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IDENTIFICATION OF NONLINEAR DYNAMICAL SYSTEMS USING VOLTERRA MODEL WITH INTERPOLATION METHOD IN FREQUENCY DOMAIN

Abstract. *The new method of nonlinear dynamical systems identification based on Volterra model is offered. This method lies in n-order differentiation of the responses of the system being identified with the amplitude of test polyharmonic signals as a main parameter. The developed hardware and software tools are used for constructing of telecommunication channel informational model as amplitude-frequency characteristics of the first and second order.*

Keywords: *identification, nonlinear dynamic systems, Volterra models, multifrequency characteristics, polyharmonic signals, telecommunication channels.*

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ИНТЕРПОЛЯЦИОННЫЙ МЕТОД ИДЕНТИФИКАЦИИ НЕЛИНЕЙНЫХ ДИНАМИЧЕСКИХ СИСТЕМ НА ОСНОВЕ МОДЕЛИ ВОЛЬТЕРРА В ЧАСТОТНОЙ ОБЛАСТИ

Аннотация. *Предложен новый метод идентификации нелинейных динамических систем на основе модели Вольтерра в частотной области, заключающийся в n–кратном дифференцировании откликов идентифицируемой системы по параметру–амплитуде тестовых полигармонических сигналов. Разработанные программно–аппаратные средства идентификации применяются для построения информационной модели телекоммуникационного канала в виде амплитудно-частотных характеристик первого и второго порядка.*

Ключевые слова: *идентификация, нелинейные динамические системы, модели Вольтерра, амплитудно-частотные характеристики, полигармонические сигналы, телекоммуникационне каналы.*

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ІНТЕРПОЛЯЦІЙНИЙ МЕТОД ІДЕНТИФІКАЦІЇ НЕЛІНІЙНИХ ДИНАМІЧНИХ СИСТЕМ НА ОСНОВІ МОДЕЛІ ВОЛЬТЕРРА В ЧАСТОТНІЙ ОБЛАСТІ

Анотація *Запропоновано новий метод ідентифікації нелінійних динамічних систем на основі моделі Вольтерра в частотній області, що полягає в n–кратному диференціюванні відгуків системи, яка ідентифікується, за параметром–амплітудою тестових полігармонічних сигналів. Розроблені програмно-апаратні засоби ідентифікації застосовуються для побудови інформаційної моделі телекомунікаційного каналу у вигляді амплітудно-частотних характеристик першого та другого порядку.*

Ключові слова: *ідентифікація, нелінійні динамічні системи, моделі Вольтерра, амплітудно-частотні характеристики, полігармонічні сигнали, телекомунікаційні канали.*

Creation of new and improvement of maintained telecommunication systems (TCS) needs the developing of new effective methods and software tools for parameters and characteristics measurement of TCS, automation of procedures of diagnostic check and forecasting of their technical conditions. The direction linked with methods of optimum reception of signals development taking into account characteristics of equipment and communication channel (CC) intensively develops. High speed data transmission in modern TCS is reached at the expense of various receptions, among which is taking into account of CC characteristics.

The CCs in modern TCS are nonlinear inertial stochastic systems. During maintenance the CC as dynamical object, depending on a design and working conditions, changes its characteristics in time, so there is a necessity for regular adjustment of mathematical model of the CC – the repeated decision of the identification problem [1].

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The technique and hardware-software tools of experimental definition of multidimensional an AFC and PFC of the CC are offered, i.e. the

determination of its multifrequency characteristics on the basis of input–output experiment data, using test polyharmonic signals. This technique is based on interpolation method of identification of the nonlinear dynamical system in type of Volterra series in the frequency domain with use of standard IBM PC hardware. The model construction of the VHF-range radio channel is offered.

Volterra models in frequency domain. In general case "input–output" type ratio for nonlinear dynamical system can be presented by Volterra series [2, 3]:

$$y[x(t)] = \sum_{n=1}^{\infty} y_n[x(t)] =$$

$$= \sum_{n=1}^{\infty} \int_0^{\infty} \dots \int w_n(\tau_1, \tau_2, \dots, \tau_n) \prod_{r=1}^{n} x(t-\tau_r) d\tau_r$$

where $x(t)$ and $y(t)$ are input and output signals of system respectively; $w_n(\tau_1, \tau_2, \dots, \tau_n)$ – weight function or n-order Volterra kernel (VK);

$y_n[x(t)]$ – n–th partial component of object response [4].

In practice, Volterra series are replaced by polynomial and generally limited to several first members of the series. Identification of nonlinear dynamical system in the form of a Volterra series consists of determination of n-dimensional weighting functions $w_n(\tau_1,\dots,\tau_n)$ or their Fourier–images $W_n(j\omega_1,\dots,j\omega_n)$ – n-dimensional transfer functions, accordingly for system modeling in time or frequency domain [5, 6].

Identification of nonlinear system in frequency domain coming to determination of absolute value and phase of multidimensional transfer function at given frequencies – multidimensional amplitude–frequency characteristics (AFC) $|W_n(j\omega_1,j\omega_2,\dots,j\omega_n)|$ and phase-frequency characteristics (PFC) $\arg W_n(j\omega_1,j\omega_2,\dots,j\omega_n)$ which are defined by formulas:

$$|W_n(j\omega_1, j\omega_2,\dots, j\omega_n)| = \sqrt{\begin{matrix}[\mathrm{Re}(W_n(j\omega_1, j\omega_2,\dots, j\omega_n))]^2 \\ +[\mathrm{Im}(W_n(j\omega_1, j\omega_2,\dots, j\omega_n))]^2\end{matrix}};$$

$$\arg W_n(j\omega_1, j\omega_2,\dots, j\omega_n) = \mathrm{arctg}\frac{\mathrm{Im}(W_n(j\omega_1, j\omega_2,\dots, j\omega_n))}{\mathrm{Re}(W_n(j\omega_1, j\omega_2,\dots, j\omega_n))}$$

where *Re* and *Im* – accordingly real and imaginary parts of a complex function of n variables.

In interpolation method of identification of the nonlinear dynamical system on a basis the Volterra series for division of the response of the nonlinear dynamical system on PC $\hat{y}_n(t)$ is used n-fold differentiation of a target signal on parameter-amplitude a of test influences [7].

If to give a test signal of a kind $ax(t)$ to input of the system, where $x(t)$ – any function; $|a|\le 1$ – scale factor for n-th order partial component allocation $\hat{y}_n(t)$ from the measured response of nonlinear dynamical system $y[ax(t)]$ it is necessary to find n-th private derivative of the response on amplitude a at a=0

$$\hat{y}_n(t) = \underbrace{\int_0^t \dots \int_0^t}_{n\ times} w_n(\tau_1,\dots,\tau_n)\prod_{l=1}^{n} x(t-\tau_l)\,d\tau_l = \frac{1}{n!}\left.\frac{\partial^n y[a\,x(t)]}{\partial a^n}\right|_{a=0}. \quad (1)$$

Using the test impacts and procedure (1) partial components of responses $\hat{y}_n(t)$ are calculated on which basis diagonal and sub diagonal sections of the Volterra kernel are defined.

Formulas for numerical differentiation using the central differences for equidistant knots $y_r = y[rhx(t)],\ r=-r_1,-r_1+1,\dots,r_2$ with step of computational mesh on amplitude $h=\Delta a$ [8] are received. For definition the Volterra kernel of the first order is calculated the first derivative at $r_1 = r_2 = 1$ or $r_1 = r_2 = 2$ accordingly

$$y_0' = y'(0) = \frac{1}{2h}(-y_{-1} + y_1),$$

$$y_0' = \frac{1}{12h}(y_{-2} - 8y_{-1} + 8y_1 - y_2).$$

For definition the Volterra kernel of the second order is calculated the second derivative at $r_1 = r_2 = 1$ or $r_1 = r_2 = 2$, accordingly

$$y_0'' = y''(0) = \frac{1}{h^2}(y_{-1} + y_1),$$

$$y_0'' = \frac{1}{12h^2}(-y_{-2} + 16y_{-1} + 16y_1 - y_2).$$

Full table of the amplitudes and corresponding coefficients are given in Table 1.

The test polyharmonic effects for identification in the frequency domain representing by signals of such type:

$$x(t) = \sum_{l=1}^{n} A_l\cos(\omega_l t + \varphi_l) \quad (2)$$

1. Amplitudes *a* and corresponding coefficients *c* of the interpolation method.

Kernel order, k	Experiments quantity, N	$a_1^{(k)}$	$a_2^{(k)}$	$a_3^{(k)}$	$a_4^{(k)}$	$a_5^{(k)}$	$a_6^{(k)}$	$c_1^{(k)}$	$c_2^{(k)}$	$c_3^{(k)}$	$c_4^{(k)}$	$c_5^{(k)}$	$c_6^{(k)}$
1	2	-1	1					-0,5	0,5				
	4	-1	-0,5	0,5	1			0,0833	-0,6667	0,6667	-0,0833		
	6	-1,5	-1	-0,5	0,5	1	1,5	-0,0167	0,15	-0,75	0,75	-0,15	0,0167
2	2	-1	1					1	1				
	4	-1	-0,5	0,5	1			-0,0833	1,3333	1,3333	-0,0833		
	6	-1,5	-1	-0,5	0,5	1	1,5	0,0111	-0,15	1,5	1,5	-0,15	0,0111
3	4	-1	-0,5	0,5	1			-0,5	1	-1	0,5		
	6	-1,5	-1	-0,5	0,5	1	1,5	0,125	-1	1,625	-1,625	1	-0,125

where n – the order of transfer function being estimated; A_l, ω_l and φ_l – accordingly amplitude, frequency and a phase of l-th harmonics. In research, it is supposed every amplitude of A_l to be equal, and phases φ_l equal to zero.

Thus, the test signal can be written in the complex form:

$$x(t) = A\sum_{l=1}^{n}\cos(\omega_l t) = \frac{A}{2}\sum_{l=1}^{n}\left(e^{j\omega_l t} + e^{-j\omega_l t}\right)$$

Then the n–th partial component in the response of system can be noted in an aspect:

$$y_n(t) = \frac{A^n}{2^{n-1}}\sum_{m=0}^{\left[\frac{n}{2}\right]} C_n^m \sum_{k_1}^{n}\ldots$$

$$\ldots\sum_{k_n}^{n}\left|W_n\left(-j\omega_{k_1},\ldots,-j\omega_{k_m}, j\omega_{k_{m+1}},\ldots j\omega_{k_n}\right)\right| \times$$

$$\times\cos\left(\left(-\sum_{l=0}^{m}\omega_{k_l} + \sum_{l=m+1}^{n}\omega_{k_l}\right)t + \arg W_n\left(-j\omega_{k_1},\ldots,-j\omega_{k_m}, j\omega_{k_{m+1}},\ldots, j\omega_{k_n}\right)\right)$$

where […] means function of extraction of an integer part of number.

The component with frequency $\omega_1+\ldots+\omega_n$ is selected from the response to a test signal (2):

$$A^n\left|W_n(j\omega_1,\ldots,j\omega_n)\right|\cos\left[(\omega_1+\ldots+\omega_n)t + \arg W_n(j\omega_1,\ldots,j\omega_n)\right].$$

In [8] it is defined that during determination of multidimensional transfer functions of nonlinear systems it is necessary to consider the imposed constraints on choice of the test polyharmonic signal frequencies which provide an inequality of combination frequencies in output signal harmonics.

Given method was tested on a nonlinear test object described by Riccati equation:

$$\frac{dy(t)}{dt} + \alpha\cdot y(t) + \beta\cdot y^2(t) = u(t).$$

Analytical expressions of AFC and PFC for the first and second order model where received:

$$|W_1(j\omega)| = \frac{1}{\sqrt{\alpha^2+\omega^2}}.\ \arg W_1(j\omega) = -\mathrm{arctg}\frac{\omega}{\alpha};$$

$$|W_2(j\omega_1, j\omega_2)| = \frac{\beta}{\sqrt{(\alpha^2+\omega_1^2)(\alpha^2+\omega_1^2)[\alpha^2+(\omega_1+\omega_2)^2]}}.$$

$$\arg W_2(j\omega_1, j\omega_2) = -\mathrm{arctg}\frac{(2\alpha^2-\omega_1\omega_2)(\omega_1+\omega_2)}{\alpha(\alpha^2-\omega_1\omega_2)-\alpha(\omega_1+\omega_2)^2}$$

Results (first order AFC and PFC) received after procedure of identification are presented in fig. 1 (number of experiments for the model N=4).

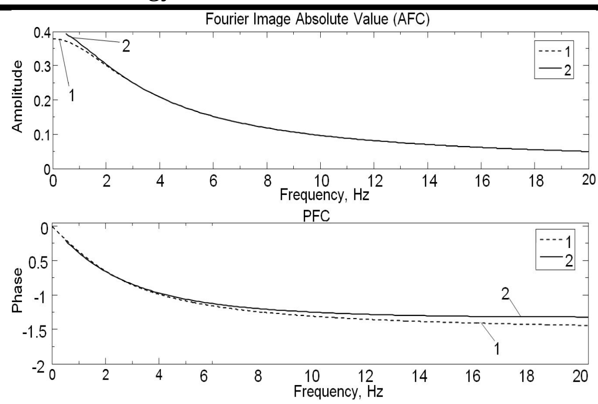

Fig. 1. First order AFC and PFC of the test object: analytically calculated values (*1*), section estimation values with number of experiments for the model N=4 (*2*)

Results (second order AFC and PFC) received after procedure of the identification are presented in fig. 2 (number of experiments for the model N=2 and N=4).

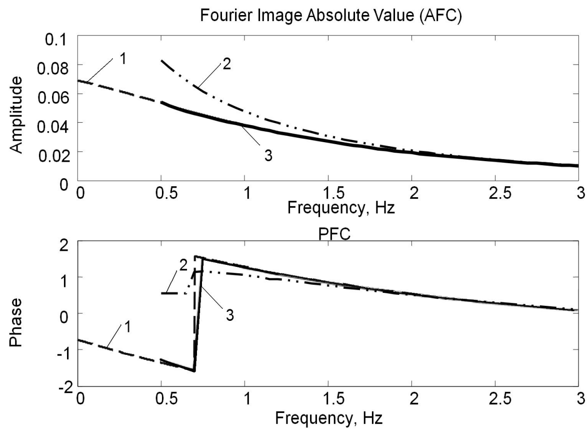

Fig. 2. Subdiagonal sections of the AFC and PFC of the test object: analytically calculated values (1), section estimation values with number of experiments N=2 (2), N=4 (3)

The hardware-software tools and the technique of radiofrequency cc identification. Experimental research of an Ultra High Frequency range CC for the purpose of identification of its multifrequency performances, characterizing nonlinear and dynamical properties of the channel are fulfilled. The Volterra model in the form of the second order polynomial is used. Thus physical CC properties are characterized by transfer functions of $W_1(j2\pi f)$ and $W_2(j2\pi f_1, j2\pi f_2)$ − by the Fourier-images of weighting functions $w_1(t)$ and $w_2(t_1, t_2)$.

Implementation of identification method on the IBM PC computer basis has been carried out using the developed software in C++ language with the usage of such classes as CWaveRecorder, CWavePlayer, CWaveReader, CWaveWriter which allow to provide rather convenient interacting with MMAPI Windows. The software allows automating the process of the test signals forming with the given parameters (amplitudes and frequencies). Also this software allows transmitting and receiving signals through an output and input section of PC soundcard, to produce segmentation of a file with the responses to the fragments, corresponding to the CC responses being researched on test polyharmonic effects with different amplitudes.

In experimental research two identical S.P.RADIO A/S, RT2048VHF VHF–radio stations (a range of operational frequencies 154,4−163,75 MHz) and IBM PC with Creative SBLive! soundcards were used. Sequentially AFC of the first and second orders were defined. The method of identification with number of experiments N=4 was applied. Structure charts of identification procedure – determinations of the 1 and 2 order AFC of CC are presented accordingly on fig. 3 and fig. 4.

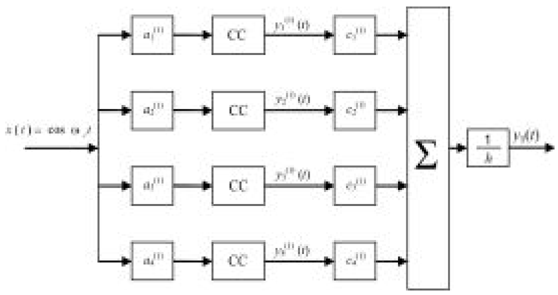

Fig. 3. The structure chart of identification using 1st order Volterra model in frequency domain, number of experiments N=4: a_1=–2h, a_2=–h, a_3=h, a a_4=2h; c_1=–1/12, c_2=–2/3, c_3=2/3, c_4=1/12.

The general scheme of a hardware–software complex of the CC identification, based on the data of input–output type experiment is presented in fig. 5.

The CC received responses $y[a_i x(t)]$ to the test signals $a_i x(t)$, compose a group of the signals,

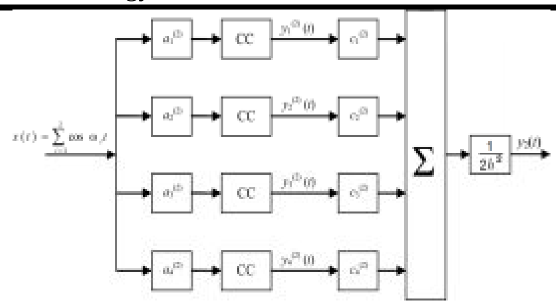

Fig. 4. The structure chart of identification using 2nd order Volterra model in frequency domain, number of experiments N=4: a_1=–2h, a_2=–h, a_3=h, a_4=2h; c_1=1/12, c_2=4/3, c_3=4/3, c_4=–1/12.

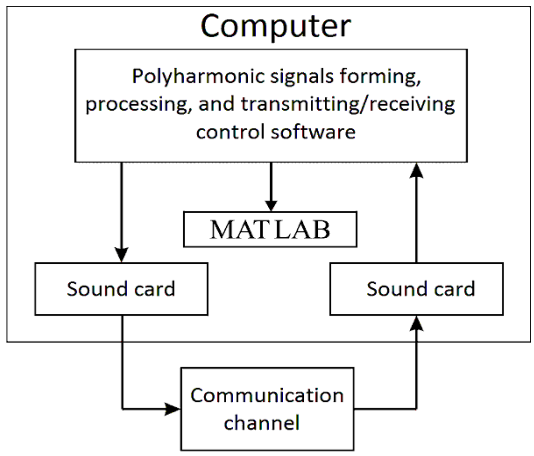

Fig. 5. The general scheme of the experiment

which amount is equal to the used number of experiments N (N=4), shown in fig.6.

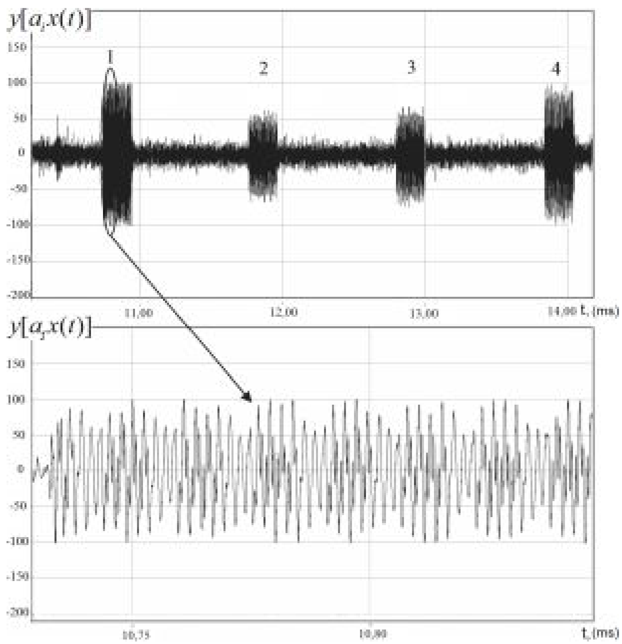

Fig. 6. The group of signals received from CC with amplitudes: -1 (*1*); 1 (*2*); -0,644 (*3*); 0,644 (*4*); N=4

In each following group the signals frequency increases by magnitude of chosen step. A cross-correlation was used to define the beginning of each received response. Information about the form of the test signals, amplitudes and corresponding to them coefficients given in [7] was used.

Maximum allowed amplitude in described experiment with use of sound card was A=0,25V (defined experimentally). The used range of frequencies was defined by the sound card pass band (20…20000 Hz), and frequencies of the test signals has been chosen from this range, taking into account restrictions specified above. Such parameters were chosen for the experiment: start frequency f_s =125 Hz; final frequency f_e =3125 Hz; a frequency change step Δf=125 Hz; to define AFC of the second order determination, an offset on frequency $\delta f=f_2-f_1$ was increasingly growing from 201 to 3401 Hz with step 100 Hz.

The weighed sum is formed from received signals – responses of each group (fig. 3 and 4). As a result we get partial component s of response of the CC $y_1(t)$ and $y_2(t)$. For each partial component of response a Fourier transform (the FFT is used) is calculated, and from received spectra only an informative harmonics (which amplitudes represents values of required characteristics of the first and second orders AFC) are taken.

The first order amplitude-frequency characteristic $|W_1(j2\pi f)|$ is received by extracting the harmonics with frequency f from the spectrum of the partial response of the CC $y_1(t)$ to the test signal $x(t)=A/2(\cos 2\pi ft)$.

The second order AFC $|W_2(j2\pi f, j2\pi(f+\delta f))|$, where $f_1=f$ и $f_2=f+\delta f$, was received by extracting the harmonics with summary frequency f_1+f_2 from the spectrum of the partial response of the CC $y_2(t)$ to the test signal $x(t)=(A/2)(\cos 2\pi f_1 t+\cos 2\pi f_2 t)$.

The wavelet denoising was used to smooth the output data of the experiment [9]. The results received after digital data processing of the data of experiments (wavelet “Coiflet” denoising) for the first and second order AFC are presented in fig. 7, 8 and 9.

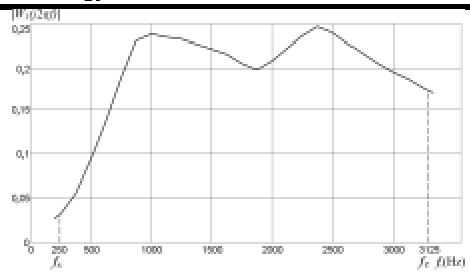

Fig.7. AFC of the first order after wavelet “Coiflet” 2nd level de–noising

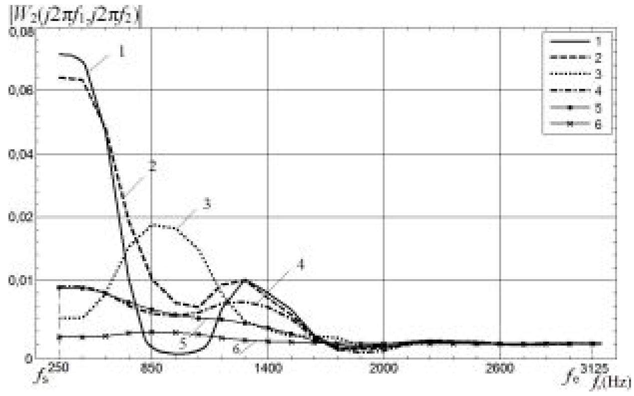

Fig. 8. Subdiagonal sections of AFCs of the second order after wavelet “Coiflet” 2nd level de-noising at different frequencies: 201 (*1*), 401 (*2*), 601 (*3*), 801 (*4*), 1001 (*5*), 1401 (*6*) Hz

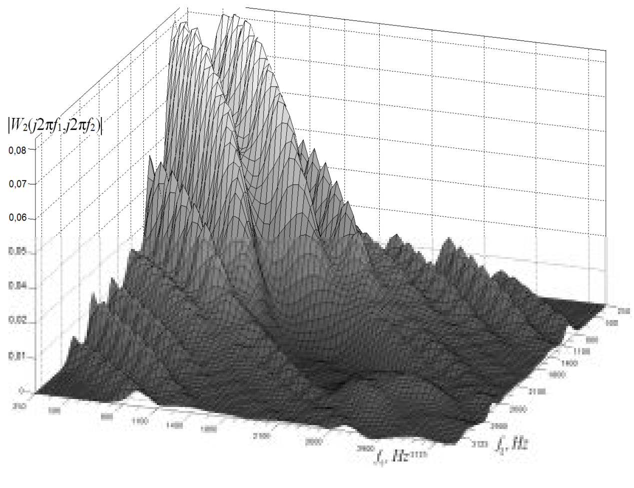

Fig. 9. Surface built of AFCs of the second order after wavelet “Coiflet” 3rd level de-noising

Conclusions. The method of determinate identification of nonlinear dynamical systems based on Volterra models using polyharmonic test signals is analyzed. To differentiate the object response for partial components we use the method based on composition of linear responses combination on test signals with different amplitudes.

New values of amplitudes are defined and they are greatly raising the accuracy of identification in compare with amplitudes and coefficients written in [10].

The modified interpolation method of identification using the hardware methodology written in [11] is applied for constructing of informational Volterra model as an APC of the first and second order for UHF band radio channel.

The received results of researches show essential nonlinearity of the CC that leads to distortions of signals in a radio broadcasting devices, reduces the important indicators of the TCS: accuracy of signals reproduction, throughput, noise immunity.

In the further researches the received frequency characteristics of the CC will be used to correct device nonlinear distortions in TCS taking into account its nonlinear characteristics.

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