_g\ell^2/EI}{\omega\ell^2\sqrt{m/EI}}$	12,62 / 14,69	12,76 / 15,11	12,81 / 15,15	12,83 / 15,10

$$2r^2 = \frac{I\omega^2 m(E+kG) + (\rho I\omega^2 + kAG)F_x}{EI(kAG+F_x)};$$

$$s^4 = \frac{\omega^2 m(\rho I\omega^2 - kAG)}{EI(kAG+F_x)};$$

$$q_y(x) = \frac{kAG - \rho I\omega^2}{kAG+F_x} q(x) - \frac{EI}{kAG+F_x} q''(x);$$

$$a_1 = 1 + \frac{F_x}{kAG};\ a_2 = \frac{m\omega^2}{kAG} + \frac{F_x}{EI};\ a_3 = \frac{1}{kAG};$$

$$a_4 = \frac{F_1}{EI}$$

$$b_1 = \frac{kAG+F_x}{kAG-\rho I\omega^2};$$

$$b_2 = \frac{I(Em + kAG\rho)\omega^2 + (kAG+\rho I\omega^2)F_x}{EI(kAG-\rho I\omega^2)};$$

$$b_3 = \frac{1}{kAG-\rho I\omega^2};$$

$$b_4 = \frac{F_1\rho I\omega^2}{EI(kAG-\rho I\omega^2)}. \qquad (6)$$

If the longitudinal forces (F1 = F2 = 0) are not taken into account in coefficients a1 - a4, b1-b4 of expressions (3), then equation (2) will describe the model of the hard rod, when the maximum deflections lie within (1/100 - 1 / 1000) ℓ. For large deflections, the longitudinal forces F1, F2 influence the bending moment and transverse force. In this regard, in Table 1, the critical forces are given on two core models - rigid (Fz) and conditionally flexible (Fg), as well as at different ratios of height and width of the section. The area of the section A = b h = 0.01 m2 at the same time did not change. Table data 1 allow to make a number of interesting conclusions.

Problem of M.Beck. The shift of the shift, the inertia of the rotation and the deformed state of the core slightly increase the critical force. In the rigid model with ℓ / h = 10, the refinement is 4.69%, while the flexible is 2.59%. Changing the ratio h / b has little effect on the magnitude of the critical force.

Problem of V.I.Reuta. The flexible model results in a significant reduction in critical force (2,12 times) compared to a rigid model. Thus, a force with a fixed line of action is more dangerous than the angle of rotation of force.

Combined problem. The joint action of the forces F1 and F2 leads to a greater critical force than the case of the action of one force F2, which is not possible with conservative compressive forces. In a rigid model, all frequencies tend to zero, i.e. a certain combination of non-conservative forces can lead to conservative problems.

Let us consider the problem when the free core is loaded at the boundary points by the forces F1 and F2 (Fig. 4).

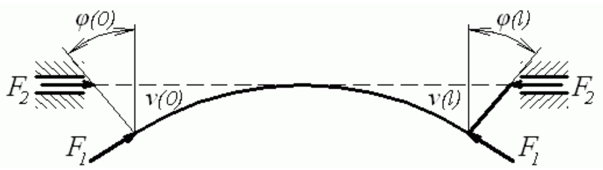

Fig. 4. Free core loaded at the boundary points

Let us consider the problem when the free core is loaded at the boundary points by the forces F1 and F2 (Fig. 4).

The boundary conditions of this problem:

$$M(0) = F_2 v(0);\ Q(0) = F_2\varphi(0); \qquad (7)$$

$$M(\ell) = F_2 v(\ell);\ Q(\ell) = F_2\varphi(\ell), \qquad (8)$$

We give the matrix of stability of the form:

$$\boldsymbol{A}_* = \begin{array}{c|c|c|} 1 & A_{11} - F_2A_{13}/EI & A_{12} - F_2A_{14}/EI \\ 2 & A_{21} - F_2A_{23}/EI & A_{22} - F_2A_{24}/EI \\ 3 & -A_{31} + F_2A_{33}/EI & -A_{32} + F_2A_{34}/EI \\ 4 & -A_{41} + F_2A_{43}/EI & -A_{42} + F_2A_{44}/EI \end{array}$$

(9)

$$\begin{array}{|c|c|c} -1 + a_4A_{13}/EI & b_2A_{14}/EI & 3 \\ a_4A_{23}/EI & -1 + b_2A_{24}/EI & 4 \\ \left[-F_2 + a_4(A_{33}-1)\right]/EI & b_2A_{34}/EI & 1 \\ a_4A_{43}/EI & \left[-F_2 + b_2(A_{44}-1)\right]/EI & 2 \end{array}$$

To exclude the zero leading elements of this matrix (in the hard model), its rows need to be rearranged in a new order, as shown by the numbers on the right. The critical forces of this problem for a square cross section and ℓ / h = 10 assume the values:

$$F_1 = 0;\ F_2 = F$$

$$F_g = 1{,}982EI/\ell^2 \text{ at } \omega = 2{,}87\sqrt{EI/m};$$

$$F_z = 30{,}88EI/\ell^2 \text{ at } \omega = 9{,}48\sqrt{EI/m}, \qquad (10)$$

i.e. for a free core the ratio of the critical forces of rigid and flexible models sharply increases in comparison with the console core,

$F_1=F_2=F;\ F_g=3{,}028 EI/\ell^2$ at $\omega=2{,}67\sqrt{EI/m}$.(11)

The remaining cases of the action of the compressive forces in Fig.4 lead to conservative problems.

Let's consider the case - a console core with a discrete arrangement of forces F1 and F2.

For definiteness, we assume that one force is applied in the middle of the span, the other at the free end. The core is sampled into two parts, where the arrows indicate their beginning and end (Fig. 5).

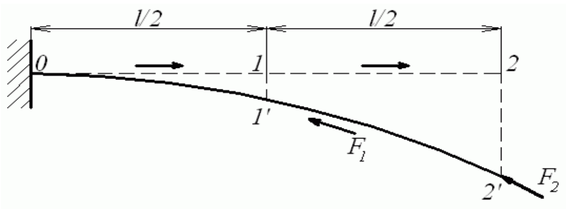

Fig. 5. Console core with a discrete force arrangement

$X_*=$ (12)

1	$EIv^{0-1}(0)=0$; $EIv^{1-2}(\ell)$
2	$EI\varphi^{0-1}(0)=0$; $EI\varphi^{1-2}(\ell)$
3	$M^{0-1}(0)$
4	$Q^{0-1}(0)$
5	$EIv^{1-2}(0)$
6	$EI\varphi^{1-2}(0)$
7	$M^{1-2}(0)$
8	$Q^{1-2}(0)$

$Y=$

1	$EI\left[v^{0-1}_{(\ell)}=v^{1-2}_{(0)}\right]-a_4A^{0-1}_{13}\cdot v^{1-2}_{(0)}-b_2A^{0-1}_{14}\cdot\varphi^{1-2}_{(0)}$
2	$EI\left[\varphi^{0-1}_{(\ell)}=\varphi^{1-2}_{(0)}\right]-a_4A^{0-1}_{23}\cdot v^{1-2}_{(0)}-b_2A^{0-1}_{24}\cdot\varphi^{1-2}_{(0)}$
3	$\left[M^{0-1}_{(\ell)}=M^{1-2}_{(0)}\right]-a_4\left(A^{0-1}_{33}-1\right)\cdot v^{1-2}_{(0)}-b_2A^{0-1}_{34}\cdot\varphi^{1-2}_{(0)}$
4	$\left[Q^{0-1}_{(\ell)}=Q^{1-2}_{(0)}\right]-a_4A^{0-1}_{43}\cdot v^{1-2}_{(0)}-b_2\left(A^{0-1}_{44}-1\right)\cdot\varphi^{1-2}_{(0)}$
5	$EIv^{1-2}_{(\ell)}-a_4A^{1-2}_{13}\cdot v^{1-2}_{(\ell)}-b_2A^{1-2}_{14}\cdot\varphi^{1-2}_{(\ell)}$
6	$EI\varphi^{1-2}_{(\ell)}-a_4A^{1-2}_{23}\cdot v^{1-2}_{(\ell)}-b_2A^{1-2}_{24}\cdot\varphi^{1-2}_{(\ell)}$
7	$\left[M^{1-2}_{(\ell)}=0\right]-a_4\left(A^{1-2}_{33}-1\right)\cdot v^{1-2}_{(\ell)}-b_2A^{1-2}_{34}\cdot\varphi^{1-2}_{(\ell)}$
8	$\left[Q^{1-2}_{(\ell)}=0\right]-a_4A^{1-2}_{43}\cdot v^{1-2}_{(\ell)}-b_2\left(A^{1-2}_{44}-1\right)\cdot\varphi^{1-2}_{(\ell)}$

After transferring the parameters from Y to X *, the frequency equation will take the form of the matrix 8X8.

The compensating elements of the matrix are determined by the expressions:

core 0-1

$$A^*_{15}=-1+a_4A_{13}/EI\ ;\ A^*_{16}=b_2A_{14}/EI\ ;$$

$$A^*_{25}=a_4A_{23}/EI\ ;\ A^*_{26}=-1+b_2A_{24}/EI\ ;$$

$$A^*_{35}=a_4\left(A_{33}-1\right)/EI\ ;\ A^*_{36}=b_2A_{34}/EI\ ;$$

$$A^*_{37}=-1\ ;\ A^*_{45}=b_2A_{43}/EI\ ;$$

$$A^*_{46}=b_2\left(A_{44}-1\right)/EI\ ;\ A^*_{48}=-1\ ; \qquad (13)$$

core 1-2

$$A^*_{51}=-1+a_4A_{13}/EI\ ;\ A^*_{52}=b_2A_{14}/EI\ ;$$

$$A^*_{61}=a_4A_{23}/EI\ ;\ A^*_{62}=-1+b_2A_{24}/EI\ ;$$

$$A^*_{71}=a_4\left(A_{33}-1\right)/EI\ ;\ A^*_{72}=b_2A_{34}/EI\ ;$$

$$A^*_{81}=a_4\,A_{43}/EI\ ;\quad A^*_{82}=b_2\left(A_{44}-1\right)/EI\ . \qquad (14)$$

Let's consider the problem when the rod is compressed by two forces F1 (Fig. 5) or a square cross section and ℓ / h = 10 it follows that

$$F_g=13{,}79EI/\ell^2 \text{ at } \omega=10{,}08\sqrt{EI/m}\ ;$$

$$F_z=15{,}02EI/\ell^2 \text{ at } \omega=10{,}62\sqrt{EI/m}\ . \qquad (15)$$

The core is compressed by two forces F2.

In this case, the compensating elements of the matrix will change:

core 0-1

$$A^*_{15}=-1;\ A^*_{16}=A^*_{25}=A^*_{36}=A^*_{45}=0;\ A^*_{26}=-1;$$

$$A^*_{35}=-F_2/EI\ ;\ A^*_{37}=-1;\quad A^*_{46}=-F_2/EI\ ;$$

$$A^*_{48}=-1; \qquad (16)$$

core 1-2

$$A^*_{51}=-1;\ A^*_{52}=A^*_{61}=A^*_{72}=A^*_{81}=0;\ A^*_{62}=-1;$$

$$A^*_{71}=-F_2/EI\ ;\quad A^*_{82}=-F_2/EI\ . \qquad (17)$$

The critical forces of this problem will be equal

$$F_g=6{,}60EI/\ell^2 \text{ at } \omega=14{,}55\sqrt{EI/m}\ ;$$

$$F_z=13{,}11EI/\ell^2 \text{ at } \omega=12{,}02\sqrt{EI/m}\ . \qquad (18)$$

The core is compressed at point 1 by force F1, at point 2 by force F2.

The compensating elements of the matrix will be equal to the expressions (10) for the core 0-1 and the expressions (11) for the core 1-2.

Critical force can only be determined for a flexible model:

$$F_g=7{,}68EI/\ell^2 \text{ at } \omega=14{,}55\sqrt{EI/m}\ . \qquad (19)$$

The core is compressed at point 1 by force F2, at point 2 by force F1.

Critical force is determined only for a flexible model

$$F_g=10{,}57EI/\ell^2 \text{ at } \omega=11{,}82\sqrt{EI/m}\ . \qquad (20)$$

For comparison, let us quote the critical force with two dead forces:

$$F_1=2{,}067EI/\ell^2\ . \qquad (21)$$

From the presented results it follows that in the combined problems the reduction of critical forces in various degrees is observed in comparison with the problems of M. Beck and V.I. Reuta.

5. CONCLUSIONS

The models of stability of cores and core systems are presented in the paper based on more precise equations of transverse oscillations of Professor Timoshenko, where the shift and inertia of rotation are additionally taken into account. The highly effective method of investigation - the numerical-analytic method of elemental elements developed by the authors - was used to obtain a number of new results on the behavior of elastic rods and structures under the action of compressive non-conservative forces. The critical strengths of the reengineering tasks are high accuracy and worthwhile, which is important in the scientific and practical terms.

Thus, we can conclude that the problems of diagnostics of the spectra of critical forces of mechanical systems have been solved with a high degree of reliability and accuracy.

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