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MATHEMATICAL FORMALIZATION OF CERTAIN PROBLEMS OF TRANSPORT LOGISTICS

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Юхименко Б.І., Малютіна С.В. Математична формалізація деяких задач транспортної логістики.

В роботі показано місце транспортної логістики в загальних логістичних проблемах. Дан перелік основних завдань, що підлягають формалізації та вирішенню методами оптимізації. Наведена деяка задача транспортування вантажів, як задача лінійного програмування. Більш пильна увага приділена вирішенню задачі визначення маршрутів доставки вантажів. Дан новий підхід вирішення задачі, що приводиться до моделі задачі про комівояжера із симетричною матрицею витрат.

Ключові слова: транспортна логістика, оптимізація, математична модель, маршрутизація, симетрична матриця витрат

Юхименко Б.И., Малютина С.В. Математическая формализация некоторых задач транспортной логистики.

В работе показано место транспортной логистики в общих логистических проблемах. Дан перечень основных задач, подлежащих формализации и решению методами оптимизации. Приведена некоторая задача транспортировки грузов как задачи линейного программирования. Более пристальное внимание уделено решению задачи определения маршрутов доставки грузов. Дан новый подход решения задачи, приводимой к модели задачи о коммивояжере с симметричной матрицей расходов.

Ключевые слова: транспортная логистика, оптимизация, математическая модель, маршрутизация, симметричная матрица расходов

Yukhymenko B.I., Maliutina S.V. Mathematical formalization of some problems of transport logistics.

The location of transport logistics in general logistical problems shows in this article. Given a list of major tasks to be formalizing and solving optimization techniques. Shows some problem of cargo transportation as a linear programming problem. The task of determining the route of delivery is given more attention. Given the new approach of solving the problem, the model driven traveling salesman problem with symmetric matrix costs.

Keywords: transport logistics, optimization, mathematical model, routing, symmetric matrix costs

In modern market conditions the quality of logistics has become one of the determining factors of competitiveness of economy of Ukraine. Special attention is paid to questions of formation of logistic systems at the level of economic systems.

It is emphasized that an important condition for effective implementation of logistics strategy of enterprise is determining the appropriate logistical capacities of all logistics operations and functional links, and effective use.

Transport logistics occupies a significant place in the total logistics. It integrates the complex planning, management and physical transportation of materials, purchases of semi-finished products, manufacturing waste in the support needed for this information flow to minimize transportation costs and time.

The main areas of transport logistics are routing, the cost of transport services, systematization of relationships with the task of inventory management.

The main objective of transport logistics is to provide the necessary transportation services in the sectors of supply and distribution. It includes sub-objectives, namely the use of vehicles and package for different types of production; inclusion of transport in the system of production planning and management, use cheaper ways of providing transport services and services in the shortest possible time [3].

Transport cost is the cost of shipping products from the production site to direct consumers. Transport costs can be considered as independent of the quantity of cargo or just run from the road, irrespective of quantity of cargo.

From mathematical standpoint it is quite different mathematical models, different computer implementation and, of course, solve the trust issues of organization of cargo transportation.

In the case that is considered an economic system in which goods transportation plays a considerable role in estimating the cost, determining the routes based on the number of transported goods is one of the key points in the definition of expenses.

In most cases, transportation logistics issues are solved intuitively, well-trained managers who know how to make the right decision in the organization and management in the field of logistics. However, the use of mathematical methods and IT technologies gives the opportunity to more quickly, accurately and financially beneficial to make these decisions.

Analysis of recent researches and publications

In modern literary sources are methods that you can use, solving logistics issues. For example [1] provides some probabilistic models estimation of the status of the solved issues of logistics. Also, in a sense described methods of expert assessment possible of the solved issues. In [4] provides a preliminary assessment to determine the optimal routes of transportation of goods, distribution of vehicles at the point of origin, determining the optimal sequence to visit the destinations of one transport unit. Quite a wide range of mathematical models is given in the work [6]. However, there is no guidance about methods of applying the existing mathematical models and possibilities of their use in addressing issues like transportation logistics and logistics in general.

The aim of the article is to give a small overview of the possibilities of using optimization models linear programming of the transport type for transportation organization. In addition, the paper outlines a new algorithm for solving the problem of determining the optimal sequence of delivery of cargoes of one transport unit. From a mathematical position is an operational algorithm for solving the traveling salesman problem with symmetric cost matrix of the transportation [5].

The main part

For the carriage of homogeneous cargo, in the sense of the cost of transportation transportation routes are defined under the auspices of "where from, where, how much cargo to transport so that the total cost of delivery to destinations was minimal". From a mathematical standpoint, this task is given to the model of linear programming problem which goes under the name of the transport problem [7].

The cost of transportation of goods also depends on the packaging. If the packaging (probably a container) to count as a unit for the delivery of goods, the transport unit can be loaded with different cargo items with different cost of transportation. In this situation, the target unit has a different wording: "where from , where, how much cargo should be transported by the transport unit, pursuing the minimization of costs". Model this problem mathematically represented as three-index transportation problem [7].

Transportation of goods also applies to transport logistics, with the so-called transit points. In other words, the goal of transportation is as follows: send → warehousing → shipping at the destination. The task from the standpoint of logistics, and from the perspective of mathematics is quite complicated. Organization and management of shipping can be considered as work on the separate economic object. However, optimization of this working object can attract a number of ineffective procedures of an organizational nature on the subsequent business objects. In this regard, strictly requires a systematic approach in optimizing the whole purpose transport logistics. A mathematical model describing a chain of delivery of the goods is reproduced by transport models going under the name "transport problem with transit points" [3].

In this paper I want to pay closer attention to the issue of a centralized delivery of goods. The situation is this, there is one point of delivery, which are delivered in one transport unit to several consumers. Alternately the cargo is delivered to destination points, without coming to the point of sending. The sequence of points of destination of goods delivery determines not only the minimization of transport costs or the efficiency of meeting demand, but also question the location of the cargo transport unit. In other words, we obtain two types of information: the optimal route to visit points of delivery, but quite good location (packing) of goods in the transport unit. This type of logistics challenge is to model the problems of integer linear programming with Boolean variables. This model belongs to the class of combinatorial optimization problems and is called the traveling salesman problem [7]. The model in discrete optimization known. However, the method of implementing the model on a computer depends on the situations in which you organize the source information for solving.

Let us give a mathematical model. There are n points of delivery of goods. Including the first paragraph is the point of sending. It is believed that after visiting all consumers transport unit returns to the point of sending. With mathematical positions of the closed loop of the bypass determines the model and the solution method, economic – in order to perform the next cycle of the curve, you need to make the loading of the transport unit.

We introduce the notation. i – denote the point of sending, and j is the point of arrival at any pair of points of dispatch and delivery. Then $\|c_{ij}\|$ – matrix expenses when moving between any pair of points visited. Variables x_{ij} – determine whether to engage in the cycle of detour crossing between the pair *(i, j)* or not. Mathematically, this variable is a Boolean type and takes values from the set {0,1}. This model is minimizing costs:

$$Z = \min \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij}$$

under constraints

$$\sum_{j=1}^{n} = 1, i = \overline{1,n}$$

with each item sent to the transport unit in one direction and arrives at the destination only once

$$\sum_{i=1}^{n} x_{ij} = 1, j = \overline{1,n};$$

$$x_{ij} \in \{0,1\}, i = \overline{1,n}; j = \overline{1,n}$$

Formal constraint, introduced to prohibit the decay cycle of a detour down into sub-cycles such

$$u_i - u_j + nx_{ij} \le n-1, \quad u_i, u_j \ge 0, \quad i=\overline{1,n}; j=\overline{1,n}.$$

Method of solution largely depends on the nature of the matrix of costs. Matrix $\|c_{ij}\|$, denoting the distance between either pair (i, j), either the fuel consumption or time spent on moving or some other costs can be either asymmetrical or symmetrical with respect to the main diagonal. In other words, the cost of $c_{ij} \neq c_{ji}$ or $c_{ij} = c_{ji}$, when $i \neq j$.

The usual method of branches and boundaries proposed by Murthy [7], implements the algorithm to solve the traveling salesman problem as a mathematical model, which provides a number of tasks of organizational management. Mathematical model for centralized delivery of goods mainly represented by the symmetric matrix costs. In addition, this logistical task the starting point is always the point that goes under the name "first" (1). It also has value for algorithmic and the rate of convergence of the solution process.

The proposed algorithm is inspired by the ideas of constructing consistent decision [2] and ideas of solving dynamic programming problems [7]. The sequence of points assignment – solution – consists of pairs of points. This pair of points is specified (defined) at each step of the algorithm. Starting from the point of dispatch you can go to any destination. Select the one for which the continuation of visiting the destinations would be best. Fixing the item, executes the next step of the algorithm selects the next item with the best continuation. Having n steps (n is the number of destinations) we get the optimal sequence of delivery of goods by one transport unit.

Method of branches and boundaries for solving the traveling salesman problem also uses the idea of sequential solution build. However, this idea is used in the decomposition of the set of variants of a detour of destinations into subsets, one of which contains a transfer between the selected pair, the other isn't. To select the subset of assessment is used, in determining the priority of choice.

Here is an example of solving the problem with symmetric matrix using the classical algorithm of the method of branches and borders and above the proposed. Given a matrix of costs when moving between any pair of destinations (Fig. 1).

	1	2	3	4
1	∞	3	2	1
2	3	∞	3	2
3	2	3	∞	3
4	1	2	3	∞

Fig.1. The matrix of cost

The decision tree method of branches and borders is shown in Fig. 2. Tops means a couple of destinations included, not included in the subset of variants – evaluation of a subset of variants.

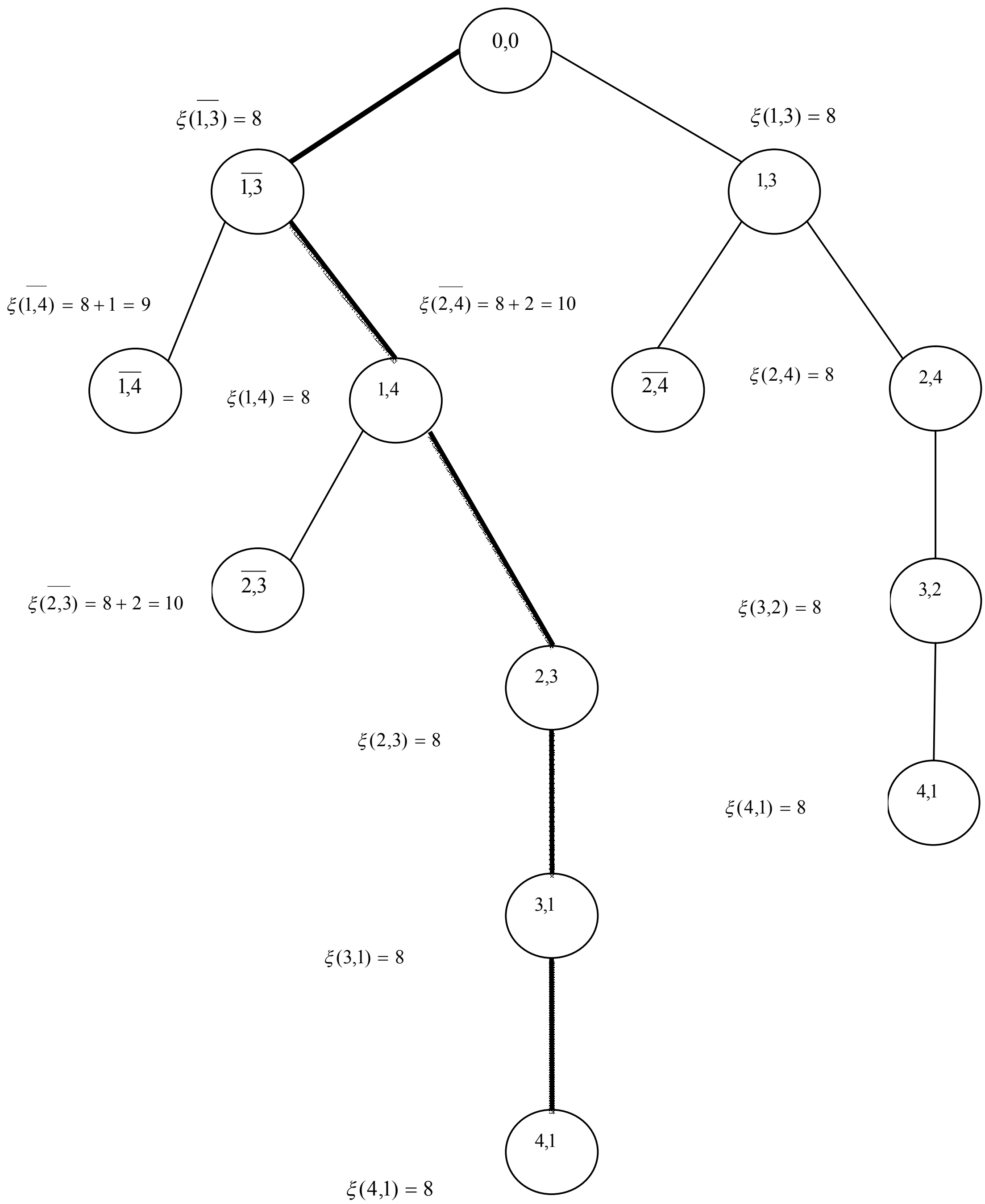

Fig. 2. The decision tree method of branches and borders

The decision tree of the proposed algorithm is shown in Fig. 3. The peaks represent specific destination, edge is the previous paragraph, R(2) – the minimum score to continue, if you selected 2.

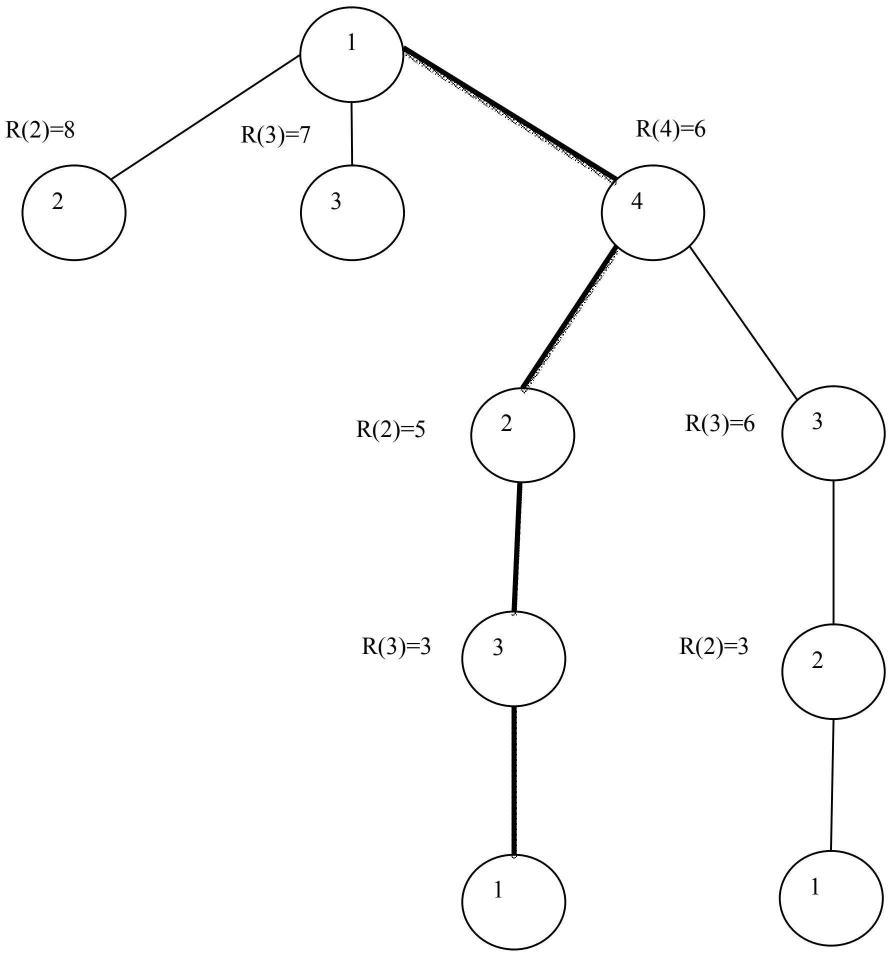

Fig. 3. The proposed decision tree algorithm

The optimal sequence to visit the destinations shown in Fig. 4.

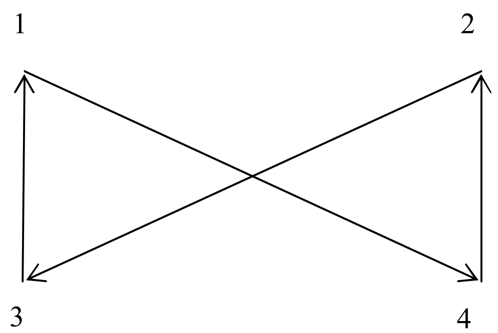

Fig. 4. The optimal sequence

The detour cost of delivery points is 8 units of money.

The symmetry of the matrix determines the value of repetition as moving from a to b and from b to a. This requires the indication of optimality of the method of branches and borders. This situation is not observed in the second algorithm. Check for the optimality of this algorithm has a different meaning. Optimality is defined for each component (pair of items) in a sequence of detours. The judgment confirms the example. However, you must prove it theoretically. A formal description will be given in the next paper.

Conclusions

It should also be noted that this is a classic method of branches and borders practically feasible if the number of destinations n<30. Therefore, to solve real problems, you should use all the specific moments of the task. In this case, the symmetry of the matrix values. In [5] mentions an algorithm for solving the traveling salesman problem with a symmetric matrix, but no results are given.

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