

UDC 621.431.74

DOI: 10.15587/1729-4061.2021.248960

# DETERMINATION OF OPTIMAL CONTROL OF A VESSEL DIESEL ENGINE DURING NON-STATIONARY TRAFFIC REGIMES

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The high pressure fuel system is the fundamental system that forms the indicator of the minimum fuel consumption per unit of the vessel's path.

The calculation of the optimal control of the vessel complex with the main diesel engine is performed according to the criterion of the minimum fuel consumption per unit path at a given average velocity of the vessel.

The propulsion of a vessel with a main diesel engine is described by equations. The equations contain a significant number of parameters, the reduction of which is performed by introducing dimensionless quantities, followed by bringing the equations into dimensionless forms. This made it possible to present a solution to the optimal control law for the main vessel diesel engine as part of the vessel complex.

Optimal control of the vessel complex under stormy navigation conditions has been investigated. The calculations of the control law of the vessel complex, which ensure the movement of the vessel with the maximum average velocity in conditions of stormy navigation, are presented. It is determined that the established law of control of the vessel complex ensures the minimum fuel consumption per mile at a given average velocity of its movement. The influence of a high-pressure fuel system on the optimal control of a vessel diesel engine has been investigated.

Thus, the calculated studies indicate that for all values of the parameters of the vessel complex according to the law of control of the fuel system  $\Phi = a + b \cdot C_2(\tau)$ , they give fuel savings up to 6 % per unit of way in comparison with the law of control of the vessel complex  $\Phi = a + b \cdot (c_1(\tau) / c_2(\tau))$ .

The obtained ratios during modeling and optimal control of the main diesel engine of the vessel complex allow using the dynamic programming method to analyze the fuel consumption per unit path with optimal control compared to the corresponding constant control

**Keywords:** vessel complex, vessel diesel engine control, optimality criteria, maximum average velocity

Received date 12.10.2021

Accepted date 01.12.2021

Published date 29.12.2021

**How to Cite:** Usov, A., Slobodianiuk, M., Nikolskyi, M. (2021). Determination of optimal control of a vessel diesel engine during non-stationary traffic regimes. *Eastern-European Journal of Enterprise Technologies*, 6 (10 (114)), 136–147.

doi: <https://doi.org/10.15587/1729-4061.2021.248960>

## 1. Introduction

The fuel consumption of a marine diesel engine is completely related to the resistance to movement of the vessel and its hydrodynamics. The rational fuel consumption of a marine diesel engine is ensured by an efficient fuel supply operation.

Investigations of the operating modes of a marine diesel engine, its systems and components during storm sailing and control commands are carried out directly on diesel engines or by the simulation method [1, 2].

The simulation method is used in studies of the high-pressure fuel supply system of a marine diesel engine, taking into account the influence of weather conditions, the inertia of the vessel complex [3, 4], wind-wave characteristics [5, 6], etc. This makes it possible to obtain theoretical calculations of fuel consumption, optimal average velocity and other characteristics of non-stationary operating modes.

The high pressure fuel supply system plays a significant role in the operation of a marine diesel engine, forms economic and energy indicators. Research into diesel engine fuel systems is based on the development of new high-pressure elements, or improvements to existing ones. The introduction

of creative views to control its technical condition during operation.

A practice request is to determine the law of control of a marine diesel engine that would ensure the minimum fuel consumption per mile at a given average velocity.

Because the high pressure fuel supply system is an integral part of the vessel diesel engine control. It forms an indicator of the minimum fuel consumption per unit of the vessel's path, which is included in the calculation of the profitability of vessel transportation, these tests require further research and are relevant.

## 2. Literature review and problem statement

Modeling of the operation of vessel power plants is carried out with the aim of developing simulators for training service personnel and research, which are associated with the multi-criteria of tasks for optimizing the vessel's propulsion system.

In [7], using the reverse control over the nonlinearity in the distribution of fuel by high-pressure elements, it is not possible to achieve optimal observation of the disturbance. In [8], the

authors investigated the dynamic processes and the influence of nonlinearity arising during injection. The results of the study indicate that modeling of the high-pressure fuel system should be performed on its individual elements. That is, high pressure fuel pump, pipeline, fuel injector, etc. But the proposed models have a correspondingly small number of variations, and therefore, when the initial conditions change, the further interaction of the elements requires additional refinements in the calculations. In addition, nonlinear disturbances arise. Therefore, there is a need to model a marine diesel engine in the general complex “engine-propeller-hull”. This approach was used in [9], but the authors did not consider the development of a control system for a marine diesel engine. In [10], the authors carried out a study of modeling a diesel engine with systems, but the issues of optimizing the operation of the complex and its control were not considered. In [11], the authors proposed a semi-empirical model for measuring the loss of velocity by a vessel when resisting sea waves. But the lost velocity must be compensated for by additional fuel injection. An inertial component arises between the acting force of the waves and the compensating actions of the engine. Therefore, appropriate control is required. In work [12], the authors highlight the issue of optimal control of a marine diesel engine, supplementing its work with an analysis as part of the hydrodynamic complex “hull-propeller shaft-diesel engine”. But the issue of managing the complex when weather conditions change, contributing to the appearance of regular and irregular waves in the marine environment, has not been considered. In [13], the author proposed a control system in which the operator selects the operating modes of the vessel's power plant. The automatic control system independently provides certain operating modes (control scenarios). In our opinion, the propulsion complex should be the main object of vessel control. In [14], the authors investigated the processes of multiphase fuel injection of a marine diesel engine. However, they did not explain the reasons for the emergence of heterogeneous forces on the propulsion complex of the vessel.

Taking into account the metrological conditions, changes in the draft of the vessel gives an estimate of the calculation of the velocity of movement of the vessel. This accounting satisfies the practical request for the optimal control of the main marine diesel engine, according to the criterion of the minimum fuel consumption per unit path, at a given average velocity of the vessel.

A review of the studies given in the literature shows that the condition of the vessel's hull, meteorological factors and draft affect the operation of the main vessel engine and economic performance, but do not correspond to the optimal use of the power plant fuel. It should be noted that adequate attention has not been paid to determining the influence of the high pressure fuel system on the control of the main marine engine.

Thus, the published materials partially solve the problem of optimal control of a marine diesel engine through the fuel system control and show the results of research in the indicated direction.

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### 3. The aim and objectives of research

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The aim of research is to ensure optimal control of a marine diesel engine according to the criterion of minimum fuel consumption per unit path at a given average velocity of the vessel under non-stationary modes of movement. This will reduce fuel consumption and make a profit when transporting cargo.

To achieve the set aim, it is necessary to solve the following objectives:

- to determine the effect of water resistance on the vessel in conditions of unsteady movement;
- to find the law of control of the vessel complex, which ensures the minimum fuel consumption per mile of track at a given average velocity of its steady motion;
- to investigate the influence of the high pressure fuel system on the optimal control of a marine diesel engine.

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### 4. Materials and methods of research

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To formulate the research problem, the following assumptions are considered. The problems are solved under the assumption of the regular nature of the excitement. In the equations to be solved, the effect of excitement on the operating mode of the power plant is taken into account by introducing periodic factors in front of the thrust and propeller torque coefficients, as well as an increase in the impedance coefficient of the water environment of the vessel's hull. It is assumed that the drag force at a given sea state is proportional to the square of the vessel's velocity, and the proportionality coefficient changes when the sea state changes. All decisions are made in relative terms.

The force of water resistance to the movement of the vessel in waves contains harmonic components of different tones. In principle, these components are obliged to cause a pulsation of the vessel's velocity. But the mass of vessels of medium and large displacement is so great that in real conditions these fluctuations are neglected. However, these components are not taken into account. When solving the problem, the averaged static, limiting, proportional and regulatory characteristics of the engines were used.

Problems are solved in dimensionless units. This makes it possible to use the results obtained on vessels of different types and classes. The method of determining the laws of optimal control is based on the averaging method, the maximum principle of L. Pontryagin and the method of dynamic programming by R. Bellman, based on the study of the entire set of optimal trajectories. Solution results are presented in the form of generalized diagrams and approximating functions. They make it possible to analyze the influence of waves on the operating parameters of the joint operation of the engine and the vessel's hull.

Due to the lack of information about the predicted ratios of the duration of non-stationary (normal) modes of vessel movement in the literature, as well as their uncertainty, this parameter is not taken into account in the problems.

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### 5. Calculation of optimal control of a vessel diesel engine according to the criterion of minimum fuel consumption

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#### 5.1. Determination of the effect of water resistance on the vessel in conditions of unsteady motion

If the diesel engine works directly and the vessel is moving on a straight course, then the propeller angular velocity and the vessel's linear velocity  $v$  are determined by the equations:

$$\begin{cases} I \frac{d\omega}{dt} = M_p - M_c, \\ m \frac{dv}{dt} = zP - R, \end{cases} \quad (1)$$

where  $I$  – moment of inertia of the masses of the moving links of the engine and transmission and the mass of the propel-

ler (including the mass of water), reduced to the propeller shaft,  $I = \text{const}$ ;  $t$  – time;  $M_p$  – driving moment reduced to the propeller shaft (taking into account the friction losses in the shaft line supports);  $M_c$  – the moment of resistance of water to the rotation of the screw;  $m$  – the mass of the vessel, including the added mass of water;  $zP$  and  $R$  – useful thrust from all propellers and the force of water resistance to the movement of the vessel's hull.

When determining the driving moment  $M_p$ , static features can be used. In this case

$$M_p = M_p(\omega, \xi),$$

where  $\xi$  – the relative cyclic fuel supply.

On calm water, the moment of resistance  $M_c$  and the propeller stop  $P$  are equal

$$\begin{cases} M_c = K_2 \frac{\rho D_s^5}{2\pi} \omega^2, \\ P = K_1 (1 - t_1) \frac{\rho D_s^5}{2\pi} \omega^2, \end{cases} \quad (2)$$

where  $K_2$  and  $K_1$  – the thrust and moment coefficients;  $\rho$  – the water density;  $D_g D_s$  – screw diameter;  $t_1$  – the suction coefficient.

Coefficients  $K_1, K_2$  can be approximately determined by the following relation vessels:

$$K_2 = K_{2\max} \left\{ \frac{\phi}{\phi_{\max}} \left[ \frac{\phi}{\phi_{\max}} \left( a_2 + b_2 \frac{\lambda}{\lambda_{2\max}} + \right) + c_2 \frac{\lambda^2}{\lambda_{2\max}^2} \right] + e_2 \frac{\lambda^2}{\lambda_{2\max}^2} \right\}, \quad (3)$$

$$K_1 = K_{1\max} \left[ \frac{\phi}{\phi_{\max}} \left( \bar{a}_1 + \bar{b}_1 \frac{\lambda}{\lambda_{1\max}} + \right) + \bar{c}_1 \frac{\lambda^2}{\lambda_{1\max}^2} \right] + \bar{e}_1 \frac{\lambda^2}{\lambda_{1\max}^2},$$

where  $K_{2\max}$  and  $K_{1\max}$  – the value of the torque coefficient and the stop coefficient on the mooring lines at the step ratio  $\phi$ , which is equal to its maximum value  $\phi_{\max}$ ;  $\lambda, \lambda_{2\max}, \lambda_{1\max}$  – respectively the current value of the relative gait of the propeller, gait of zero moment and gait of zero stop,

$\lambda = \frac{v_1 (1 - \vartheta) \pi}{D_s \omega}$ ;  $\vartheta$  – coefficient of the associated flow;  $a_2,$

$b_2, c_2, e_2, \bar{a}_1, \bar{b}_1, \bar{c}_1, \bar{e}_1$  – the coefficients that have become in separate sections of the change in  $\lambda$  and  $\phi$ .

If to take into account only the main component of the perturbation of the moment of resistance and stop on regular waves – due to a change in the immersion  $h$  of the screw axis under water, then the moment of resistance and stop of the screw can be taken equal to

$$\begin{aligned} M_c &= M_c \cdot C_2(t), \\ \bar{P} &= P \cdot C_1(t), \end{aligned} \quad (4)$$

where

$$C_2 = 1 + \beta(t),$$

$$C_1 = 1 + 1.2\beta(t),$$

$$\beta(t) = \alpha(t) \frac{1 - \text{sign}\alpha(t)}{2},$$

$$\alpha(t) = \frac{h_0}{2.4R_s} - \frac{l\psi}{2.4R_s} \sin pt - 0.5,$$

$h_0$  – immersion of the propeller axis in calm water;  $l\psi$  and  $p$  – the amplitude and frequency of the rotor vibrations in the vertical plane;  $R_s$  – radius of the screw.

The force of water resistance to the movement of the vessel in waves in a small range of velocity variation can be considered proportional to the square of the velocity

$$R = k \cdot v^2, \quad (5)$$

where  $k$  – the coefficient established for a given sea state.

Substituting expressions (2)–(5) into equation (1), let's obtain

$$I \frac{d\omega}{dt} = M_p(\omega, \xi) - \frac{\rho D_s^5}{2\pi} C_2(t) K_{2\max} \times \left\{ \frac{\phi}{\phi_{\max}} \left[ \frac{\phi}{\phi_{\max}} \left( a_2 \omega^2 + b_2' \frac{v\omega}{\lambda_{2\max}} + \right) + c_2' \frac{v^2}{\lambda_{2\max}^2} \right] + e_2' \frac{v^2}{\lambda_{2\max}^2} \right\}, \quad (6)$$

$$m \frac{dv}{dt} = z \frac{\rho D_s^4}{(2\pi)^2} C_1(t) K_{1\max} \times \left[ \frac{\phi}{\phi_{\max}} \left( a_1 \omega^2 + b_1' \frac{v\omega}{\lambda_{1\max}} + \right) + \bar{c}_1 \frac{v^2}{\lambda_{1\max}^2} \right] - kv^2,$$

where

$$b_2' = b_2 \frac{(1 - \vartheta) \pi}{D_s},$$

$$c_2' = c_2 \frac{(1 - \vartheta)^2 \pi^2}{D_s^2},$$

$$e_2' = e_2 \frac{(1 - \vartheta)^2 \pi^2}{D_s^2},$$

$$a_1 = \bar{a}_1 (1 - t_1),$$

$$b_1' = \bar{b}_1 \frac{(1 - \vartheta)(1 - t_1) \pi}{D_s},$$

$$c_1' = \bar{c}_1 \frac{(1 - \vartheta)^2 (1 - t_1) \pi^2}{D_s^2},$$

$$e_1' = \bar{e}_1 \frac{(1 - \vartheta)^2 (1 - t_1) \pi^2}{D_s^2}.$$

Equations (6) contain a significant number of parameters  $I, \rho, D_s, K_{2\max}, a_i, b_i, c_i, e_i, m, z, K_{1\max}$ , and others. It is possible to reduce their number by entering dimensionless quantities.

**5.2. Control of the vessel complex with the determination of the minimum fuel consumption**

Let's refer the variables  $\omega, v, M_p, M_c, P, R$  to their values  $\omega_0, v_0, M_{p0}, M_{c0}, P_0, R_0$  in the initial mode – in calm water. Let's introduce dimensionless time

$$\tau = pt, \tag{7}$$

so that the perturbing functions  $C_2(\tau)$  and  $C_1(\tau)$  are  $2\pi$  periodic. Equation (6) in dimensionless form can be written as

$$p\omega_0 I \frac{d\omega}{d\tau} = \frac{M_p}{M_{p0}} M_{p0} - \frac{K_{2\max} \rho D_s^5 \omega^2}{(2\pi)^2} \times \left[ \frac{\Phi}{\Phi_{\max}} \left( \frac{\omega^2}{\omega_0^2} + b'_2 \frac{v}{v_0} \cdot \frac{\omega}{\omega_0} \cdot \frac{v_0}{\omega_0} \cdot \frac{1}{\lambda_{2\max}} + c'_2 \frac{v^2}{v_0^2} \cdot \frac{v_0^2}{\omega_0^2} \cdot \frac{1}{\lambda_{2\max}^2} + e'_2 \frac{v^2}{v_0^2} \cdot \frac{v_0^2}{\omega_0^2} \cdot \frac{1}{\lambda_{2\max}^2} \right) + \right] C_2(\tau), \tag{8}$$

$$pv_0 m \frac{dv}{d\tau} = zK_{1\max} \frac{\rho D_s^4 \omega_0^2}{(2\pi)^2} \times \left[ \frac{\Phi}{\Phi_{\max}} \left( a_1 \frac{\omega^2}{\omega_0^2} + b' \frac{v}{v_0} \cdot \frac{\omega}{\omega_0} \cdot \frac{v_0}{\omega_0} \cdot \frac{1}{\lambda_{1\max}} + c'_1 \frac{v^2}{v_0^2} \cdot \frac{v_0^2}{\omega_0^2} \cdot \frac{1}{\lambda_{1\max}^2} + e'_1 \frac{v^2}{v_0^2} \cdot \frac{v_0^2}{\omega_0^2} \cdot \frac{1}{\lambda_{1\max}^2} \right) + \right] \times C_1(\tau) - k_0 v_0 \frac{k}{k_0} \cdot \frac{v^2}{v_0^2},$$

where  $k_0$  – the value of the coefficient  $k$  in calm water. If to take into account that in the output mode  $M_p = M_{c0}$ ,  $zP_0 = R_0$  and put  $t_1 = \text{const}$ ,  $V = \text{const}$ , then from eq. (8) let's obtain

$$\frac{d\Omega}{d\tau} = \frac{N}{A} \left[ \frac{\Phi_0}{\Phi_{\max}} \left[ \frac{\Phi_0}{\Phi_{\max}} \left( a_2 + b_2 \frac{\lambda_0}{\lambda_{2\max}} + c_2 \frac{\lambda_0^2}{\lambda_{2\max}^2} \right) + e_2 \frac{\lambda_0^2}{\lambda_{2\max}^2} \right] M_p^0 - \Phi \left( a_2 \Omega^2 + b_2 \frac{\lambda_0}{\lambda_{2\max}} \Omega V + c_2 \frac{\lambda_0^2}{\lambda_{2\max}^2} V^2 \right) + e_2 \frac{\lambda_0^2}{\lambda_{2\max}^2} V^2 \right] C_2(\tau), \tag{9}$$

$$\frac{dV}{d\tau} = \frac{1}{A} \left[ \Phi \left( a_1 \Omega^2 + b_1 \frac{\lambda_0}{\lambda_{1\max}} \Omega V + c_1 \frac{\lambda_0^2}{\lambda_{1\max}^2} V^2 \right) + e_1 \frac{\lambda_0^2}{\lambda_{1\max}^2} V^2 \right] C_1(\tau) - \left[ \frac{\Phi_0}{\Phi_{\max}} \left( a_1 + b_1 \frac{\lambda_0}{\lambda_{1\max}} + c_1 \frac{\lambda_0^2}{\lambda_{1\max}^2} V^2 \right) + e_1 \frac{\lambda_0^2}{\lambda_{1\max}^2} V^2 \right] \chi V^2$$

$$\Omega(0) = 1, \quad V(0) = 1, \tag{10}$$

where  $\Omega = \frac{\omega}{\omega_0}$ ,  $N = \frac{M_\chi v_0 m}{z P_\chi \omega_0 I}$ ,  $A = \frac{p v_0 m}{z P_\chi}$ ,  $M_\chi = K_{2\max} \frac{\rho D_s^5}{(2\pi)^2} \omega_0^2$ ,  $P_\chi = K_{1\max} \frac{\rho D_s^4}{(2\pi)^2} \omega_0^2$ ,  $\tag{11}$

$\lambda_0$  and  $\phi_0$  – the values of the relative gait  $\lambda$  and the step angle  $\phi$  on calm water;

$$M_p^0 = \frac{M_p}{M_{p0}}, \quad \Phi = \frac{\phi}{\phi_{\max}}, \quad V = \frac{v}{v_0}, \quad a_1 = \bar{a}_1(1-t_1),$$

$$b_1 = \bar{b}_1(1-t_1), \quad c_1 = \bar{c}_1(1-t_1), \quad e_1 = \bar{e}_1(1-t_1), \quad \chi = \frac{k}{k_0}.$$

Let's denote

$$M_p = M_c^0(1, 1, \Phi_0) \cdot C_2(0) M_p^0(\Omega, \xi), \quad M_c = M_c^0 \cdot C_2(\tau), \tag{12}$$

$$M_c^0 = \Phi \left[ \Phi \left( a_2 \Omega^2 + b_2 \frac{\lambda_0}{\lambda_{2\max}} \Omega V + c_2 \frac{\lambda_0^2}{\lambda_{2\max}^2} V^2 \right) + e_2 \frac{\lambda_0^2}{\lambda_{2\max}^2} V^2 \right],$$

$$\tilde{P} = P^0 C_1(\tau),$$

$$P^0 = \Phi \left( a_1 \Omega^2 + b_1 \frac{\lambda_0}{\lambda_{1\max}} \Omega V + c_1 \frac{\lambda_0^2}{\lambda_{1\max}^2} V^2 \right) + e_1 \frac{\lambda_0^2}{\lambda_{1\max}^2} V^2.$$

Obviously

$$\frac{\Phi_0}{\Phi_{\max}} \left[ \frac{\Phi_0}{\Phi_{\max}} \left( a_2 + b_2 \frac{\lambda_0}{\lambda_{2\max}} + c_2 \frac{\lambda_0^2}{\lambda_{2\max}^2} \right) + e_2 \frac{\lambda_0^2}{\lambda_{2\max}^2} \right] = M_c^0(1, 1, \Phi_0),$$

$$\frac{\Phi_0}{\Phi_{\max}} \left( a_1 + b_1 \frac{\lambda_0}{\lambda_{1\max}} + c_1 \frac{\lambda_0^2}{\lambda_{1\max}^2} \right) + e_1 \frac{\lambda_0^2}{\lambda_{1\max}^2} = P^0(1, 1, \Phi_0),$$

$$\frac{\Phi_0}{\Phi_{\max}} \left( a_1 + b_1 \frac{\lambda_0}{\lambda_{1\max}} + c_1 \frac{\lambda_0^2}{\lambda_{1\max}^2} \right) + e_1 \frac{\lambda_0^2}{\lambda_{1\max}^2} = P^0(1, 1, \Phi_0),$$

where  $\Phi_0 = \Phi_0 / \Phi_{\min}$ .

Taking into account expressions (12), equations (9) will take the form:

$$\begin{aligned} \frac{d\Omega}{d\tau} &= \frac{N}{A} (M_p - M_c), \\ \frac{dV}{d\tau} &= \frac{1}{A} (\tilde{P} - \chi P^0(1, 1, \Phi_0) C_1(0) V^2). \end{aligned} \tag{13}$$

Moreover, here the relative driving moment  $M_p^0 = (\Omega, \xi)$  given by the restrictive, partial and regulatory characteristics, can be found from the following approximate dependencies:

$$M_p^0(\Omega, \xi) = \begin{cases} \left. \begin{aligned} &\xi \frac{1 + \left(\frac{\omega_p}{\omega_0} - \Omega\right) \psi}{1 + \left(\frac{\omega_p}{\omega_0} - 1\right) \psi} \text{ at } 0.4 \leq \Omega \leq \frac{\omega_p}{\omega_0}, \\ &\xi - 1 + \frac{\tilde{n} - \frac{\omega_0}{\omega_p} \Omega}{\tilde{n} - 1} \\ &1 + \left(\frac{\omega_p}{\omega_0} - 1\right) \psi \end{aligned} \right\} \frac{\omega_0}{\omega_p} < 1.0, \\ \left. \begin{aligned} &\text{at } \frac{\omega_p}{\omega_0} < \Omega \leq \frac{\xi(\tilde{n}-1)+1}{\frac{\omega_0}{\omega_p}}, \\ &0 \text{ at } \Omega > \frac{\xi(\tilde{n}-1)+1}{\frac{\omega_0}{\omega_p}}, \end{aligned} \right\} \\ \left. \begin{aligned} &\xi \frac{1 + \left(\frac{\omega_p}{\omega_0} - \Omega\right) \psi}{\tilde{n} - \frac{\omega_0}{\omega_p}} (\tilde{n} - 1) \\ &\text{at } 0 \leq \Omega \leq \frac{\omega_p}{\omega_0}, \\ &\frac{(\tilde{n}-1)\xi + \left(1 - \frac{\omega_0}{\omega_p} \Omega\right)}{c - \frac{\omega_0}{\omega_p}} \end{aligned} \right\} \frac{\omega_0}{\omega_p} \geq 1.0, \\ \left. \begin{aligned} &\text{at } \frac{\omega_p}{\omega_0} < \Omega \leq \frac{\xi(\tilde{n}-1)+1}{\frac{\omega_0}{\omega_p}}, \\ &0 \text{ at } \Omega > \frac{\xi(\tilde{n}-1)+1}{\frac{\omega_0}{\omega_p}}, \end{aligned} \right\} \end{cases}$$

where  $\omega_p$  – the angular velocity of the propeller shaft, on which the regulator adjustment element is installed;

$$\psi = \text{tg}\beta, \quad c = (2 + \delta_p) : (2 - \delta_p),$$

where  $\beta$  – the angle of inclination of the limiting characteristic (variable parameter);  $\delta_p$  – the degree of non-uniformity of the regulator at a given setting of the organ of its adjustment.

It is assumed that the friction loss in the shaft line supports in calm water and in waves are the same.

Equation (13) is the main one for determining the optimal control law for the power unit of the vessel complex.

### 5.3. Influence of the high pressure fuel system on the optimal control of a vessel diesel engine

When studying the issue of optimal control of the vessel complex, first of all, two variants of the problem should be considered, each of which will be considered in the future as an independent problem:

Problem I. Find the control law of the vessel complex that ensures (for a given value of the characteristics of a vessel diesel engine  $\xi$ ) the movement of a given vessel with the maximum average velocity in a steady state.

Problem II. Find the control law for the vessel complex, which provides (for a given value of the characteristics of a vessel diesel engine  $\xi$ ) the minimum fuel consumption per mile for a given average velocity of its steady motion.

Of course, these two problems do not exclude others that meet other optimality criteria.

To solve Problem II, it is necessary to choose a functional dependence for the fuel consumption per mile of the vessel's path.

The average (for the period T of the vessel's swing) fuel consumption per mile is equal to

$$\bar{B} = \frac{\int_0^{\tau} B dt}{\int_0^{\tau} v dt}, \tag{14}$$

where  $B = g_e \cdot N_e$ ;  $g_e$  – specific effective fuel consumption;  $N_e$  – the effective power of the motors.

Passing in expression (14) to relative values, there is

$$\bar{B}^0 = \frac{\int_0^{2\pi} B^0 d\tau}{\int_0^{2\pi} v d\tau}, \tag{15}$$

where  $\bar{B}^0 = \bar{B} \frac{v^0}{B_0}$ ,

$$\begin{aligned} B^0 &= \frac{g_e}{g_{e0}} \cdot \frac{N_e}{N_{e0}} = g_e^0 M_p^0(\Omega) \Omega, \\ g_e^0 &= \frac{g_e}{g_{e0}}, \quad M_p^0(\Omega) \Omega = \frac{N_e}{N_{e0}}, \end{aligned} \tag{16}$$

$B_0, N_{e0}$  and  $g_{e0}$  – the values of  $B, N_e$  and  $g_e$  in calm water.

Specific effective fuel consumption  $g_e$  depends on many parameters: average effective pressure, shaft velocity, air pressure and temperature at the engine inlet, gas pressure at the outlet, etc. For given external conditions, the main parameter that determines the value of  $g_e$  is the effective engine power or the product of the average effective pressure and the number of shaft revolutions.

Fig. 1, *a* shows typical dependences  $g_e/g_{en}=f(N_e/N_{en})$ , (where the index *n* indicates the nominal mode) for some marine diesel engines:

- I – 8DP 43/61 ( $N_{en}=2000$  HP at  $n=250$  rpm);
- II – 1274-VTBF-160 ( $N_{en}=15000$  HP at  $n=120$  rpm);
- III – 9IEC 75/150 ( $N_{en}=12000$  HP at  $n=120$  rpm).

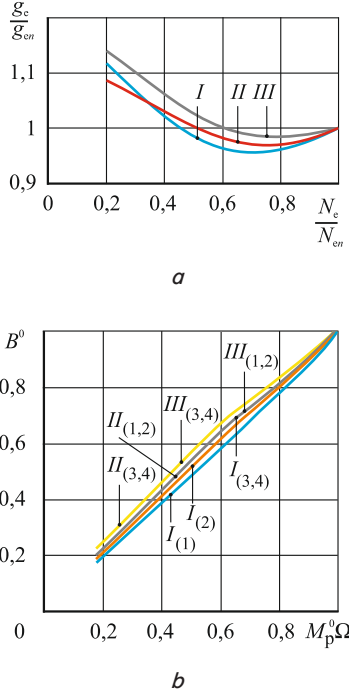


Fig. 1. Dependencies and corresponding functions  $B^0$  for marine diesel engines: *a* – typical dependences  $g_e/g_{en}=f(N_e/N_{en})$ ; *b* – constructed functions  $B^0 = B^0(M_p^0\Omega)$  for different values of the initial power (designation of curves in the text)

The nature of the dependence  $g_e/g_{en}=f(N_e/N_{en})$ , contained in expression (16), is determined not only by the type of the engine itself, but also by the power value  $N_{e0}$  in the initial mode.

In Fig. 1, *b* according to dependences I, II, III Fig. 1, constructed functions  $B^0 = B^0(M_p^0\Omega)$  for different values of the initial power  $N_{e0}$  are constructed:

- 1 –  $\frac{N_{e0}}{N_{en}} = 1.0$ ;
- 2 –  $\frac{N_{e0}}{N_{en}} = 0.75$ ;
- 3 –  $\frac{N_{e0}}{N_{en}} = 0.85$ ;
- 4 –  $\frac{N_{e0}}{N_{en}} = 0.65$ .

As seen from Fig. 1, the change in power within real limits is not strongly reflected in the law of change in the relative fuel consumption. Therefore, in the study, the value can be set by curve II and set  $N_{e0}=N_{en}$ .

Problems I, II can be formulated as follows:

Problem I. For a given value of  $\xi$ , find the power unit control law that provides

$$\max_{0 \leq \Phi(\tau) \leq 1} \frac{1}{2\pi} \int_0^{2\pi} V(\tau) d\tau,$$

so

$$\max_{0 \leq \Phi(\tau) \leq 1} \bar{V}, \tag{17}$$

with the satisfaction of equations (13) and the limiting conditions

$$\Omega(0)=\Omega(2\pi), V(0)=V(2\pi), \tag{18}$$

where

$$\bar{V} = \frac{1}{2\pi} \int_0^{2\pi} V(\tau) d\tau. \tag{19}$$

Problem II. For a given value of  $\xi$ , find the power unit control law that provides

$$\min_{0 \leq \Phi(\tau) \leq 1} \frac{\int_0^{2\pi} B^0 d\tau}{\int_0^{2\pi} V d\tau},$$

so

$$\min_{0 \leq \Phi(\tau) \leq 1} \bar{B}_0, \tag{20}$$

upon satisfaction of the condition

$$\frac{1}{2\pi} \int_0^{2\pi} V d\tau = V_1, \tag{21}$$

equations (13) and boundary conditions (18).

Let's note that, due to requirements (21) and relation (16), the expression can be used in the form

$$\bar{B} = \frac{1}{2\pi} \int_0^{2\pi} \frac{M_p^0(\Omega)\Omega g_e^0(M_p^0(\Omega)\Omega)}{V_1} d\tau. \tag{22}$$

Determination of dependencies for the average velocity  $\bar{V}$ , fuel consumption  $\bar{B}^0$  and other indicators was carried out using computer simulation. Studies of the influence of the  $N/A$  parameter on the operating parameters  $\Omega_{\min}$ ,  $\bar{B}^0$ ,  $\Omega_{\min}$ ,  $\Phi_{\min}$ ,  $V_0$ ,  $V_{\min}$ ,  $M_{c\min}$ ,  $M_{c\min}$ ,  $M_{p\min}$ ,  $M_{p\min}$ ,  $P_{\min}$ ,  $P_{\min}$  were carried out. The resulting typical dependences for these indicators as a function of  $N/A$  are shown in Fig. 2–4. They correspond to the parameter values:

$$\alpha=2; \Phi_0=1.0; \xi=0.9; \omega_0/\omega_p=1.0;$$

$$\lambda_0/\lambda_{1\min}=0.6; \lambda_{1\min}/\lambda_{2\min}=0.935,$$

and for Fig. 2 –  $\Phi_0=\Phi$ ;  $h_0/R_s=1.2$ ;  $l\psi_0/R_s=1.9$ ; for Fig. 3 –  $\Phi=\Phi_{\min}$ ;  $h_0/R_s=1.2$ ;  $l\psi_0/R_s=1.0$   $V_1=V(\Phi_0)$ ; for Fig. 4, *a* –  $\Phi=\Phi_{\min}$ ;  $h_0/R_s=1.2$ ;  $l\psi_0/R_s=1.9$   $V_1=V(\Phi_0)$ ; for Fig. 4, *b* –  $\Phi=\Phi_{\min}$ ;  $h_0/R_s=1.2$ ;  $l\psi_0/R_s=1.9$   $V_1<V(\Phi_0)$ .

From Fig. 2–4, it follows that with an increase in the parameter  $N/A$ , the values of the variable operating parameters asymptotically approach some boundaries. The intensity of the change in indicators depends on the value  $\Delta C_2 = \max C_2(\tau) - \min C_2(\tau)$ , which in turn is determined by the values of the parameters  $h\psi_0/R_s$  and  $h_0/R_s$ . The more  $\Delta C_2$ , the more the indicators change. For  $\Delta C_2 < 0.5$  (Fig. 3) the indices  $V_0, V_{\max}, \Phi_{\min}, \Phi_{\max}$  practically do not depend on the parameter  $N/A$ . Therefore, in the future, the values of  $\Phi_{\min}, \Phi_{\max}$  at arbitrary  $N/A$  can be found under the assumption that  $N/A$  is much less than unity 1.0.

For optimal control of the power unit, let's use the maximum principle of L. Pontryagin and the dynamic programming equations of R. Bellman [4]. If the movement of the vessel complex is described by differential equations:

$$\frac{dx_i}{dt} = f_i(x_1, x_2, \dots, x_n, u_1, u_2, \dots, u_m), \quad (i=1, 2, \dots, n), \quad (23)$$

where  $x_1, x_2, \dots, x_n$  – phase variables;

$u_1, u_2, \dots, u_m$  – control functions.

As  $u_i(t)$ , there is the amount of fuel supplied per unit path. It is necessary to transfer the vessel complex from a given initial state:

$$x_i(t_0) = x_{i0}, \quad (i=1, 2, \dots, n). \quad (24)$$

In some finite, for which the values of only a part of the phase variables are given:

$$x_i(t_k) = x_{ik},$$

$$(i=1, 2, \dots, r < n).$$

The control for the case of  $N/A$  by much more than 1.0 and  $N/A$  by much less than 1.0 is close to the optimal one implemented by the function:

$$\Phi(\tau) = a + b \cdot C_2(\tau), \quad (25)$$

which can be simply implemented and, in addition, has extreme points for the problem  $0 \leq \Phi(\tau) \leq 1$  which ensures the minimum fuel consumption per unit path of the main diesel engine of the vessel complex at an average velocity of its movement, Fig. 2–5.

In Fig. 2 curves 1–11 indicate the dependence of operating parameters on the parameter  $N/A$  1 –  $P_{\min} = f_1(N/A)$ ; 2 –  $M_{c \min} = f_2(N/A)$ ; 3 –  $M_{d \min} = f_3(N/A)$ ; 4 –  $M_{d \max} = f_4(N/A)$ ; 5 –  $0.5B^0 = f_5(N/A)$ ; 6 –  $V^0 = f_6(N/A)$ ; 7 –  $P_{\max} = f_7(N/A)$ ; 8 –  $P_c \max = f_8(N/A)$ ; 9 –  $\Phi_{\min} = f_9(N/A)$ ; 10 –  $\Omega_{\min} = f_{10}(N/A)$ ; 11 –  $\Omega_{\max} = f_{11}(N/A)$ .

In Fig. 3 curves 1–13 characterize the dependence of the main operating parameters on the  $N/A$  parameter at other values: 1 –  $M_{c \min} = f_1^*(N/A)$ ; 2 –  $P_{\min} = f_2^*(N/A)$ ; 3 –  $M_{d \min} = f_3^*(N/A)$ ; 4 –  $M_{d \max} = f_4^*(N/A)$ ; 5 –  $M_{c \max} = f_5^*(N/A)$ ; 6 –  $P_{\max} = f_6^*(N/A)$ ; 7 –  $V_0 = f_7^*(N/A)$ ; 8 –  $V_{\max} = f_8^*(N/A)$ ; 9 –  $\Phi_{\min} = f_9^*(N/A)$ ; 10 –  $\Phi_{\min} = f_{10}^*(N/A)$ ; 11 –  $\Omega_{\min} = f_{11}^*(N/A)$ ; 12 –  $\Omega_{\max} = f_{12}^*(N/A)$ ; 13 –  $B^0_{\min} = f_{13}^*(N/A)$ .

In Fig. 4 and curves 1–11 characterize the dependence of the force parameters on the complex parameter  $N/A$ : 1 –  $M_{c \min} = \phi_1(N/A)$ ; 2 –  $M_d = \phi_2(N/A)$ ; 3 –  $M_b = \phi_3(N/A)$ ; 4 –  $M_{\min} = \phi_4(N/A)$ ; 5 –  $0.5B^0 = \phi_5(N/A)$ ; 6 –  $P_{\min} = \phi_6(N/A)$ ; 7 –  $V_0 = \phi_7(N/A)$ ; 8 –  $\Phi_{\min} = \phi_8(N/A)$ ; 9 –  $\Phi_{\min} = \phi_9(N/A)$ ; 10 –  $\Omega_{\min} = \phi_{10}(N/A)$ ; 11 –  $\Omega_{\min} = \phi_{11}(N/A)$ .

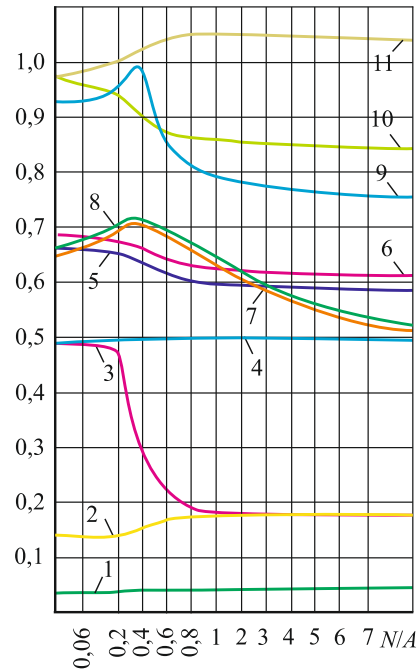


Fig. 2. Dependence of the main performance indicators on the  $N/A$  parameter (designation of curves in the text)

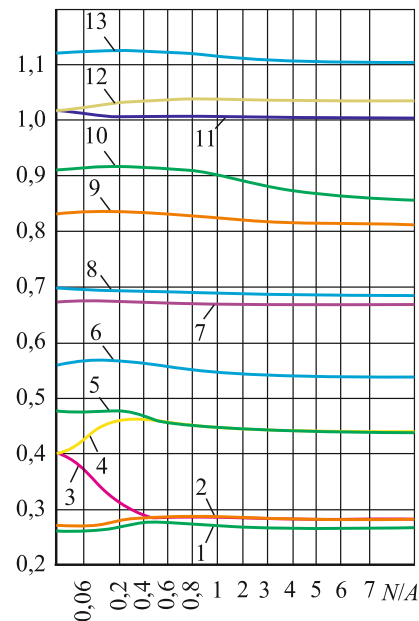


Fig. 3. Dependence of the main performance indicators on the  $N/A$  parameter:  $\Phi = \Phi_{\min}, V_1 = V(\Phi_0), h/R = 1,2$  (designation of curves in the text)

In Fig. 4, *b*, curves 1–10 characterize the dependence of the main operating parameters on the complex parameter  $N/A$ : 1 –  $P_{\min} = \phi_1^*(N/A)$ ; 2 –  $M_{c \min} = \phi_2^*(N/A)$ ; 3 –  $M_{d \min} = \phi_3^*(N/A)$ ; 4 –  $M_{d \max} = \phi_4^*(N/A)$ ; 5 –  $M_{c \max} = \phi_5^*(N/A)$ ; 6 –  $P_{\max} = \phi_6^*(N/A)$ ; 7 –  $V_0 = \phi_7^*(N/A)$ ; 8 –  $\Phi_{\min} = \phi_8^*(N/A)$ ; 9 –  $B_0 = \phi_9^*(N/A)$ ; 10 –  $\Omega_{\min} = \phi_{10}^*(N/A)$ .

The extreme values of the variables  $\Omega, M_p, M_c, P$ , as well as the indices  $V_0, \Phi_{\max}, \Phi_{\min}, B^0, V_{\max}$  almost do not differ from their limiting values. With an increase in  $N/A$ ,

the shift of the maximum of the function  $\Phi(\tau)$  relative to the minimum of the function  $C_2(\tau)$  changes from  $\pi/2$  to 0, Fig. 5.

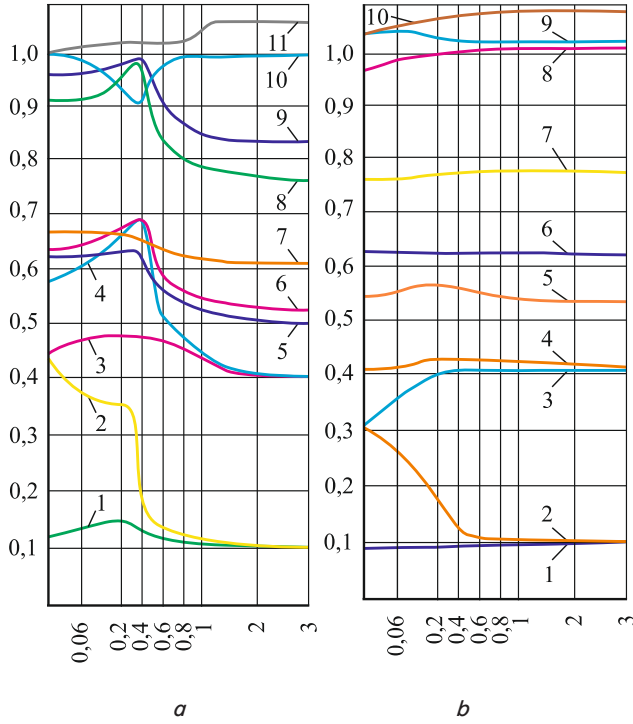


Fig. 4. Dependence of power parameters and performance indicators on the complex parameter  $N/A$ :  $a$  – dependence of power parameters on the complex parameter  $N/A$ ;  $b$  – dependence of the main performance indicators on the complex parameter  $N/A$  (designation of curves in the text)

At  $\Delta C_2 > 0.5$  (Fig. 2, 4), the values of the  $\Phi_{\max}$  and  $\Phi_{\min}$  indices reach maxima in the interval  $N/A = 0.1 - 1.0$ , and then monotonically decreases with an increase in the  $N/A$  parameter. At  $\Delta C_2 < 0.5$  (Fig. 3) all characteristics are monotonic functions of the  $N/A$  parameter. From Fig. 4,  $b$  it follows that even at  $\Delta C_2 > 0.5$ , the value of  $\Phi_{\min}$  is practically independent of  $N/A$ , if the value of the average velocity  $\bar{V}$  for large  $N/A$  and  $\Phi = 1.0$  is taken as  $V_1$ . In this case, as for small  $\Delta C_2$ , all operating parameters are monotonic functions of  $N/A$  (Fig. 4,  $b$ ).

Curves 1–11 in Fig. 5 represent the results of solving equations (13) with limiting conditions (10): 1 –  $P(1)$ ; 2 –  $P(0.5)$ ; 3 –  $P(0.25)$ ; 4 –  $(0.125)$ ; 5 –  $P(0.02)$ ; 6 –  $\Omega(1)$ ; 7 –  $\Omega(0.5)$ ; 8 –  $\Omega(0.25)$ ; 9 –  $\Omega(0.125)$ ; 10 –  $\Omega(0.02)$ ; 11 –  $C_2(\tau)$ .

Thus, let's obtain a solution to problems I and II for small and large values of  $N/A$ . For a large interval of variation of the parameters of the vessel complex, it is possible to obtain upper and lower estimates of the main operating parameters. In addition, these solutions can be used if the interpolation is performed to approximate the performance during  $N/A$  times that are within the  $N/A$  range is significantly greater than 1.0 and the  $N/A$  is significantly less than 1.0.

When determining the law of optimal control, the function:

$$\Phi(\tau) = a + b \cdot C_2(\tau),$$

can be realized and have extreme points. Fig. 6 shows the results of calculations for the problem, the solution of which

provides the minimum fuel consumption per mile of the vessel's path. At a given average velocity of the set motion according to the control laws, let's obtain the expression:  $\Phi = a + b \cdot (c_1(\tau)/c_2(\tau))$  – solid curves and  $\Phi(\tau) = a + b \cdot C_2(\tau)$  – dashed curves. It should be noted that the calculations were performed for  $N/A$  significantly less than 1.0,  $\Phi_0 = 1.0$ ,  $\lambda/\lambda_{1\max} = 0.2$ ,  $\xi = 0.6$ ,  $\kappa = 3.0$ .

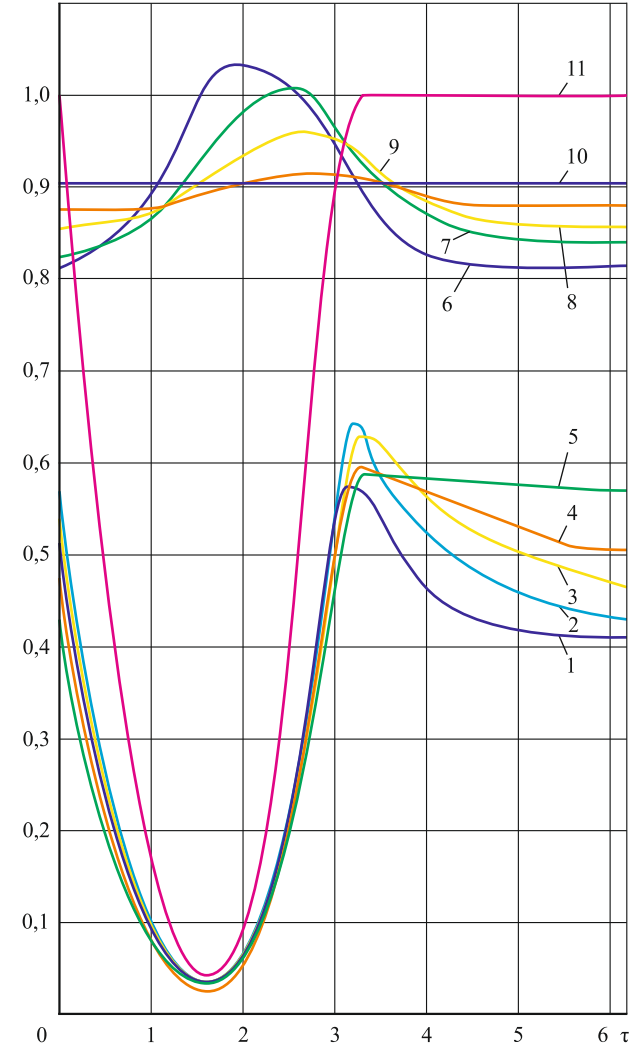


Fig. 5. Graphic representation of the result of solving equation (13) for the condition when  $\Phi = \Phi_0$ . The nature of changes in the quantities:  $\Omega$ ,  $P$ ,  $C_2$ , in time  $\tau$  for  $\Phi = \Phi_0$ ,  $h_0/R = 1.2$ ,  $h_T/R_s = 1.9$ ,  $\xi = 0.8$  and various values of  $N/A$  (designation of curves in the text)

$$\delta B_1 = \frac{B^0(\Phi_{\min}) - B^0\left(a + b \cdot \frac{c_1(\tau)}{c_2(\tau)}\right)}{B^0(\Phi_{\min})}; \tag{26}$$

$$\delta B_1^0 = \frac{B^0(\Phi_{\min}) - B^0(a + b \cdot C_2(\tau))}{B^0(\Phi_{\min})}. \tag{27}$$

Fig. 6,  $a$  can be used to determine the gain in fuel consumption using arbitrary values:  $\Phi_0$ ;  $\kappa = k/k_0$ ;  $\lambda_0/\lambda_{1\max}$ ;  $\xi$ , lying in the real range of their changes, when  $N/A$  is much less than 1.0.



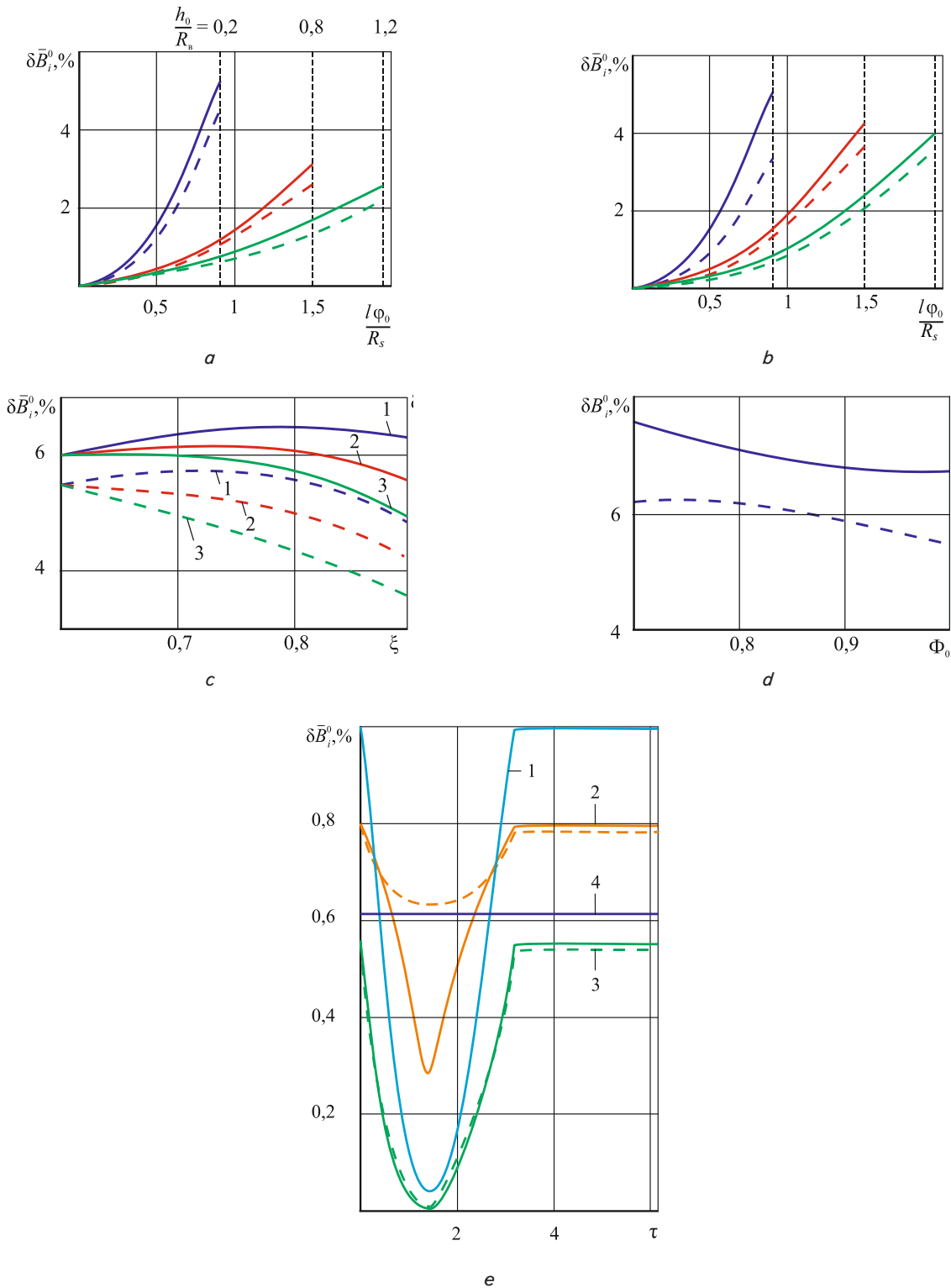


Fig. 6. Results of solutions for control laws  $\Phi = a + b \cdot (c_1(\tau) / c_2(\tau))$  and  $\Phi = a + b \cdot C_2(\tau)$ : *a* –  $\delta B_1^0$  dependence on immersion, calculated for control laws  $\Phi = a + b \cdot (c_1(\tau) / c_2(\tau))$  – dashed curves,  $\Phi = a + b \cdot C_2(\tau)$  – solid curves at  $N/A$  significantly less than 1.0; *b* –  $\delta B_1^0$  dependence on immersion, calculated for the control laws  $\Phi = a + b \cdot (c_1(\tau) / c_2(\tau))$  – dashed curves,  $\Phi = a + b \cdot C_2(\tau)$  – solid curves at  $N/A$  are much larger 1.0; *c* –  $\delta B_2^0$  dependence (dashed curves) for  $N/A$  is much more than 1.0 at the values of the parameters  $\Phi_0 = 1.0$ ,  $\kappa = 3.0$ ,  $\lambda \psi_0 / R_s = 0.9$ ,  $h_0 / R_s = 0.2$ ,  $\lambda_0 / \lambda_{1max} = 0.6, 0.4, 0.2$  (curves 1, 2, 3, respectively), and solid curves for the control law  $\Phi = a + b \cdot C_2(\tau)$ ; *d* –  $\delta B_2^0$  dependence when  $N/A$  is significantly greater than 1.0 for  $\xi = 0.8$ ,  $\kappa = 3.0$ ,  $\lambda \psi_0 / R_s = 0.9$ ,  $h_0 / R_s = 0.2$ ,  $\lambda_0 / \lambda_{1max} = 0.6$ , solid curves – control law  $\Phi = a + b \cdot (c_1(\tau) / c_2(\tau))$ , dashed curves – control law  $\Phi = a + b \cdot C_2(\tau)$ ; *e* –  $\delta B_2^0$  dependences according to the control  $\Phi = a + b \cdot C_2(\tau)$  – dashed curves,  $\Phi = a + b \cdot C_2(\tau)$  – solid curves when  $N/A$  is much whiter than 1.0,  $\kappa = 2.0$ ,  $\Phi_0 = 1.0$ ,  $\lambda_0 / \lambda_{1max} = 0.6$ ,  $\xi = 0.9$ ,  $\lambda \psi_0 / R_s = 0.9$ ,  $h_0 / R_s = 0.2$ , curves 1– $C_1(\tau)$ , 2– $\Phi(\tau)$ , 3– $P(\tau)$ , 4– $V(\tau)$

Fig. 6, *b–d* shows the dependences for  $\delta B_1^0$  – solid curves and  $\delta B_2^0$  – dashed curves, when  $N/A$  is significantly greater than 1.0 for the parameter values: *b* –  $\Phi_0=1.0$ ,  $\lambda_0/\lambda_{1\max}=0.6$ ,  $\xi=0.9$ ,  $h_0/R_s=2$ ; *c* –  $\Phi_0=1.0$ ,  $\kappa=3.0$ ,  $l\psi_0/R_s=0.9$ ,  $h_0/R_s=2$ ,  $\lambda_0/\lambda_{1\max}=0.6$ ; 0.4; 0.2 (curves 1, 2, 3, respectively); *d* –  $\xi=0.8$ ,  $\kappa=3.0$ ,  $l\psi_0/R_s=0.9$ ,  $h_0/R_s=0.2$ ,  $\lambda_0/\lambda_{1\max}=0.6$ .

Fig. 6, *e* shows the solutions according to the control: 1, 2, 3, 4 – dashed curves and solid curves when  $N/A$  is much greater than 1.0,  $\kappa=2.0$ ,  $\Phi_0=1.0$ ,  $\lambda_0/\lambda_{1\max}=0.6$ ,  $\xi=0.9$ ,  $l\psi_0/R_s=0.9$ ,  $h_0/R_s=0.2$ . These parameter values correspond to the area of the largest error values  $\Delta B_{12}^0$ . Curves in Fig. 6, *e* represent: 1 –  $C_1(\tau)$ ; 2 –  $\Phi(\tau)$ ; 3 –  $P(\tau)$ ; 4 –  $V(\tau)$ .

From Fig. 6, *a – e* it is possible to find that the value  $\delta B_1^0$ ,  $\delta B_2^0$ ,  $\Delta B_{12}^0 = \delta B_1^0 - \delta B_2^0$  is greater at smaller immersions  $h_0/R_s$ , and an increase occurs with an increase in the amplitude  $l\psi_0/R_s$  of fluctuations in the water medium. The value  $\Delta B_{12}^0$  does not exceed 1%, and, as a consequence, control law (25) gives a fairly good approximation to the optimal one.

Thus, the calculations showed that for all values of the parameters of the main diesel engine of the vessel control complex (25), with an error of 1%, it gives a fuel economy of up to 6% per unit path.

The obtained ratios in modeling and optimal control of the main diesel engine of vessel complexes allow using the dynamic programming method to analyze the fuel consumption per unit path in comparison with the optimal constant control.

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## 6. Discussion of the results of the study of the influence of the high-pressure fuel system on the optimal control of a vessel diesel engine

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The results were obtained on the study of the operation of the engine, fuel system and the hull of the vessel as a single complex and the characteristics of its elements in various operating conditions. The constructed mathematical model of the constituent mechanical processes made it possible to obtain optimal control of the main diesel engine of its fuel system of the vessel complexes.

A feature of the proposed method and the results obtained in comparison with the existing ones is the construction of an optimal control model based on the averaging method and the maximum principle of L. Pontryagin using the method of dynamic programming by R. Bellman. Based on them (formulas (23), (24)), a control close to optimal, implemented by the function  $\Phi(\tau)=a+b\cdot C_2(\tau)$ , is determined. This control can be realized, since the specified function has extreme points that ensure the minimum fuel consumption per unit path of the main diesel engine of the vessel complex at an average velocity of its movement (Fig. 2–5).

Fig. 6 shows the results of calculations for the problem, the solution of which provides the minimum fuel consumption per mile of the vessel's path.

Studies given in literary sources [13, 14] indicate that the condition of the vessel's hull, meteorological factors

and draft affect the operation of the main vessel engine and economic performance, but do not correspond to the optimal use of the power plant fuel. It should be noted that adequate attention has not been paid to determining the influence of the high pressure fuel system on the control of the main marine engine.

The limitations proposed in the study include the assumption of the regular nature of waves that affect the operating mode of power equipment.

The disadvantages of the study carried out include the obtained as a result of solving the optimal control equations. This equation is open. This allows it to be used only when the boat is moving on regular undulations.

In real conditions, the navigation of the vessel due to the irregularity of the control waves should be closed.

Further development of these studies consists in solving the problem of optimal control in a closed form. The technical implementation of the results of the study of optimal control is associated with the development of an additional device to the high-pressure fuel system of a marine diesel engine, which will allow obtaining performance indicators based on the results of vessel tests.

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## 7. Conclusions

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1. A mathematical model of the constituent mechanical processes in the high-pressure fuel supply system, taking into account the dynamics of the vessel complex, provided the parameters of the vessel's movement relative to the optimal average velocity with obtaining the ratio of fuel consumption to power. Under the conditions of a non-stationary mode of motion with a certain control law of the fuel system  $\Phi=a+b\cdot C_2(\tau)$ , it is possible to achieve fuel savings of up to 6% per unit path

2. The obtained ratios in modeling, the optimal control of the power units of vessel complexes made it possible to analyze the fuel consumption per unit path using the dynamic programming method. When comparing the optimal control with the corresponding constant control, up to 6% fuel economy per unit path was obtained.

3. The influence of water environment disturbances on the fuel system of the vessel complex with optimal and constant control was established. This, in turn, will improve the reliability and performance of the fuel system of a marine diesel engine by reducing the load on it due to fuel economy.

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## Acknowledgments

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The authors express their gratitude to the Chief Engineer of the State Enterprise Scientific Research Design Institute of the Marine Fleet of Ukraine for their support in the implementation of experimental test results, confirmed by obtaining a patent for a useful model.

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