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V.E. Volkov, DSc, Odessa National Academy of Food Technologies

MATHEMATICAL SIMULATION OF LAMINAR-TURBULENT TRANSITION AND THE TURBULENCE SCALE ESTIMATION

Introduction. In fluid mechanics the turbulence represents a flow pattern characterized by chaotic property changes [1]. This mode causes a rapid pressure and velocity variation in space and time. As opposed to laminar flow, the turbulent fluid stream does not flow in parallel layers with no disruption between them (it flows chaotically).

The turbulent flow is characterized by such parameters as irregularity, diffusivity (the characteristic responsible for the enhanced mixing and increased rates for transports of mass, momentum and energy in a flow) and dissipation (the turbulence dissipates rapidly as the kinetic energy is converted into internal energy through viscous shear stresses). Moreover, in turbulent flow a lot of unsteady vortices do appear and mutually interact.

There exist numerous examples of turbulence both natural (most of the atmospheric circulations, intense oceanic currents etc.) and artificial (present at various engineering fields: fluids flowing inside of machinery, circumfluent external flows over all kind of vehicles such as cars, airplanes and ships, etc.). The engineering often does require a comprehensive understanding of turbulence phenomena and knowledge of the laminar-turbulent transition laws. In many cases the turbulence scale also is a contributing crucial factor.

Analysis of reference sources. On the one hand the turbulence is rather well studied [1, 2] at a first sight. Such features of turbulence as its irregularity, diffusivity and dissipation are described from various points of view.

Turbulent flows are always highly irregular ones. For this reason, turbulence problems are usually treated rather from statistical side than applying a deterministic approach. Generally the turbulence investigations are based on the statistical theory of Kolmogorov [2], which, in its turn, departs from the Richardson's notion of turbulence [1].

On the other hand the turbulence still remains only partially understood despite multiple efforts of many leading scientists applied for well over a century [1, 3].

One of the most interesting (from practical point of view) problems of fluid dynamics is the one of laminar flow to turbulent one transition [4]. This problem is still unsolved. Up to present date we did not found a theorem correlating the non-dimensional Reynolds number to turbulence [1, 4]. Also still unresolved remains the turbulence scale estimation problem.

Research objective. This research goal consists in developing the principles for laminar-turbulent transition mathematical modeling and methods for estimating the turbulence scale.

Main Body.

Mathematical model. The continuous medium (fluid or gas) motion is described by the Euler equations (for inviscid flows) or by the Navier-Stokes equations (for viscous flows) combined with the continuity equation. In the case of compressible medium this set of equations is expanded with the energy conservation law (or the first law of thermodynamics).

The mathematical reason for a laminar flow transition into the turbulent one relates to instability of hydrodynamic equations solutions corresponding to a steady laminar flow. This instability is known as the instability of steady (laminar) flow, but its nature is mathematical (not physical).

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Despite its intricacy the mathematical theory of laminar flows stability may be generally stated in such a simple way:

— small nonstationary perturbations imposed on the one-dimensional (as a rule) stationary solution, which corresponds to steady laminar flow;

— those perturbations time-dependent increase does mean both the stability loss and transition to turbulence.

Numerous hydrodynamic stability problems can be considered as two-dimensional ones because of their symmetry.

In the two-dimensional case the perturbations are set in such a form:

$$\sim \exp(ihy + \omega t)\,, \tag{1}$$

where $h = 2\pi / \lambda$ (2)

λ — wave length;

i — unit imaginary number ($i^2 = -1$);

ω — complex number (eigen-value);

y — spatial coordinate;

t — time parameter.

Such a choice for the perturbations represented is due to the fact that every linearized perturbation can be represented (by spatial coordinate y) as a Fourier series or a Fourier integral, that does mean we can proceed to this perturbation as to the exponential type $\exp(ihy)$ elementary waves' superposition. Necessary boundary condition for perturbations is their finitude at infinity ($t \to \infty$).

As a result there is an eigen-value problem for the linearized set of the hydrodynamic equations.

If the solution of the eigen-value problem leads to inequality

$$\mathrm{Re}\omega > 0\,, \tag{3}$$

that means that we observe an instability and transition to turbulence takes its place as a result.

If the solution of the eigen-value problem leads to inequality

$$\mathrm{Re}\omega < 0\,, \tag{4}$$

that means that the flow is stable to perturbations of the exponential type (1). But this fact is not a guarantee of the flow absolute stability.

If the solution of the eigen-value problem leads to the equality

$$\mathrm{Re}\omega = 0\,, \tag{5}$$

that means that this case is "neutral" from the point of view of the stability theory, i.e. necessary is to change the problem posing (as a rule the problem formulation needs to use another — and more complicated — model) to solve the laminar flow stability problem and to define conditions for the autoturbulization.

Thus classical theory of hydrodynamic instability allows solving the laminar-turbulent transition possibility problem, considering that the inequality (2) embodies a sufficient criterion for such transition. But this theory development allows also to estimate approximately the scale of turbulence Λ.

Characteristic (secular) equation for the eigen-value ω (in the most general case) is:

$$F(\omega,\ \lambda) = 0\,, \tag{6}$$

where F — the multiparameter function (polynomial or quasipolynomial for ω as usual).

If function F does not depend on the wave length λ then the adherence of the sufficient condition (3) for instability means so-called an absolute instability, i.e., instability to perturbations with every possible wave lengths. For example such situation takes place for the stability problem of the laminar plane flame in the inviscid incompressible medium.

If the function F represents an explicit function of the wave length λ then formally two cases are possible:

1) absolute instability;

2) instability in respect to perturbations with the bounded spectrum of the wave lengths.

The second case is the most probable one. In this case the instable wave lengths spectrum may be either discrete or continuous ones. Therewith the number of instable ($\mathrm{Re}\omega > 0$) roots of the secular equation (6) may be either finite (for example if function F is polynomial), or infinite one (for example if function F is quasipolynomial). It is possible to find out the wave length λ_m that corresponds to the perturbation with the fastest amplitude growth rate. The wave length λ_m can be taken as an approximate estimate for the turbulence scale Λ.

If the stability problem is solved for viscous medium then critical (transition) Reynolds number Re_λ^* may be calculated by the wave length λ_m. That is

$$\mathrm{Re}_\lambda = \frac{\lambda_m \nu}{u_1}, \quad (8)$$

and

$$\mathrm{Re}_\lambda^* = \frac{\lambda_m \nu}{u_1}, \quad (9)$$

where ν — kinematic-viscosity coefficient,

u_1 — characteristic process velocity (for example velocity of the constant laminar flow, burning velocity or detonation velocity).

If

$$\mathrm{Re}_\lambda > \mathrm{Re}_\lambda^*, \quad (10)$$

then we observe the development of instability and the transition of laminar flow to turbulence.

For every particular case the wave length λ_m (and the turbulence scale Λ) is connected with the global typical size of the problem (such as a tube diameter, a channel width, a flame sphere radius etc.)

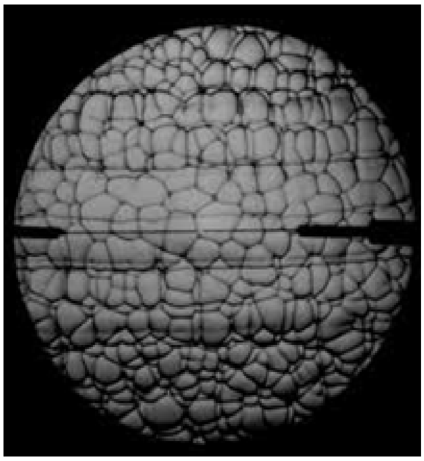

Fig. 1. A sample of the cellular flame surface

Results. The above-mentioned method for the turbulence scale estimation is applied to the research of the cellular flame (Fig. 1) as the turbulent flame well-known species. The stability problem is solved for the viscous incompressible medium with resulting two-dimensional time-dependent solutions of the Navier-Stokes equations obtained [5] analytically. The theoretical results issuing for λ_m [5] are in good agreement with the cell sizes Λ in experiments.

The turbulent flame structure investigation provides the possibility to study deflagration-to-detonation transition (DDT), that represents interest both for explosion safety and for the pulse detonation engine designing [6]. Thus analytical estimates for DDT run-up distance and for the detonation wave formation time are obtained [5] by rather simple algebraic formulae.

The stability and structure of the self-sustaining detonation wave propagating in a cylindrical tube (considered as a model combustor) is studied [7]. The stability problem is therefore solved for an inviscid compressible medium. The method used for analysis of perturbation development in the detonation wave provides a satisfactory prediction for the detonation structure. According to the effected analysis an integer number of nonuniformities having a mean size λ_m is packed in the tube cross-section. This number can be found precisely. Thus the solution for single-head (Fig. 2), double-head and multihead detonations can be obtained. In such a way obtained results are also in good agreement with experimental data [8] and with different numerical simulations for the turbulent structure of gaseous detonation.

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Fig. 2. Photograph of the single-head detonation in gas mixture. Picture taken through a slot, positioned in parallel to the tube axis

The suggested method never takes in consideration the laminar-turbulent transitions details that represents the main imperfection of the elaborated method.

By now there exist a huge number of different models (including semi-empirical) for the detailed turbulent flow calculations. Almost all of these models are implemented in programme codes (for example in Open source Field Operation And Manipulation — Open FOAM, which is a C^{++} toolbox for the solution of continuum mechanics problems, including computational fluid dynamics). But such programmes implementation takes a lot of computing time. And the suggested method is so simple that it is rather useful for problems of turbulent scale estimating without flow detalization. Its main advantage relates to the possibility of turbulence scale Λ calculations in the real time mode.

Conclusions. The laminar-turbulent transition mathematical model and the turbulence scale estimating methods elaborated are based on the hydrodynamic stability problem solution.

The suggested mathematical model is universal, but its implementation in every particular case requires to get a characteristic equation in an explicit form and to solve this equation (analytically or numerically).

The elaborated model is mathematical but it is not physical (or mechanical). It can't explain the real mechanism of the laminar-turbulent transition.

The suggested method does not allow to get the detailed picture of turbulent flow but provides a possibility to calculate the turbulence scale in a short period of time.

Specific results are obtained for the laminar-turbulent transitions in such flows as:

1) combustion wave propagating in viscous medium;

2) self-sustaining detonation wave propagating in inviscid medium.

In both cases the calculation data are in good agreement with experimental ones. This fact proves the suggested theory correctness and feasibility.

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АНОТАЦІЯ / АННОТАЦИЯ / ABSTRACT

В.Е. Волков. **Математичне моделювання переходу ламінарної течії в турбулентну та оцінка масштабу турбулентності.** Досліджуваний в даній роботі процес переходу ламінарної течії в турбулентну викликає інтерес для різних галузей науки і техніки. Одним з найважливіших з практичної зору питань є питання про масштаб турбулентності. Метою статті є розвинення детермінованого математичного моделювання переходу ламінарної течії в турбулентну. Розроблено оригінальний метод оцінювання масштабу турбулентності на основі розв'язання задачі про стійкість ламінарної течії і обчислення довжини хвилі збурення з найбільшою швидкістю зростання. Дієвість методу демонструє його застосування до дослідження нестійкості та структури хвиль горіння і детонації.
Ключові слова: : ламінарність, турбулентність, нестійкість, математична модель, горіння, детонація.

В.Э. Волков. **Математическое моделирование перехода ламинарного течения в турбулентное и оценка масштаба турбулентности.** Исследуемый в данной работе процесс перехода ламинарного течения в турбулентное представляет интерес для различных областей науки и техники. Одним из важнейших с практической точки зрения является вопрос о масштабе турбулентности. Целью статьи является развитие детерминированного математического моделирования перехода ламинарного течения в турбулентное. Разработан оригинальный метод оценки масштаба турбулентности на основе решения задачи об устойчивости ламинарного потока и вычисления длины волны возмущения с наибольшей скоростью роста. Действенность метода демонстрирует его применение к исследованию неустойчивости и структуры волн горения и детонации.
Ключевые слова: ламинарность, турбулентность, неустойчивость, математическая модель, горение, детонация**.**

V.E. Volkov. **Mathematical simulation of laminar-turbulent transition and the turbulence scale estimation.** The laminar-turbulent transition, analyzed in this work represents a particular interest for various branches of science and engineering. From the practical point of view, the turbulence scale represents one of the most important problems. This article goal of the article consisted in developing a deterministic mathematical model of laminar-turbulent transition effect. Elaborated is an original method for estimating the turbulence scale on the basis of the laminar flow stability problem solution and calculation of the wave length that corresponds to the fastest growth rate perturbation. This method efficiency is demonstrated by its application to investigate the flame waves and detonations structure and instability.
Keywords: laminarity, turbulence, instability, mathematical model, deflagration, detonation.

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