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# ANALYSIS OF NONLINEAR SYSTEM IDENTIFICATION ACCURACY BASED ON VOLTERRA MODEL IN FREQUENCY DOMAIN

*Abstract. The accuracy of the interpolation method for identification of nonlinear dynamical systems based on the Volterra model in the frequency domain is studied. To highlight the n-th partial component in the response of the system to the test signal the n-th partial derivative of the response using the test signal amplitude is found and its value is taken at zero. The polyharmonic signals are used as test ones. The algorithmic and software toolkit is developed for identification processes. This toolkit is used for informational model of test system building. The model is built as a first, second and third order amplitude–frequency characteristics and phase–frequency characteristics. The comparison of obtained characteristics with standard is given.*

*Keywords: identification, nonlinear dynamic systems, Volterra models, multifrequency characteristics, polyharmonic signals.*

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# АНАЛИЗ ТОЧНОСТИ ИДЕНТИФИКАЦИИ НЕЛИНЕЙНОЙ СИСТЕМЫ НА ОСНОВЕ МОДЕЛИ ВОЛЬТЕРРА В ЧАСТОТНОЙ ОБЛАСТИ

*Аннотация. Проведено исследование точности интерполяционного метода идентификации нелинейных динамических систем на основе модели Вольтерра в частотной области. Данный метод заключается в n–кратном дифференцировании откликов идентифицируемой нелинейной системы по значению параметра–амплитуды тестовых полигармонических сигналов. Был разработан программный комплекс идентификации, который используется для построения информационной модели нелинейной системы в виде АЧХ и ФЧХ первого, второго и третьего порядков.*

*Ключевые слова: идентификация, нелинейные динамические системы, модели Вольтерра, амплитудно-частотные характеристики, полигармонические сигналы.*

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# АНАЛІЗ ТОЧНІСТІ ІДЕНТИФІКАЦІЇ НЕЛІНІЙНОЇ СИСТЕМИ НА ОСНОВІ МОДЕЛІ ВОЛЬТЕРРА В ЧАСТОТНІЙ ОБЛАСТІ

*Анотація. Проведено дослідження точності інтерполяційного методу ідентифікації нелінійних динамічних систем на основі моделі Вольтерра в частотній області. Даний метод полягає в n-кратному диференціюванні відгуків нелінійної системи, яка ідентифікується за значенням параметру–амплітуди тестових полігармонійних сигналів. Розроблено програмний комплекс ідентифікації, який використовується для побудови інформаційної моделі нелінійної системи у вигляді АЧХ та ФЧХ першого, другого та третього порядків.*

*Ключові слова: ідентифікація, нелінійні динамічні системи, моделі Вольтерра, амплітудно-частотні характеристики, полігармонічні сигнали.*

It is necessary to consider technical conditions of the communication channels (CC) operation for effective data transfer. Changes in environmental conditions cause reducing the transmission data rate: in the digital CC - up to a full stop of the transmission, in analog CC - to the noise and distortion of the transmitted signals. The new methods and supporting toolkit are developing to automate the measurement of parameters and taking into account the characteristics of the CC. This toolkit allows obtaining the informational and mathematical model of such nonlinear dynamic object, as the CC [1],

i.e. to solve the identification problem.

Modern continuous CCs are nonlinear stochastic inertial systems. The model in the form of integro-power Volterra series used to identify them [2–5].

The nonlinear and dynamic properties of such system are completely characterized by a sequence of multidimensional weighting functions – Volterra kernels).

Building a model of nonlinear dynamic system in the form of a Volterra series lies in the choice of the test actions form. Also it uses the developed algorithm that allows determining the Volterra kernels and their Fourier-images for the measured responses (multidimensional am-

plitude–frequency characteristics (AFC) and phase–frequency characteristics (PFC)) to simulate the CC in the time or frequency domain, respectively [7].

The additional research of new method of nonlinear dynamical systems identification, based on the Volterra model in the frequency domain is proposed. This method lies in n-fold differentiation of responses of the identifiable system by the amplitude of the test polyharmonic signals. The developed identification toolkit is used to build information model of the test nonlinear dynamic object in the form of the first, second and third order model [8].

**Volterra models in frequency domain.** Generally, "input–output" type ratio for nonlinear dynamical system can be presented by Volterra series [2, 3]:

$$y[x(t)] = \sum_{n=1}^{\infty} y_n[x(t)] =$$

$$= \sum_{n=1}^{\infty} \int_0^{\infty} \ldots \int w_n(\tau_1, \tau_2, \ldots, \tau_n) \prod_{r=1}^{n} x(t - \tau_r) d\tau_r$$

where $x(t)$ and $y(t)$ are input and output signals of system respectively; $w_n(\tau_1, \tau_2, \ldots, \tau_n)$ – weight function or $n$-order Volterra kernel; $y_n[x(t)]$ – $n$–th partial component of object response [4].

In practice, Volterra series are replaced by polynomial and generally limited to several first members of the series. Identification of nonlinear dynamical system in the form of Volterra series consists of determination of $n$-dimensional weighting functions $w_n(\tau_1, \ldots, \tau_n)$ or their Fourier–images $W_n(j\omega_1, \ldots, j\omega_n)$ – $n$-dimensional transfer functions, accordingly to system modeling in time or frequency domain [5, 6].

Identification of nonlinear system in frequency domain consists in determination of absolute value and phase of multidimensional transfer function at given frequencies – multidimensional AFC $/W_n(j\omega_1, j\omega_2, \ldots, j\omega_n)/$ and PFC arg $W_n(j\omega_1, j\omega_2, \ldots, j\omega_n)$ which are defined by formulas:

$$|W_n(j\omega_1, j\omega_2, \ldots, j\omega_n)| = \sqrt{\begin{array}{l}[\operatorname{Re}(W_n(j\omega_1, j\omega_2, \ldots, j\omega_n))]^2 \\ + [\operatorname{Im}(W_n(j\omega_1, j\omega_2, \ldots, j\omega_n))]^2\end{array}},$$

$$\arg W_n(j\omega_1, j\omega_2, \ldots, j\omega_n) = \operatorname{arctg}\frac{\operatorname{Im}(W_n(j\omega_1, j\omega_2, \ldots, j\omega_n))}{\operatorname{Re}(W_n(j\omega_1, j\omega_2, \ldots, j\omega_n))}$$

where Re and Im – accordingly real and imaginary parts of a complex function of n variables.

An interpolation method of identification of the nonlinear dynamical system based on Volterra series is used [8–9]. It is used *n*-fold differentiation of a target signal on parameter–amplitude *a* of test actions to separate the responses of the nonlinear dynamical system on partial components $\hat{y}_n(t)$ [9].

In [10] it is defined that during determination of multidimensional transfer functions of nonlinear systems it is necessary to consider the imposed constraints on choice of the test polyharmonic signal frequencies. It provides an inequality of combination frequencies in output signal harmonics: $\omega_1 \neq 0$, $\omega_2 \neq 0$ and $\omega_1 \neq \omega_2$ for the second order identification procedure, and $\omega_1 \neq 0$, $\omega_2 \neq 0$, $\omega_3 \neq 0$, $\omega_1 \neq \omega_2$, $\omega_1 \neq \omega_3$, $\omega_2 \neq \omega_3$, $2\omega_1 \neq \omega_2 + \omega_3$, $2\omega_2 \neq \omega_1 + \omega_3$, $2\omega_3 \neq \omega_1 + \omega_2$, $2\omega_1 \neq \omega_2 - \omega_3$, $2\omega_2 \neq \omega_1 - \omega_3$, $2\omega_3 \neq \omega_1 - \omega_2$, $2\omega_1 \neq -\omega_2 + \omega_3$, $2\omega_2 \neq -\omega_1 + \omega_3$ и $2\omega_3 \neq -\omega_1 + \omega_2$ for the third order identification procedure.

Given method was fully tested on a nonlinear test object described by Riccati equation:

$$\frac{dy(t)}{dt} + \alpha y(t) + \beta y^2(t) = u(t).$$

Analytical expressions of AFC and PFC for the first, second and third order model were received:

$$|W_1(j\omega)| = \frac{1}{\sqrt{\alpha^2 + \omega^2}}, \quad \arg W_1(j\omega) = -\operatorname{arctg}\frac{\omega}{\alpha};$$

$$|W_2(j\omega_1, j\omega_2)| = \frac{\beta}{\sqrt{(\alpha^2 + \omega_1^2)(\alpha^2 + \omega_2^2)[\alpha^2 + (\omega_1 + \omega_2)^2]}},$$

$$\arg W_2(j\omega_1, j\omega_2) = -\operatorname{arctg}\frac{(2\alpha^2 - \omega_1\omega_2)(\omega_1 + \omega_2)}{\alpha(\alpha^2 - \omega_1\omega_2) - \alpha(\omega_1 + \omega_2)^2}.$$

$$|W_3(j\omega_1, j\omega_2, j\omega_3)| = \sqrt{[\text{Re}(W_3(j\omega_1, j\omega_2, j\omega_3))]^2 + [\text{Im}(W_3(j\omega_1, j\omega_2, j\omega_3))]^2} = \frac{2\beta^2}{3} \frac{1}{\sqrt{[\alpha^2 + (\omega_1 + \omega_2 + \omega_3)^2](\alpha^2 + \omega_1^2)(\alpha^2 + \omega_2^2)(\alpha^2 + \omega_3^2)}} \times$$

$$\times \frac{\sqrt{[3\alpha^2 - (\omega_1 + \omega_3)(\omega_1 + \omega_2) - (\omega_1 + \omega_2)](\omega_2 + \omega_3) - (\omega_1 + \omega_3)(\omega_2 + \omega_3)]^2 + 16\alpha^2(\omega_1 + \omega_2 + \omega_3)^2}}{\sqrt{[\alpha^2 + (\omega_1 + \omega_2)^2][\alpha^2 + (\omega_1 + \omega_3)^2][\alpha^2 + (\omega_2 + \omega_3)^2]}}$$

$$\arg W_3(j\omega_1, j\omega_2, j\omega_3) = \text{arctg} \frac{\text{Im} W_3(j\omega_1, j\omega_2, j\omega_3)}{\text{Re} W_3(j\omega_1, j\omega_2, j\omega_3)} = -\text{arctg} \frac{DA - CB}{AB + CD},$$

where $A = 3\alpha^2 - 3\omega_1\omega_2 - 3\omega_2\omega_3 - 3\omega_1\omega_3 - \omega_1^2 - \omega_2^2 - \omega_3^2$; $C = 4\alpha(\omega_1 + \omega_2 + \omega_3)$; $B = uw - vx$; $D = vw + ux$;

$$u = \alpha^3 - \alpha\omega_1\omega_3 - \alpha\omega_2\omega_3 - \alpha(\omega_1 + \omega_2 + \omega_3);$$
$$v = (\omega_1 + \omega_2 + \omega_3)(2\alpha^2 - \omega_1\omega_3 - \omega_2\omega_3);$$
$$w = (\alpha^2 - \omega_1\omega_2 - \omega_2\omega_3)(\alpha^2 - \omega_1\omega_2 - \omega_1\omega_3) - \alpha^2(\omega_1 + \omega_2 + \omega_3)^2;$$
$$x = \alpha(\omega_1 + \omega_2 + \omega_3)(2\alpha^2 - 2\omega_1\omega_2 - \omega_1\omega_3 - \omega_2\omega_3).$$

**The techniques of test object identification.** The main purpose was to identify the multifrequency performance which characterized nonlinear and dynamical properties of CC as a nonlinear test object. Volterra model in the form of the second order polynomial is used. Thus, physical CC properties are characterized by transfer functions of $W_1(j\omega)$, $W_2(j\omega_1, j\omega_2)$, $W_3(j\omega_1, j\omega_2, j\omega_3)$ − by Fourier-images of weight functions $w_1(t)$, $w_2(t_1, t_2)$ and $w_3(t_1, t_2, t_3)$.

Structure charts of identification procedure – determinations of the 1–, 2– and 3–order AFC of CC are presented accordingly on fig. 1–3.

The weighted sum is formed from received signals – responses of each group (fig. 1–3). As a result the partial components of CC responses $y_1(t)$, $y_2(t)$ and $y_3(t)$ are got. For each partial component of response the Fourier transform (the FFT is used) is calculated, and from received spectrum only an informative harmonics (which amplitudes represent values of required characteristics of the first, second and third orders AFC) are taken.
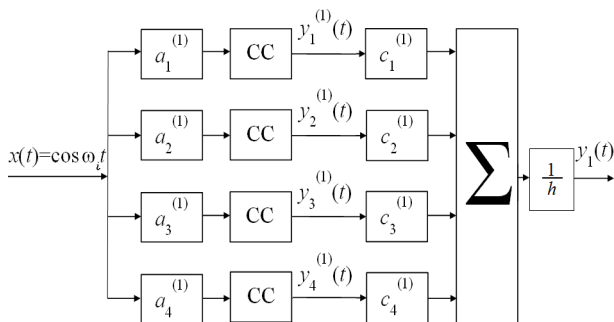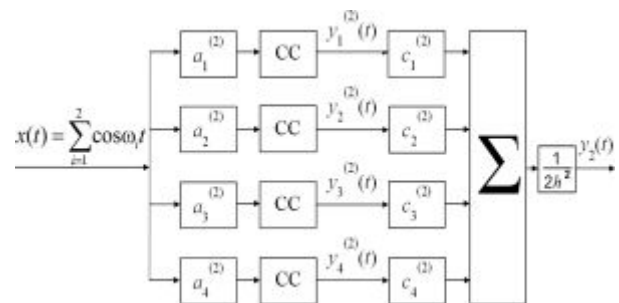


Fig. 2. The structure chart of identification procedure using the second order Volterra model in frequency domain, number of experiments N=4
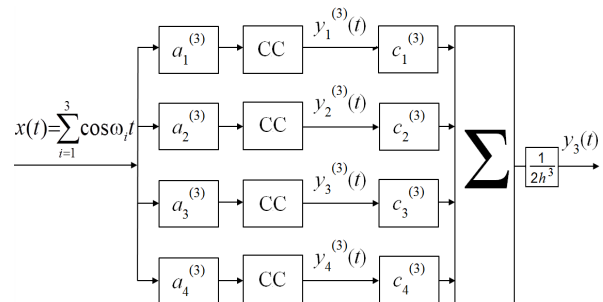


Fig. 3. The structure chart of identification procedure using the third order Volterra model in frequency domain, number of experiments N=4

The first order AFC $|W_1(j\omega)|$ and PFC $\arg W_1(j\omega)$ is received by extracting the harmonics with frequency f from the spectrum of the CC partial response $y_1(t)$ to the test signal $x(t)=A/2(\cos\omega t)$.

The second order AFC $|W_2(j\omega, j(\omega + \delta_1\omega))|$ and PFC $\arg W_2(j\omega, j(\omega + \delta_1\omega))$, where $\omega_1=\omega$ и $\omega_2=\omega + \delta_1\omega$, was received by extracting the har-



Fig. 1. The structure chart of identification procedure using the first order Volterra model in frequency domain, number of experiments N=4

monics with summary frequency $\omega_1+\omega_2$ from the spectrum of the CC partial response $y_2(t)$ to the test signal $x(t)=(A/2)(\cos\omega_1 t+\cos\omega_2 t)$.

The third order AFC $|W_3(j\omega,j(\omega+\delta_1\omega)),j(\omega+\delta_2\omega))|$ and PFC $\arg W_3(j\omega,j(\omega+\delta_1\omega)),j(\omega+\delta_2\omega))$, where $\omega_1=\omega$, $\omega_2=\omega+\delta_1\omega$ and $\omega_3=\omega+\delta_2\omega$, was received by extracting the harmonics with summary frequency $\omega_1+\omega_2+\omega_3$ from the spectrum of the CC partial response $y_2(t)$ to the test signal $x(t)=(A/2)(\cos\omega_1 t+\cos\omega_2 t+\cos\omega_3 t)$.

The results (first, second and third order AFC and PFC) which had been received after procedure of identification are represented in fig. 4–6 (number of experiments for the model $N$=4).
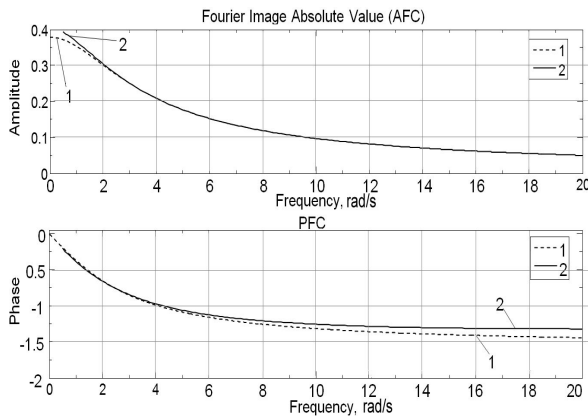


Fig. 4. First order AFC and PFC of the test object: analytically calculated values (1), section estimation values with number of experiments for the model $N$=4 (2)
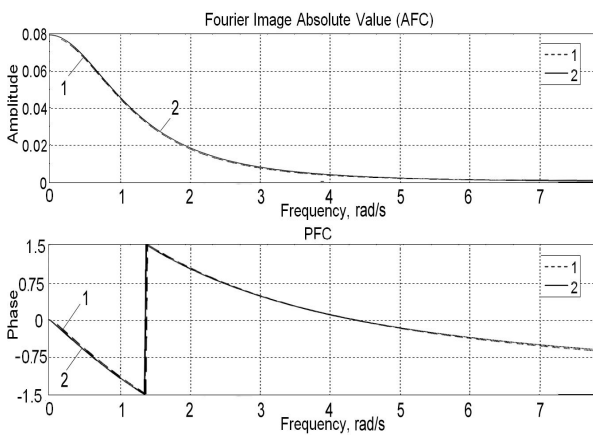


Fig. 5. Second order AFC and PFC of the test object: analytically calculated values (1), subdiagonal cross-section values with number of experiments for the model $N$=4 (2), $\delta_1\omega$=0,01 rad/s
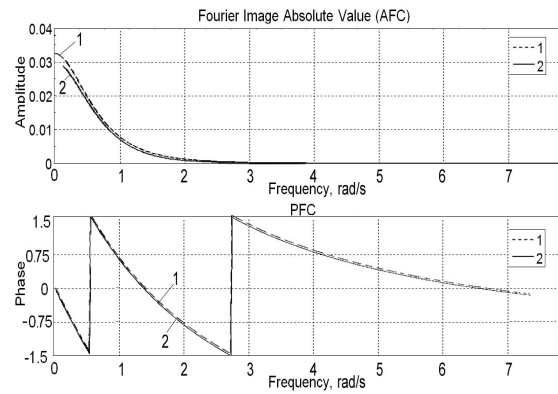


Fig. 6. Third order AFC and PFC of the test object: analytically calculated values (1), subdiagonal cross-section values with number of experiments for the model $N$=6 (2), $\delta_1\omega$=0,01 rad/s, $\delta_2\omega$=0,1 rad/s

The surfaces shown on fig. 7–10 are built from subdiagonal cross-sections which were received separately. $\delta_1\omega$ was used as growing parameter of identification with different value for each cross–section in second order characteristics. Fixed value of $\delta_2\omega$ and growing value of $\delta_1\omega$ were used as parameters of identification to obtain different value for each cross-section in third order characteristics.

The second order surfaces for AFC and PFC which had been received after procedure of the test object identification are shown in fig. 7–8 (number of experiments for the model $N$=4).

The third order surfaces for AFC and PFC which had been received after procedure of the test object identification are presented in fig. 9-10 (number of experiments for the model $N$=6).
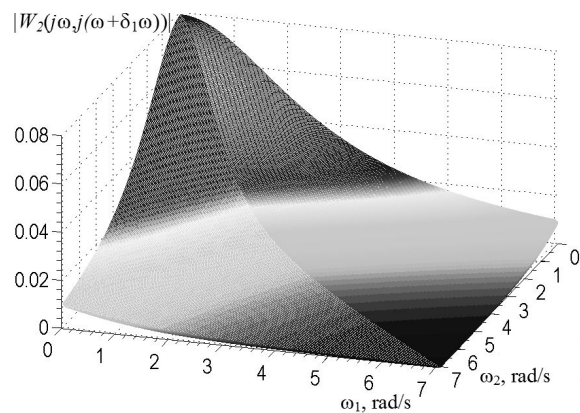


Fig. 7. Surface of the test object AFC built of the second order subdiagonal cross-sections received for $N$=4, $\delta_1\omega$=0,01 rad/s

$\arg W_2(j\omega, j(\omega+\delta_1\omega))$
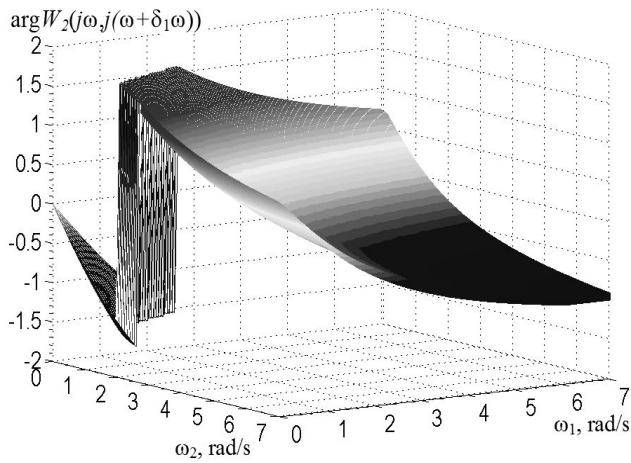


Fig. 8. Surface of the test object PFC built of the second order subdiagonal cross-sections received for $N$=4, $\delta_1\omega$=0,01 rad/s

$|W_3(j\omega, j(\omega+\delta_1\omega)), j(\omega+\delta_2\omega))|$



Fig. 9. Surface of the test object AFC built of the third order subdiagonal cross-sections received for $N$=6, $\delta_1\omega$=0,01 rad/s, $\delta_2\omega$=0,1 rad/s

$|W_3(j\omega, j(\omega+\delta_1\omega)), j(\omega+\delta_2\omega))|$
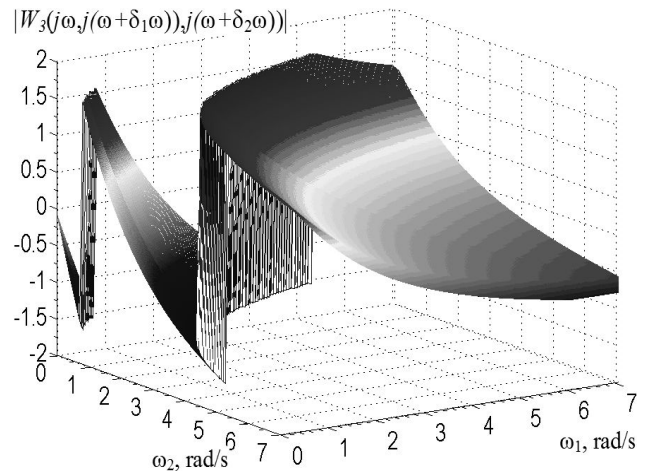


Fig. 10. Surface of the test object PFC built of the third order subdiagonal cross-sections received for $N$=6, $\delta_1\omega$=0,01 rad/s, $\delta_2\omega$=0,1 rad/s
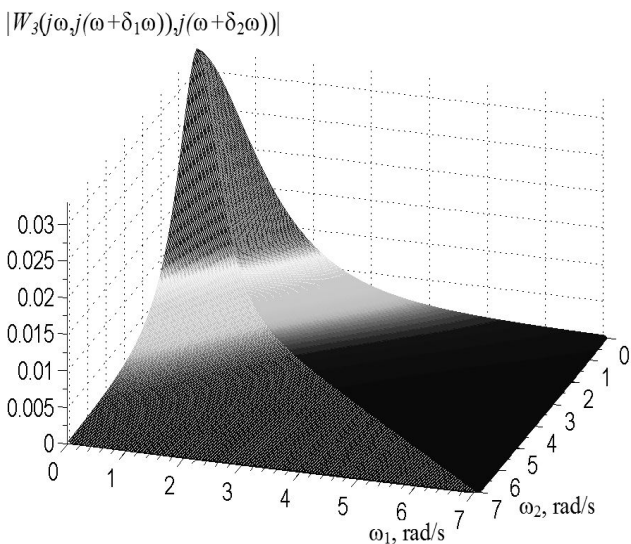
Numerical values of identification accuracy using interpolation method for the test object are represented in table.

**Conclusions.** The method based on Volterra model using polyharmonic test signals for identification nonlinear dynamical systems is analyzed. To differentiate the responses of object for partial components we use the method based on composition of linear responses combination on test signals with different amplitudes.

New values of test signals amplitudes were defined and model were validated using the test object. Excellent accuracy level for received model is achieved as in liner model so in in nonlinear ones.

Received results had confirmed significant nonlinearity of the test object characteristics that leads to distortions of signals in different type radio devices.

1. Numerical values of identification accuracy using interpolation method

| Kernel order, $k$ | Experiments quantity / approximation order, $N$ | AFC relative error, % | PFC relative error, % |
|---|---|---|---|
| 1 | 2 | 2.1359 | 2.5420 |
| | 4 | 0.3468 | 2.0618 |
| | 6 | 0.2957 | 1.9311 |
| 2 | 2 | 30.2842 | 76.8221 |
| | 4 | 2.0452 | 3.7603 |
| | 6 | 89.2099 | 5.9438 |
| 3 | 4 | 10.981 | 1.628 |
| | 6 | 10.7642 | 1.5522 |

References

1. Giannakis, G. B., Serpedin, E. A bibliography on nonlinear system identification and its applications in signal processing, communications and biomedical engineering. Signal Processing – EURASIP, Elsevier Science B.V. – 81(3). – 2001. – P. 533–580 [in English].

2. Westwick, D. T. Methods for the Identification of Multiple–Input Nonlinear Systems, Departments of Electrical Engineering and Biomedical Engineering, McGill University, Montreal, Quebec, Canada, 1995 [in English].

3. Doyle, F. J, Pearson, R. K., Ogunnaike B. A. Identification and Control Using Volterra Models. Published Springer Technology & Industrial Arts. – 2001. – 314 p. [in English].

4. Boyd, S., Jang, Y. S., Chua, L. O. Measuring Volterra Kernels. IEEE Trans. on Circuits and Systems. – 1983. –Vol. CAS-30. – No.8. – P. 571–578 [in English].

5. Peddanarappagari, K. V., Brandt-Pearce M. Volterra Series Approach for Optimizing Fiber–Optic Communications System Designs. J. Lightwave Tech., 16 (11). – 1998. – P. 2046–2055 [in English].

6. Chang, K. F., Binh, L. N., Cambrell, G. K., Dabke, K. P. Nonlinear frequency domain modeling of optical fibre transmission systems by Volterra series. IOOC/OECC Conference, Sydney, July 2001.– P. 360–362 [in English].

7. Pavlenko, V. D. Identification of Nonlinear Dynamic Systems in the Form of the Volterra Kernels on the Basis of the Data of Pulse Response Measurements. Electronic Modeling. – 2010. – Vol. 32. – № 3. – P. 3–18. [in Russian].

8. Pavlenko, V. D., Speranskyy, V. O. Identification of Nonlinear Dynamical Systems Using Volterra Model with Interpolation Method in Frequency Domain // Electrotechic and Computer Systems. – 2012. – № 05 (81). – P. 229–234 [in English].

9. Schetzen, M. The Volterra and Wiener Theories of Nonlinear Systems. Wiley & Sons. – New York : – 1980. –531 p. [in English].

10. Pavlenko, V. D., Issa, S. I. M. The Limitation of the choice frequencies test polyharmonic signals for the identification nonlinear system. Proceedings of Odessa Polytechnic University, Ukraine. – 1 (31). – 2008. – P. 107–113 [in Russian].

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