ISSN 2076-2429 (print)
ISSN 2223-3814 (online)

UDC 621.039

V.A. Tarasov, DSc, Prof.,
S.A. Chernezhenko, PhD,
A.A. Kakaev,
V.V. Urbanevich
Odessa National Polytechnic University, 1 Shevchenko Ave., Odessa, Ukraine, 65044; e-mail: vtarasov@ukr.net

THE NEUTRON MODERATION THEORY, TAKING INTO ACCOUNT A MODERATING MEDIUM TEMPERATURE

В.О. Тарасов, С.А. Чернеженко, А.О. Какаев, В.В. Урбаневич. **Теорія сповільнювання нейтронів, що враховує температуру сповільнюваючого середовища.** У даній роботі в рамках газової моделі на основі рішення кінематичної задачі про пружне розсіяння нейтрона на ядрі в «Л» – системі в загальному випадку, тобто, коли до розсіювання не тільки нейтрон, а й ядро володіє довільно заданим вектором швидкості в «Л» – системі, вперше отримано аналітичний вираз для закону розсіювання нейтронів, що включає температуру уповільнюваючого середовища як параметр, а також, отримані щільності потоку і спектри уповільнення нейтронів для ізотропного джерела нейтронів у реакторному середовищі, також залежні від температури середовища і справедливі для всіх енергій нейтронів спектра поділу (за винятком енергій порівнянних з енергіями міжатомного або міжмолекулярної взаємодії для уповільнюючого середовища, тобто, при необхідності виходу за рамки газової моделі). Отримані вирази для спектрів нейтронів, що уповільнюються, дозволяють по-новому інтерпретувати фізичну природу процесів, що визначають вид спектра нейтронів в тепловій області нейтронів.
Ключові слова: теорія уповільнення, нейтрони, ядерні реактори, температура середовища

V.A. Tarasov, S.A. Chernezhenko, A.A. Kakaev, V.V. Urbanevich. **The neutron moderation theory, taking into account a moderating medium temperature.** In this paper, in the framework of the gas model, based on the solution of the kinematic problem of the neutron elastic scattering on the nucleus in the “L” system in the general case, that is, when prior to the scattering not only the neutron, but also the kernel has an arbitrary given velocity vector in the “L” system, an analytical expression for the neutron scattering law was first obtained, which includes the temperature moderating medium as a parameter, as well as there were received flux densities and neutron decay spectra for an isotropic neutron source in the reactor medium, also dependent on the temperature of environment and fair for all neutron energy spectrum division (excluding energy of interatomic or intermolecular interactions for moderating medium, if necessary to go beyond the gas model). The expressions obtained for the spectrums of moderating neutrons allow reinterpreting the physical nature of the processes that determine the form of the neutron spectrum in the thermal neutrons.
Keywords: moderation theory, neutrons, nuclear reactors, the temperature of medium

1. Introduction. An important link in the theory of neutron physics cycle of nuclear reactors is the theory of neutron moderation [1 – 7]. Traditional for today's nuclear reactor physics the neutron moderation theory developed in the framework of the gas model, that is within the framework of this model and neglecting the interaction between neutrons and the nuclei moderating medium, although attempts were made that included the interaction between the nuclei of the moderating medium, for example, in [2, 3]. The traditional theory of neutron moderation is based on the law of probability of the energy distribution of elastically scattered neutrons in the laboratory coordinate system (“L” – system) (neutron scattering law, for example, [4, 6]), which is based on solving the problem of the kinematic elastic scattering of neutrons by nuclei of the reactor active zone [1 – 6]. Kinematic problem of elastic neutron scattering on a nucleus in the “L” – system refers to the kinematic problem of two-particle system and has been solved. However beautiful and compact analytical solution of this problem can be obtained only in the case when the nucleus rests in the “L” – system before scattering . In the general case, when not only the neutron scattering, but the nucleus has arbitrarily given velocity vector in the “L” – system, the solution of this problem is a set of cumbersome expressions, due to the fact that an intermediate solution of the problem includes cosines of the angles between velocity vectors of the neutron and the nucleus located in the “C” – system, and that is why the final solution of the problem in the “L” – system requires a transformation of the cosines of the “C” – system to “L” – system, but that is several relations transforming the unit vectors, defining the direction of the velocity

DOI 10.15276/opu.2.52.2017.12

vectors of the neutron and the nucleus after scattering, from the "C" – system to the "L" – system. Therefore, reduction of the solving problem to a single analytical expression in this case loses all meaning. However, the solution of the problem in this case has proved itself well for computational operations and result may be obtained by computer calculation. This approach to finding a solution of kinematic problem of elastic neutron scattering on a nucleus today is implemented in software codes reactor, for example, MCNP4, GEANT4 and others. Indeed, this approach allows us to obtain the spectrum of the neutrons slowed down by the Monte Carlo method, that is producing a computer calculation of kinematic problem of elastic neutron scattering on a nucleus, which takes huge amount of time, asking for each calculation of initial velocity of the neutron and the nucleus via a random counter and getting the resulting spectrum of slowing-down neutrons by averaging all realizations of all accumulated results of the calculations. It should be noted that today mainly reactor program codes are used in practical calculations requiring neutron spectra.

However, the traditional neutron moderation theory of modern physics of nuclear reactors due to lack of alternative except for computer simulations, is based on the above analytical solution of the kinematic problem of elastic neutron scattering on a nucleus, which is obtained for the case when the nucleus scattering rests in the "L" – system, that is the traditional theory of neutron moderation neglects the thermal motion of nuclei of the moderating medium, which is physically acceptable, if we assume that the kinetic energies of moderated neutrons are much higher than the energies of thermal motion of the nuclei. As a consequence of this, the neutron scattering law and analytical expressions for Fermi neutron moderation spectrum arising from it do not contain moderating medium temperature. In order to somehow close this significant drawback of the theory of neutron moderation, until today nothing else was offered except an artificial way (in the sense that it does not have strict manner of scattering spectrum of moderated neutrons from the scattering law) to complement Fermi neutron moderation spectrum in thermal neutron energy with Maxwell neutron spectrum, and, for the tasks of Maxwell neutron spectrum in the first place we need to recalculate the moderating medium temperature T in the temperature of the neutron gas T_n according to the formula $T_n = T\left[1 + 1.8\dfrac{\Sigma_a(kT)}{\xi\Sigma_s}\right]$ (where $\Sigma_a(kT)$ – macro cross-section neutron absorption for moderating medium, taken at an energy of neutrons kT, $\xi\Sigma_s$ – slowdown of the ability of the moderating medium for neutrons with energies of 1 eV), which according to [1] was obtained by the numerical approximation of the experimental spectra of several different types of nuclear reactors available at that time, and which is still widely used in the physics of nuclear reactors, for example, [5 – 9]. Note also that the coefficient in front of the second term in brackets in the conversion temperature of the medium in the temperature of the neutron gas, which is often chosen by developers, depends on the type of reactor, for example, [5].

In this paper, based on the solution of the kinematic problem of elastic neutron scattering on a nucleus in the "L" – system in general case, that is, when not only the neutron scattering, but also the nucleus has arbitrarily given velocity vector in the "L" – system, an analytical expression for the law of neutron scattering for an isotropic source of neutrons was obtained, which includes moderating medium temperature as a parameter, as well as the spectra of neutron moderation for different moderating mediums, also depending on the moderating medium temperature and fair for almost all energy spectrum of fission neutrons (excluding energy of interatomic or intermolecular interactions for moderating medium, if necessary to go beyond the gas model). The expressions obtained for the spectrums of moderating neutrons allow reinterpreting the physical nature of the processes that determine the form of the neutron spectrum in the thermal neutrons.

2. Kinematics of elastic neutron scattering on a moderating medium nucleus. We consider the elastic scattering of neutrons on the nucleus of moderating medium. Moderating neutrons medium was described within the gas model, assumed that the nucleuses do not interact between itself, but possess kinetic energy due to their thermal motion.

Important for achievement of the aim (formulation of a neutron moderation theory in analytical form, taking into account thermal motion of a moderating medium nucleus) was that the authors ini-

 ISSN 2076-2429 (print)
ISSN 2223-3814 (online)

tially adopted a provision stating that the shape of the desired solution of the kinematic problem of elastic neutron scattering on a nucleus must be similar to the solution of this problem used in the traditional neutron moderation theory. Therefore, in order to solve the problem, that is considered as a special case of the solution, which is based on the traditional theory of neutron moderation, search for a solution of this problem is conveniently carried out by introducing two laboratory systems:

– resting laboratory coordinate system, which we call the laboratory coordinate system "L";

– moving laboratory coordinate system with respect to a constant rate equal to the rate of the thermal motion of the nucleus moderating medium at the neutron scattering, which will be called the laboratory frame "L'".

It should be noted that we consider the special case when the spatial orientation of the coordinate axes of the laboratory coordinate system "L" and "L'" is the same, as well as the radius vector of the start of the laboratory coordinate system "L'" in the laboratory coordinate system "L" coincides with the radius vector of the moderating medium nucleus, at which the neutron scattering in the laboratory coordinate system "L" (that is the moderating medium nucleus in the laboratory frame "L'") is at rest.

We introduce the following notation: $m_1 = m_n$ – mass of a neutron; $m_2 = m_N$ – mass of a nucleus; $\vec{r}_1^{(\mathrm{L})}$ – radius vector of a neutron in the laboratory "L"; $\vec{r}_2^{(\mathrm{L})}$ – radius vector of a nucleus in the laboratory "L"; $\vec{r}_C^{(\mathrm{L})}$ – radius vector of the center of mass in the laboratory "L"; $\vec{r}_1^{(\mathrm{L}')}$ – radius vector of a neutron in the laboratory "L′"; $\vec{r}_2^{(\mathrm{L}')}$ – radius vector of a nucleus in the laboratory "L'"; $\vec{V}_{10}^{(\mathrm{L})}$ – speed of a neutron in the "L" system before a collision with a nucleus; $\vec{V}_1^{(\mathrm{L})}$ – speed of a neutron in the "L" system after a collision with a nucleus; $\vec{V}_{20}^{(\mathrm{L})}$ – speed of a nucleus in the "L'" system before a collision with a neutron; $\vec{V}_2^{(\mathrm{L})}$ – speed of a nucleus in the "L" system after a collision with a neutron; $\vec{V}_{10}^{(\mathrm{L})}$ – speed of a neutron in the "L'" system before a collision with a nucleus; $\vec{V}_1^{(\mathrm{L}')}$ – speed of a neutron in the "L'" system after a collision with a nucleus; $\vec{V}_{20}^{(\mathrm{L}')}$ – speed of a nucleus in the "L′" system before a collision with a neutron; $\vec{V}_2^{(\mathrm{L}')}$ – speed of a nucleus in the "L′" system after a collision with a neutron; $\vec{V}_C^{(\mathrm{L}')}$ – speed of the center of mass in the laboratory "L′".

Now, because of the limitations on the article volume we missed of the well-known intermediate kinematic calculations (all intermediate kinematic calculations see in [10]) and immediately proceed to the well-known ratio of neutron speeds squares before and after the interaction with the nucleus (this ratio is also equal to the ratio between the kinetic energy of the neutron after the collision and before scattering at the nucleus), which is given in the standard moderation theory (for example, [5, 6]), and which is valid for neutron scattering on a stationary nucleus, that corresponds to our consideration of the scattering process in the "L′" coordinate system:

$$\frac{(V_1^{(\mathrm{L}')})^2}{(V_{10}^{(\mathrm{L}')})^2} = \frac{E_2^{(\mathrm{L}')}}{E_1^{(\mathrm{L}')}} = \frac{A^2 + 2A\cos\theta + 1}{(A+1)^2}, \tag{1}$$

where E_1 and E_2 – the kinetic energy of the neutron, respectively before and after the collision in the laboratory coordinate system "L′",

θ – neutron emission angle in the coordinate system of the center of mass "*C*", introduced for the consideration of the scattering in the "L′" coordinate system.

After entering the parameter $\alpha = \left(\frac{A-1}{A+1}\right)^2$ the expression (1) can be given the following well-known form [5, 6]:

$$\frac{E_2^{(\mathrm{L}')}}{E_1^{(\mathrm{L}')}} = \frac{1}{2}[(1+\alpha) + (1-\alpha)\cos\theta]. \tag{2}$$

Now, using the relation between the neutron velocity in the laboratory coordinate system "L′" and the laboratory coordinate system "L" and the ratio (1), the ratio (2) can be given as follows:

$$\frac{(\vec{V}_1^{(L)} - \vec{V}_{20}^{(L)})^2}{(\vec{V}_{10}^{(L)} - \vec{V}_{20}^{(L)})^2} = \frac{A^2 + 2A\cos\theta + 1}{(A+1)^2} = \frac{1}{2}[(1+\alpha) + (1-\alpha)\cos\theta]. \quad (3)$$

From (3) after algebraic manipulations we can find the ratio of the neutron velocity squares before and after interaction with the nucleus in the laboratory coordinate system "L", which is also equal to the ratio of the kinetic energy of the neutrons before and after the interaction:

$$\frac{(\vec{V}_1^{(L)})^2}{(\vec{V}_{10}^{(L)})^2} = \frac{E_1^{(L)}}{E_{10}^{(L)}} = \frac{1}{2}[(1+\alpha) + (1-\alpha)\cos\theta] - [(1+\alpha) + (1-\alpha)\cos\theta]\frac{V_{10}^{(L)} V_{20}^{(L)} \cos\beta}{(V_{10}^{(L)})^2} +$$
$$+2\frac{V_1^{(L)} V_{20}^{(L)} \cos\gamma}{(V_{10}^{(L)})^2} - \left\{1 - \frac{1}{2}[(1+\alpha) + (1-\alpha)\cos\theta]\right\}\frac{1 \cdot E_{20}^{(L)}}{A \cdot E_{10}^{(L)}}, \quad (4)$$

where $\cos\beta$ – the cosine of the angle between the vectors $\vec{V}_{10}^{(L)}$ and $\vec{V}_{20}^{(L)}$ which is obtained after the scalar multiplication of these vectors $(\vec{V}_{10}^{(L)}, \vec{V}_{20}^{(L)})$;

$\cos\gamma$ – cosine of the angle between the vectors $\vec{V}_1^{(L)}$ and $\vec{V}_{20}^{(L)}$ which is also obtained after the scalar multiplication of these vectors $(\vec{V}_1^{(L)}, \vec{V}_{20}^{(L)})$.

It should be noted that as seen from the derived expression (4), since the $\cos\beta$ and $\cos\gamma$ may take not only positive, but also negative values, then the energy of neutron scattered by the nucleus, may not only be less than the initial energy, as in the standard moderation theory, but it may be greater than its initial energy, in other terms, part of the kinetic energy of the nucleus in the scattering can be transmitted to the neutron.

For our purposes, as shall be seen below, we can limit ourselves to the expression (4) and not take into account in this article the further transformations of the expression (4) related with substitution of the expressions for the $\cos\beta$ and $\cos\gamma$, that must lead to a final form of expression (4), which gives us the final analytical form of exact solution of the kinematic problem of elastic neutron scattering on a nucleus taking into account the thermal motion of the nucleus.

3. The neutron scattering LAW, taking into account thermal motions of the nuclei moderating medium. According to the results of the kinematics of a neutron scattering by a nucleus moderating medium shown in the expression (4) of section 2, the probability, that a neutron with kinetic energy $E_{10}^{(L)}$ before scattering on a nucleus in the laboratory coordinate system "L", after scattering will have a kinetic energy in the range from $E_1^{(L)}$ to $E_1^{(L)} + dE_1^{(L)}$, should be written in the following form:

$$P(E_1^{(L)})dE_1^{(L)} = P(\theta, \beta, \gamma, E_N^{(L)})d\theta d\beta d\gamma dE_N^{(L)} =$$
$$= P(\theta)d\theta \cdot P(\beta)d\beta \cdot P(\gamma)d\gamma \cdot P(E_N^{(L)})dE_N^{(L)}. \quad (5)$$

Since the scattering of neutrons in the center of mass coordinate system is spherically symmetric (isotropic), then for $P(\theta)d\theta$ we obtain:

$$P(\theta)d\theta = \int_0^{2\pi}[P(\theta,\varphi)d\theta]d\varphi = \int_0^{2\pi}\frac{r\sin\theta d\varphi \cdot r d\theta}{4\pi r^2} = \frac{\sin\theta d\theta}{4\pi}\int_0^{2\pi}d\varphi = \frac{1}{2}\sin\theta d\theta, \quad (6)$$

where φ – the azimuth angle indicated by the usual spherical coordinates r, θ, φ entered in the center of mass coordinate system.

Since the thermal motion of nuclei moderating medium is chaotic and neutron source is isotropic (neutron source emits neutrons group with a given energy and an isotropic spatial distribution of the directions of the vectors of their velocities), the distribution of the directions of velocity vectors in the

 ISSN 2076-2429 (print)
ISSN 2223-3814 (online)

space for the neutrons after the collision is set equally probable in the corners β and γ, included in the expression (4), that is also spherically symmetric (isotropic), thus we obtain the same as above:

$$P(\beta)d\beta = \frac{1}{2}\sin\beta d\beta, \tag{7}$$

$$P(\gamma)d\gamma = \frac{1}{2}\sin\gamma d\gamma . \tag{8}$$

By calculating the average of neutron kinetic energy after scattering at the nucleus given by the expression (4) for a spherically symmetric distribution of velocities of the thermal motion of the moderating medium nuclei and isotropic source of neutrons (for isotropic spatial distribution of the neutron velocity vectors with a given energy emitted neutron source), we obtain the following expression:

$$\bar{E}_1^{(\mathrm{L})} = \int_0^{\pi}\int_0^{\pi} E_1^{(\mathrm{L})} P(\theta)\,d\theta P(\beta)d\beta P(\gamma)d\gamma =$$

$$= \bar{E}_{10}^{(\mathrm{L})}\left\{\frac{1}{2}[(1+\alpha)+(1-\alpha)\cos\theta] - \left[1-\frac{1}{2}[(1+\alpha)+(1-\alpha)\cos\theta]\frac{E_N^{(\mathrm{L})}}{A\cdot E_{10}^{(\mathrm{L})}}\right]\right\}. \tag{9}$$

In this expression $E_N^{(\mathrm{L})}$ is given by the Maxwell distribution [11], depending on the temperature of the moderating medium, as a parameter, and having the following form:

$$P_M(E_N^{(\mathrm{L})})dE_N^{(\mathrm{L})} = \frac{2}{\sqrt{\pi(kT)^3}} e^{-\frac{E_N^{(\mathrm{L})}}{kT}} \sqrt{E_N^{(\mathrm{L})}}\, dE_N^{(\mathrm{L})} . \tag{10}$$

After averaging the expression (9) over the Maxwell distribution of the thermal motion of nuclei moderating medium (10), considering that $\bar{E}_{10}^{(\mathrm{L})} = E_{10}^{(\mathrm{L})}$ and using the well-known result $\bar{E}_N^{(\mathrm{L})} = \int_0^{\infty} E_N^{(\mathrm{L})} P_M(E_N^{(\mathrm{L})})dE_N^{(\mathrm{L})} = \frac{3}{2}kT$ [9], we obtain the following expression:

$$\bar{\bar{E}}_1^{(\mathrm{L})} = \int_0^{\infty} \bar{E}_1^{(\mathrm{L})} P_M(E_N^{(\mathrm{L})})dE_N^{(\mathrm{L})} =$$

$$E_{10}^{(\mathrm{L})}\left\{\frac{1}{2}[(1+\alpha)+(1-\alpha)\cos\theta] - \left[1-\frac{1}{2}[(1+\alpha)+(1-\alpha)\cos\theta]\right]\frac{\int_0^{\infty} E_N^{(\mathrm{L})} P_M(E_N^{(\mathrm{L})})dE_N^{(\mathrm{L})}}{AE_{10}^{(\mathrm{L})}}\right\} = \tag{11}$$

$$= E_{10}^{(\mathrm{L})}\left\{\frac{1}{2}[(1+\alpha)+(1-\alpha)\cos\theta] - \left[1-\frac{1}{2}[(1+\alpha)+(1-\alpha)\cos\theta]\frac{\frac{3}{2}kT}{AE_{10}^{(\mathrm{L})}}\right]\right\}.$$

Thus, as the functional relationship between $\bar{\bar{E}}_1^{(\mathrm{L})}$ and θ is unambiguous as follows from (11), then the probabilities that neutrons, with kinetic energy $E_{10}^{(\mathrm{L})}$ before scattering on a nucleus in the laboratory frame "L", after scattering by randomly moving the moderating medium nuclei will have the kinetic energy averaged over the thermal motion of the nuclei to a given range from $\bar{\bar{E}}_1^{(\mathrm{L})}$ to $\bar{\bar{E}}_1^{(\mathrm{L})} + dE_1^{(\mathrm{L})}$ $P(\bar{\bar{E}}_1^{(\mathrm{L})})dE_1^{(\mathrm{L})}$, are given in the expression (6). Therefore we obtain the following relation (here for simplicity we omit the signs of averaging and the laboratory coordinate system "L", i.e., denoting $P(\bar{\bar{E}}_1^{(\mathrm{L})})dE_1^{(\mathrm{L})} = P(E_1)dE_1$):

$$P(E_1)dE_1 = P(\theta)d\theta = P(\theta)\left|\frac{d\theta}{dE_1}\right|dE_1 = \frac{dE_1}{\left[E_{10}^{(\mathrm{L})} + \frac{1}{A}\frac{3}{2}kT\right](1-\alpha)}. \quad (12)$$

Thus, we obtained the neutron scattering law, taking into account nuclei thermal motion of the moderating medium:

$$P(E_1)dE_1 = \frac{dE_1}{\left[E_{10}^{(\mathrm{L})} + \frac{1}{A}\frac{3}{2}kT\right](1-\alpha)} at \alpha\left(E_{10}^{(\mathrm{L})} + \frac{1}{A}\frac{3}{2}kT\right) \le E_1 \le \left(E_{10}^{(\mathrm{L})} + \frac{1}{A}\frac{3}{2}kT\right)$$

and (13)

$$P(E_1) = 0 \ \text{ at } \ E_1 < \alpha\left(E_{10}^{(\mathrm{L})} + \frac{1}{A}\frac{3}{2}kT\right) \ \text{ and } \ E_1 > \left(E_{10}^{(\mathrm{L})} + \frac{1}{A}\frac{3}{2}kT\right).$$

In conclusion of this section we emphasize that the new law of scattering (13) is written for the average neutron energy after scattering. Averaging was carried out of the neutron energy on to thermal (chaotic) motion of the nuclei moderating medium and isotropic source of neutrons. And as it follows from the law of scattering (13) all the neutrons, emitted by an isotropic source of neutrons and having the energy $E_{10}^{(\mathrm{L})}$ before scattering on moderating medium nuclei, with a probability given by equation (13), after scattering on the moderating medium nuclei will have energy E_1, averaged over thermal (chaotic) motion of the moderating medium nuclei and isotropy of the neutron source. Although (as noted above in section 2) from the relation (4) it is implied that the individual neutrons at the scattering on the thermalized medium nuclei may both lose and gain energy, but as follows from the resultant moderating law for an isotropic source of neutrons (13), averaged over the thermal chaos of nuclei moderating medium and neutron source isotropy, the energy of group neutron after scattering is always smaller than the averaged energy of group neutron before scattering, that is it is the law of slowing down neutrons, but now the thermal motion of the moderating medium nuclei and the isotropy neutron source is taken into account.

The standard for the physics of nuclear reactors scattering law, not taking into account the thermal motion of the moderating medium nuclei and close in shape to the law (13) (see. [4 – 6]) is formulated for the neutron energy after scattering on a nucleus, which is at rest, and gives the probability that the neutrons having energy $E_{10}^{(\mathrm{L})}$ before scattering will have energy E after scattering.

4. Neutron moderation in hydrogen media that don't absorb neutrons. According to the neutron scattering law (13), which takes into account nuclei thermal motion of the moderating medium, the law of a neutron moderation in non-absorbing neutrons hydrogen media ($\alpha = 0$ and A=1) has the following form:

$$P(E_1)dE_1 = \frac{dE_1}{\left[E_{10}^{(\mathrm{L})} + \frac{3}{2}kT\right]}. \quad (14)$$

If we carry out calculations similar to those shown [4 – 6, 10] for the moderation law (14), then we find the following expression for the flux of moderating neutrons (here in order to simplify the new notation $E = E_{10}^{(\mathrm{L})}$ and $dE = dE_1$):

$$\Phi(E) = \frac{\int_E^{\infty} Q(E)dE}{\left[E + \frac{3}{2}kT\right]\Sigma_s(E)} + \frac{Q(E)}{\Sigma_s(E)}, \quad (15)$$

where $Q(E)$ – the number of generated neutrons with energy E per unit, volume per unit of time.

ISSN 2076-2429 (print)
ISSN 2223-3814 (online)

Given that the neutron density $n(E)$ is equal (for example, [6]) $n(E) = \Phi(E)\big/\sqrt{2E/m_n}$, we get the following expression for the probability density function of the distribution of moderated neutrons in their energy:

$$\rho(E) = \frac{n(E)dE}{\int_0^\infty n(E)dE} = \frac{\frac{1}{\sqrt{2E/m_n}}\left\{\frac{\int_E^\infty Q(E)dE}{\left[E+\frac{3}{2}kT\right]\Sigma_s(E)} + \frac{Q(E)}{\Sigma_s(E)}\right\}dE}{\int_0^\infty \left\{\frac{1}{\sqrt{2E/m_n}}\left\{\frac{\int_E^\infty Q(E)dE}{\left[E+\frac{3}{2}kT\right]\Sigma_s(E)} + \frac{Q(E)}{\Sigma_s(E)}\right\}\right\}dE}. \tag{16}$$

Fission spectrum of fissile nuclide (or their combination) is given for reactor fission media $Q(E)$, which, according to [1, 12] can be defined by the following expression:

$$Q(E) = c\exp(-aE)sh\sqrt{bE}\,, \tag{17}$$

where *c*, *a* and *b* – constants [1, 10, 12],

E – energy of neutrons per 1 MeV, $Q = \int_0^\infty Q(E)dE$ – total number of neutrons generated per unit, volume per unit of time.

Fig. 1 is a graph showing the energy spectrum of moderated neutrons produced by the expression (16), at the source of fission neutrons given by expression (17) for uranium 235 [1, 10, 12], at a moderator temperature equal to 1000 K. When calculating the spectrum of micro cross-section for elastic scattering of neutrons on hydrogen the data was taken from the base ENDF / B–VII.0 (see. [10]).

Analysis of the energy spectrum shown in Fig. 1 demonstrates that a single expression (16) in a complete manner and physically correctly describes the energy spectrum of the neutrons, which are slowed down taking into account the temperature moderating medium (see [10]).

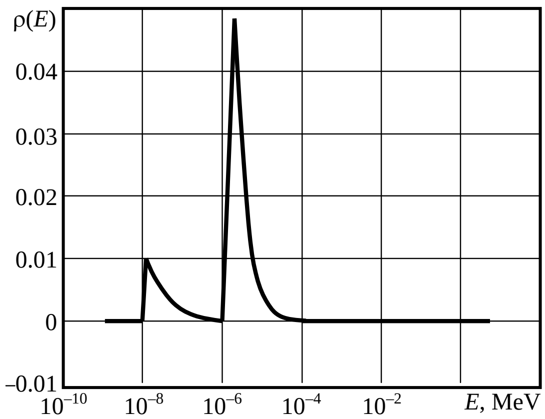

Fig. 1. The energy spectrum of neutrons (0 – 5 MeV), calculated by the expression (16) at the source of fission neutrons given by the expression (17) for uranium 235, moderator temperature equal to 1000 K

5. Neutron moderation in an absorbing neutrons moderating medium containing several varieties of nuclides. In this case, the law of neutron scattering is also given by (13). In this case carrying out calculations similar to those shown in [4 – 6, 10, 13], we find the following expression for the flux of moderating neutrons:

$$\Phi(E) = \left\{\frac{\int_E^\infty Q(E)dE}{\left[E+\frac{1}{A}\frac{3}{2}kT\right]\Sigma_t(E)\bar{\xi}} + \frac{Q(E)}{\Sigma_s(E)}\right\}\exp\left[-\int_E^\infty \frac{\Sigma_a(E')dE'}{\bar{\xi}\Sigma_t(E')\left[E'+\frac{1}{A}\frac{3}{2}kT\right]}\right], \tag{18}$$

where Σ_s^i – macro cross-section of a scattering for the *i*-th nuclide,

$\Sigma_t = \sum_i \Sigma_s^i + \Sigma_a^i$ – full macro cross-section of fission material,

$\Sigma_s = \sum_i \Sigma_s^i$ – full macro cross-section of a scattering of the fission medium,

Σ_a – macro cross-section of a absorption,

$\overline{\xi}$ – averaging all types of nuclei of moderating medium of logarithmic energy decrement.

It should be noted that in (18) the expression for the probability function that neutrons will not be absorbed by resonances is introduced, which now also contains a temperature of moderating medium in contrast to the standard theory of moderation [4 – 6, 10, 13]:

$$\varphi(E') = \exp\left[-\int_E^{\infty} \frac{\Sigma_a(E')dE'}{\overline{\xi}\,\Sigma_t(E')\left[E' + \frac{1}{A}\frac{3}{2}kT\right]}\right]. \tag{19}$$

Knowing the expression for the moderating neutrons flux density (18), in the same way (Section 4, the expression (16)) we can easily obtain the expression for the probability density function of the moderating neutrons distribution by their energy, which is as follows:

$$\rho(E) = \frac{\dfrac{1}{\sqrt{2E/m_n}}\left(\left[\dfrac{\int_E^{\infty} Q(E)dE}{\left[E + \frac{3}{2}kT\right]\Sigma_s(E)} + \dfrac{Q(E)}{\Sigma_s(E)}\right]\varphi(E)\right)dE}{\int_0^{\infty} \dfrac{1}{\sqrt{2E/m_n}}\left(\left[\dfrac{\int_E^{\infty} Q(E)dE}{\left[E + \frac{3}{2}kT\right]\Sigma_s(E)} + \dfrac{Q(E)}{\Sigma_s(E)}\right]\varphi(E)\right)dE}. \tag{20}$$

Analysis of the energy spectrum represented by the expression (20), as well as its comparison with rare in the literature scheme of a full energy spectrum of moderated neutrons, presented in [14] and [10], shows that the single physically correct and complete expression describes the energy spectrum of slowing-down neutrons, also already taking into account the temperature of moderating medium.

Indeed, at high neutron energies ($E_n \geq 100$ keV) the second term in the brackets for the neutron flux density (18), which is included in the expression (20), will be significantly larger than the first and therefore the energy spectrum of moderated neutrons in this part of the energies is the same as the fission neutron spectrum (maximum is in the high part of the spectrum).

With further decrease of neutron energy ($10 \text{ eV} \leq E_n \leq 100 \text{ keV}$), both terms in the curly brackets of the expression (18) are approximately the same, and therefore this part of the energy spectrum of the moderating neutron can be called a "transition area" because it is formed by the contributions of the two terms in the curly brackets of the expression (18), i.e., the amount of fission spectrum and Fermi spectrum ($\sim 1/E_n$).

If the decrease of neutron energy continues ($E_n \leq 10$ eV), the first term in the curly brackets of the expression (18) will be significantly greater than the second, and so the energy spectrum of the moderating neutrons in this energy region is defined by this term. However, due to the fact that the denominator of the first term in the curly brackets of the expression (18) has a term $\frac{3}{2}kT$, we can select an area of the neutron energy $\frac{3}{2}kT \sim E_n \leq 10$ eV, which in this part of the energy spectrum of

ISSN 2076-2429 (print)
ISSN 2223-3814 (online)

the moderating neutrons will coincide with the Fermi spectrum ($\sim 1/E_n$), we also can select an energy region near energy of neutrons $\sim \frac{3}{2}kT$, i.e., some "transitional region" from the Fermi spectrum to a low-energy spectrum, and a low-energy region of neutrons, i.e., the low-energy part of the spectrum of moderating neutron.

According to expression (18) or (20), the low-energy part of the neutron spectrum (supposing that $\overline{\xi}$ and $\Sigma_t = \sum_i \Sigma_s^i + \Sigma_a^i$ – constants) should be a constant, since the integral in the numerator of the first term in the curly brackets of (18) with a decrease of neutron energy in this area is almost unchanged. However, as it turns out at these energies, elastic micro-cross section for neutron scattering grows sharply exponentially with decreasing neutron energy (for example, according to the ENDF/B-VII.0 data and [7] for hydrogen the exponent index increases 1000 times, and for uranium it increases 100 times). This behavior of the elastic scattering cross section leads to the fact that according to the expression (18) or (20) the neutron spectrum will have the second maximum, but in the low-energy part of the spectrum. According to the above it is clear that the nature of this peak is associated with the process of slowing down of non-equilibrium system of neutrons, that are emitted by an isotropic source of neutrons on the thermalized system of nuclei of a moderating medium and could not be explained only by a thermalized part of neutron system, i.e., by a thermal equilibrium part of the neutron system and therefore could not be described by Maxwell distribution.

It should be noted that in contrast to the above, according to the solution for the flux of slowing neutrons, one and the same expression (18), given in [4 – 6] and obtained for the traditional law of scattering in the low-energy part of the spectrum, has the form of the Fermi spectrum ($\sim 1/E_n$) and, consequently, for neutron energies tending to zero, tends to infinity, i.e., there is no low-energy maximum. Therefore, in order to somehow fit the experimental data in the framework of the traditional theory of neutron moderation in the low-energy part of the neutron spectrum, the Fermi spectrum ($\sim 1/E_n$) is set to a certain boundary energy (E_{boundary}) below which the spectrum of moderated neutrons is given by the Maxwell spectrum, the form of which is defined by the temperature of the neutron gas, which in turn is calculated by the empirical formula, linking it with the temperature of the fissile medium (see Introduction**).**

In our analysis of the obtained expressions for the flux density and spectrum of moderated neutrons we have left out of account the effect of the probability function for the unabsorbed neutron resonance (19). This function will affect the ratio of amplitudes of the two maximums of the flux density and the neutron moderating spectrum, it also shows a thin resonant structure of the neutron moderating spectrum in regions of resonance energies of the moderating neutrons (similar to that shown in scheme of full energy spectrum of the moderating neutrons is presented in [14] and [10]).

6. Conclusion. In general case, for the first time the analytical expression for the law of neutron scattering for isotropic source of neutrons was obtained, which includes a temperature of a moderating medium as a parameter. Also the analytical expressions for a neutron flux density and a spectrum of neutron moderation were obtained, which also depends on the temperature of a reactor medium.

The expressions obtained for the spectra of moderate neutrons allow reinterpreting the physical nature of the processes that determine the form of the neutron spectra in the region of the thermal neutrons. The influence of the behavior of the cross sections for the elastic scattering of neutrons on the formation of a neutron moderation spectrum maximum in the low-energy part of the spectrum was established. According to the above it is clear that the nature of this maximum is associated with the process of slowing down of neutrons in a non-equilibrium neutron system, which is generated by an isotropic source of neutrons in thermalized system of nuclei of moderation medium, and cannot be explained only by thermalized part of neutron system, i.e., thermal equilibrium part of the neutron system, and cannot be described by Maxwell distribution.

In conclusion, it should be noted that substantially different behavior of elastic scattering cross sections of neutrons for the nuclei of different reactor moderation media (for example, see ENDF / B-

VII.0 or [7]) opens the possibility for an experimental study of the effect of the behavior of the elastic scattering cross sections of neutrons for different moderating media on the formation of moderating neutron spectrum maximum in the low-energy part of the spectrum and experimental verification.

Література

1. Вейнберг, А. Физическая теория ядерных реакторов / А. Вейнберг, Е. Вигнер. – М.: Изд-во иностр. лит., 1961. – 732 с.
2. Ахиезер, А.И., Введение в теорию нейтронных мультиплицирующих систем (реакторов) / А.И. Ахиезер, А. Померанчук. – М.: ИздАТ, 2002. – 367 с.
3. Галанин, А.Д. Теория ядерных реакторов на тепловых нейтронах / А.Д. Галанин. – М.: Атомиздат, 1971. – 529 с.
4. Фейнберг, С.М. Теория ядерних реакторов Т. 1 / С.М. Фейнберг, С.Б. Шихов, В.Б. Троянский. – М.: Атомиздат, 1978. – 400 с.
5. Бартоломей, Г.Г. Основы теории и методы расчета ядерних энергетических реакторов / Г.Г. Бартоломей, Г.А. Бать, В.Д Байбаков., М.С. Алхутов. – М.: Энергоатомиздат, 1989. – 512 с.
6. Широков, С.В. Физика ядерних реакторов / С.В. Широков. – К.: «Наукова думка», 1998. – 288 с.
7. Stacey, W.M. Nuclear reactor physics / W.M. Stacey. – New York: A Wiley-Intersience Publication, 2001. – 707 p.
8. Traveling Wave Reactor and Condition of Existence of Nuclear Burning Soliton-like Wave in Neutron-Multiplicating Media / V.D. Rusov, E.P. Linnik, V.A. Tarasov, et al. // Energies Special Issue "Advances in Nuclear Energy". – 2011. – № 4. – P. 1337 – 1361. doi: 10.3390/en4091337.
9. Русов, В.Д. Режимы с обострением в уран-плутониевой делящейся среде технических ядерных реакторов и георeактора / В.Д. Русов, В.А. Тарасов, С.А. Чернеженко // Вопросы атомной науки техники. Сер. физика радиац. повреждений и радиац. материаловедение. – 2011. – № 2(97). – С. 123 – 131.
10. The neutron moderation theory, taking into account thermal motions of moderating medium nuclei [Електронний ресурс] / V.D. Rusov, V.A. Tarasov, S.A. Chernezhenko, A.A. Kakaev // Nuclear Theory, 1-26. Режим доступу: https://arxiv.org/abs/1612.06838v1 – 2016. – P. 1 – 26.
11. Левич, В.Г. Курс теоретической физики Т. 1 / В.Г. Левич. – М.: «Наука», 1969. – 910 с.
12. Краткий справочник инженера-физика. – М.: «Государственное изд-во литературы в области атомной науки и техники», 1961. – 507 с.
13. Функция вероятности резонансного поглощения для нейтрона и мультипликативный интеграл / В.Д. Русов, В.А. Тарасов, С.И. Косенко, С.А. Чернеженко // Вопросы атомной науки техники. Сер. физика радиац. повреждений и радиац. материаловедение. ─ 2012. ─ Вып. 2(78). ─ С. 68 – 72; arXiv:1208.1019v1 [nucl-th].
14. Владимиров, В.И. Практические задачи по эксплуатации ядерных реакторов / В.И. Владимиров – М.: Энергатомиздат, 1986. – 304 с.

References

1. Weinberg, A.M., & Wigner, E.P. (1961). *Fizicheskaya teoriya yadernykh reaktorov [The physical theory of nuclear reactors]*. Moscow: Publishing foreign literature.
2. Akhiezer, A.I., & Pomeranchuk, I.Ya. (2002) *Vvedeniye v teoriyu neytronnykh mul'tiplitsiruyushchikh sistem (reaktorov) [Introduction into the theory of neutron multiplication systems (reactors)]*. Moscow; IzdAT.
3. Galanin, A.D. (1971). *Teoriya yadernykh reaktorov na teplovykh neytronakh [The theory of nuclear reactors on thermal neutrons]*. Moscow; Atomizdat.
4. Feinberg, S.M., Shikhov, S.B. & Troyanskii, V.B. (1978). *Teoriya yadernikh reaktorov [The theory of nuclear reactors]*, Vol. 1. Moscow; Atomizdat.
5. Bartolomey, G.G., Bat', G.A., Baibakov, V.D., & Altukhov, M.S. (1989). *Osnovy teorii i metody rascheta yadernikh energeticheskikh reaktorov [Basic theory and methods of nuclear power installation calculations]*. Moscow: Energoatomizdat.
6. Shirokov, S.V. (1998). *Fizika yadernikh reaktorov [The nuclear reactor physics]*. Kiev: Naukova Dumka.
7. Stacey, W.M. (2001). *Nuclear reactor physics*. New York: A Wiley-Intersience Publication.
8. Rusov, V.D., Linnik, E.P., Tarasov, V.A., Zelentsova, T.N., Vaschenko, V.N., Kosenko S.I., & at al. (2011). Traveling Wave Reactor and Condition of Existence of Nuclear Burning Soliton-like Wave in

Neutron–Multiplicating Media. *Energies Special Issue "Advances in Nuclear Energy"*, 4, 1337–1361; DOI: 10.3390/en4091337.
9. Rusov, V.D., Tarasov, V.A., & Chernezhenko, S.A. (2011). Rezhimy s obostreniyem v uran-plutoniyevoy delyashcheysya srede tekhnicheskikh yadernykh reaktorov i georeaktora [Blow-up modes in uranium-plutonium fissile medium in technical nuclear reactors and georeactor]. *Voprosy atomnoy nauki tekhniki. Ser. fizika radiats. povrezhdeniy i radiats. Materialovedeniye – Problems of Atomic Science and Technology, Series: Physics of Radiation Effect and Radiation Materials Science*, Vol. 2(97), 123–131.
10. Rusov, V.D., Tarasov, V.A., Chernezhenko, S.A., & Kakaev, A.A. (2016). The neutron moderation theory, taking into account thermal motions of moderating medium nuclei. *Nuclear Theory*, 1–26. Retrieved from: https://arxiv.org/abs/1612.06838v1.
11. Levich, V.G. (1969). *Kurs teoreticheskoy fiziki [The course of theoretical physics]*. Vol. 1. Moscow: Nauka.
12. *Kratkiy spravochnik inzhenera-fizika [The engineer-physicist short reference guide]*. (1961). Moscow: State publishing of literature in the field of nuclear science and technology.
13. Rusov, V.D., Tarasov, V.A., Kosenko, S.I., & Chernezhenko, S.A. (2012). Funktsiya veroyatnosti rezonansnogo pogloshcheniya dlya neytrona i mul'tiplikativnyy integral [The resonance absorption probability function for neutron and multiplicative integral] *Voprosy atomnoy nauki tekhniki – Problems of Atomic Science and Technology*, Vol. 2(78), 112–121. arXiv:1208.1019v1 [nucl-th].
14. Vladimirov, V.I. (1986). *Prakticheskiye zadachi po ekspluatatsii yadernykh reaktorov [The physics nuclear reactors practical problems in their operation]*. Moscow: Energoatomizdat.

Received July 17, 2017

Accepted July 24, 2017