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DEVELOPMENT OF THE METHOD OF APPROXIMATE SOLUTION TO THE NONSTATIONARY PROBLEM ON HEAT TRANSFER THROUGH A FLAT WALL

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Для дослідження процесу нестаціонарної теплопередачі через плоску стінку був розроблений наближений аналітичний метод рішення задачі в зосередженій постановці. Працездатність методу продемонстрована на прикладі рішення задач, що послідовно ускладнюються, для плоскої пластини: симетричного, несиметричного нагріву і нестаціонарної теплопередачі. Показана адекватність отриманих рішень в рамках точності інженерних розрахунків

Ключові слова: нестаціонарна теплопередача, акумуляція енергії, плоска стінка, аналітичний розрахунок, наближене рішення

Для исследования процесса нестационарной теплопередачи через плоскую стенку разработан приближенный аналитический метод решения задачи в сосредоточенной постановке. Работоспособность метода продемонстрирована на примере решения последовательно усложняющихся задач для плоской пластины: симметричного, несимметричного нагрева и нестационарной теплопередачи. Показана адекватность полученных решений в рамках точности инженерных расчетов

Ключевые слова: нестационарная теплопередача, аккумуляция энергии, плоская стенка, аналитический расчет, приближенное решение

1. Introduction

The inertiality of change in the thermophysical parameters during operation process of heat power equipment is one of the reasons for the instability of its functioning and its deviations from the optimal state. The magnitude of delay in changing controlled magnitudes following external disturbances and controlling impacts depends on the accumulating properties of equipment under operation. This effect manifests itself in various parts of the equipment by changing the flow of energy compared with the flow that arrives with fuel. Energy flows are predetermined by the flows of substance (a working body) and thermal fields in the elements of the structure.

Working bodies are typically gases. Gaseous products have low density and, as a consequence, small accumulating properties of thermal energy. There is a different pattern with design elements that have significant masses and considerable accumulating capacity. In thermal power equipment, one can distinguish parts with regenerative and recuperative heat transfer conditions. Conditions for regenerative heat exchange are implemented for elements that are not heat-transfer surfaces: enclosures, fasteners. The main working elements, for example, in the boiler equipment, are the surfaces of heat exchangers where the recuperative heat transfer conditions are implemented.

In all cases, creation of effective control systems requires tools for the estimation of temperature fields and accumulating properties of elements of heat power equipment under non-stationary conditions of heat exchange. At present, solution to a nonstationary problem can be obtained based on universal methods of numerical realization. At the design stage, however, when setting up control systems, of importance are the results of analytical studies. The existence of results of analytical studies becomes even more relevant when using fuel of variable composition [1], characterized by a sharp increase in the number of transition processes.

2. Literature review and problem statement

For the elements with regenerative heat transfer conditions, papers [2, 3] developed analytical methods for determining a nonstationary temperature field at any point of bodies that have complex shapes. Among the considered geometric primitives are an infinite plate, a cylinder, a sphere, under conditions of heating or cooling with determining the nonstationary dimensionless temperature fields. Subsequently, by the geometric intersection of primitives, one can model bodies of complex shapes with their temperature fields obtained by multiplying the dimensionless temperature fields of the respective primitives.

To date, research in this direction have either tackled modifications of methods for solving the original model, which do not have fundamental advantages over each other, or have been considering a new subject for application in the form of modern equipment. Thus, authors of [4] employ the Trefftz functions that produce an approximate solution. Paper [5] considers a thermal field in one of the primitives – a cylinder. To solve the problem, authors apply a particular approach that employs the cylindrical coordinate system, multiple integrals, and Laplace transforms to solve them. In addition, boundary conditions of the first and second kind are applied, while the most common are conditions of the third kind.

In paper [6], mathematical model of the heat transfer process is based on the variational approach. Heat transfer through the anisotropic body is investigated, which is interesting. However, [6] solves a stationary problem. In article [7], authors examine nonstationary heat transfer in a recuperative heat exchanger. In this work, they obtained results based on numerical studies, which binds them to a specific form of equipment and initial conditions. This makes it difficult or even excludes the possibility of their application at the design stage, when control systems are set up, where the results of analytical studies are important. Authors of paper [8], based on the Laplace transfor, search for analytical solution to one of the variants of a nonstationary problem on heat transfer. They search for a time delay in the transfer of heat pulse through the heat exchange surface. The result found, however, does not make it possible to solve the problem on accumulation of heat energy. Article [9] addresses the problem on cooling a display, and [10] considers cooling a plate with an internal heat source. In the latter case, special feature is the consideration of a multi-layer plate.

Recuperative heat exchange conditions in thermal power equipment are implemented in the heat-exchange devices. Main heat-transfer surfaces are a plate and a wall of the hollow cylinder (pipe). In the latter case, at the ratio of external cylinder radius to the internal cylinder radius of $r_2/r_1<2$, the results obtained based on models for a cylindrical wall and a flat wall differ by less than 4 %. That is why, in the vast majority of cases, in order to solve nonstationary problems, it will suffice to be able to determine temperature fields at heat transfer through a flat wall.

Attempts to obtain at least intermediate results in this direction were made in papers [2, 3]. They considered, as one of the examples, the asymmetrical heating of a flat plate. This case, due to the similarity of temperatures on both sides of the plate, meets the conditions for regenerative heat exchange. Different adopted heat transfer coefficients could provide a possible path for solving the problem on nonstationary heat transfer. Paper [2] outlined only a direction for possible solution without giving its form. Work [3], published later than [2], and repeating its results in many ways, described the solution to determine a dimensionless temperature field. Following a certain period of time, at the end of the examined process, albeit asymmetric, but still of heating, all points of the plate must have the same temperature equal to the ambient temperature. The expression, given in [3], does not show this.

Subsequently, solutions to problems on nonstationary heat transfer come down to various kinds of numerical studies. However, even given the easiest implementation [11] with the minimum possible number of estimated nodes, the results obtained do not demonstrate the degree of commonality that would distinguish analytical calculations.

In [12], authors applied an approximate solution to the problem of nonstationary heat transfer. The proposed approach is applicable, for example, in the study of processes in the boiler equipment. The authors employed a special feature in the heat transfer process from gaseous products of fuel combustion to the liquid heat carrier (water). This process is characterized by a large difference between heat transfer coefficients at different sides of the heat transfer surface. The magnitudes of coefficients may vary by 2 3 orders. In addition, heat-transfer surfaces are characterized by low thickness, high thermal conductivity and, consequently, have low thermal resistance. As a result, temperature in the wall is considered constant. The magnitude of temperature is accepted to be equal to the surface temperature from the side of the liquid heat carrier and close to the temperature of the heat carrier itself.

There are a number of cases when such assumptions are not acceptable. Heat transfer coefficients on different sides of the heat-transfer surface may turn out to be different but still comparable. An example is the cooling of combustion chamber of gas turbine engine, when gaseous combustion products are on one side and air is on the other side. A similar situation could arise in direct flow boilers at shift in the interface of steam water. In this case, on one side are the gaseous combustion products, on the other side is steam. Superheaters also belong here. In all cases, the temperature and thermal-physical properties of heat-carrier (air, steam) are close to constant. The same parameters for the products of combustion when utilizing fuel of variable composition change considerably. One of them is the heat transfer coefficient whose magnitude may change by several times [13]. These peculiarities lead to an increase in the quantity of transitional processes and increase the influence of energy accumulation in the design elements on manageability of the occurring thermophysical processes.

Solving a problem of nonstationary heat transfer is complicated not only by the distributed nature of a mathematical model that describes processes in the body with two differentiated media, but also because of the need to take into consideration the processes that occur in the media that surrounds it. In other words, a given problem is conjugate. The accuracy of solution to a conjugate problem is limited by the largest error in the description of any of the elements of the original model.

In many cases, specifically in engineering applications, in order to obtain a solution, they change a general model of the process by simplifying some of its elements. As far as the examined problem is concerned, the description of processes of heat transfer from the environment to the dividing surface is replaced by assigning boundary conditions. Boundary conditions of the third kind are the most common and frequently used. In this case, the entire description of a complex process comes down to assigning one heat transfer coefficient. This coefficient is typically calculated using the criterial equations, which can lead to an error of its determining at 15 20 % [14]. Another, a later, work [15] argues that this error can be even larger and amount to 30–50 %. Under these conditions, there is no need to search for an exact analytical solution to the problem of nonstationary heat transfer. Approximate solution is acceptable.

The analysis that we conducted reveals the need to develop a method for the analytical solution to problems on nonstationary heat transfer. The method can be approximate, but the calculation error must not exceed possible error in determining original data (heat transfer coefficients). The method must be capable of solving problems on both regenerative and recuperative heat transfer conditions. The devices for which these conditions are implemented are typically elements of the same equipment and are exposed to the same influences. Given this, it is desirable that the problems, even under different conditions of influence, should have the same or similar solutions.

3. The aim and objectives of the study

The aim of present study is to develop a method for solving analytically the problems on heat exchange under different conditions of influence. This should provide the possibility of a unified approach both under conditions of nonstationary regenerative (heating or cooling) and recuperative heat exchange (heat transfer).

To accomplish the aim, the following tasks have been set:

– using symmetric heating of a plate as an example representing the simplest case, to work out a principle of approximate solution to nonstationary problems of heat exchange;

– using asymmetrical heating of a plate as an example, to devise an approach that would make it possible to apply the devised principle to solve asymmetric problems;

– employing the approach applied to solving a problem on asymmetric heating, to devise a method for solving the problem on nonstationary heat transfer through a flat wall.

4. Method for constructing an approximate model of the problem on nonstationary heat transfer

An approximate solution can be obtained in a variety of ways. As a minimum:

– through the simplification of the more accurate and complex model;

– through the simplification of the model itself with its subsequent exact solution.

An analysis of the scientific literature did not reveal analytical solutions to the problem on nonstationary heat transfer, which could have formed the basis for their simplification. Therefore, we shall focus on the second way.

In order to maximize the power of signal transmitted from the source to the receiver, it is necessary to harmonize the load (convergence or equalization of the source and the load resistances). By analogy, it can be assumed that in order to maximize the informativeness of solution, it is required to adjust the degree of complexity (detail) in the models of parts of the conjugate problem. At present, there exists a contradiction in contemporary techniques for solving the problem on nonstationary heat transfer. On the one hand, they employ a differential model of thermal conductivity in partial derivatives (one-dimensional for an infinite plate). On the other hand, it is not about the concentrated statement, but a zero-dimensional model of heat transfer from the environment to the plate in the form of a constant heat transfer coefficient. If we do not consider the possibility of complicating the model of heat transfer, there is a way to simplify a thermal conductivity model by reducing it to be at least in the form of concentrated statement. However, given such an approach, there is a contradiction in terms of the impossibility of accounting for temperature distribution through the thickness of the plate.

Approximate solution can be regarded as the approximation of the exact one. With respect to this position, it is required to choose a functional dependence that will be applied to perform the approximation. A nonstationary heat transfer process is characterized by the accumulation of thermal energy. In other words, it is inertial and, accordingly, can be described using an exponential dependence. In this case, it is possible to consider a temperature distribution through the thickness of the plate. The transition to the model in a concentrated statement is achieved by using the value of temperature averaged for the thickness of the entire body.

5. The solution, and the assessment of its adequacy, to the problem on nonstationary heat transfer based on the approximated model

The solution will be sought for by considering three problems that are consistently complicated:

5. 1. Symmetric heating of an infinite plate;

5. 2. Asymmetric heating of an infinite plate;

5. 3. Nonstationary heat transfer through a flat wall.

The first problem has an analytic solution already based on a more complex model in a distributed statement [2, 3]. Considering it within the framework of a new approach is aimed at working out the proposed method of solution. The existence of precise analytical solution will make it possible to assess adequacy of the new results. Solution to the second problem should make it possible to consider an approach to solving the desired problem, stated in point three. Assessment of adequacy of the results of solution to the second and third problems will be carried out by comparing them with the results of numerical calculations.

5. 1. Symmetrical heating of an infinite plate

We investigate heating under the same temperature and heat transfer coefficients on both sides of the plate. Due to symmetry, the calculations are performed for half the thickness of the plate. The coordinate origin is positioned in the center of the plate with the count directed to the surface. At a starting point, body temperature and the ambient temperature are the same and are equal to t_o. At a certain point of time, the ambient temperature abruptly changes to the magnitude t_s=const. Temperature on the surface t_n and in the center of the plate t_c are time-dependent variables. Temperature inside the plate t_x depends on the coordinate of the point at which it is determined, and on time. Calculation will be carried out for deviations of temperature from the body temperature at initial time t_o:

$$\theta_x = t_x - t_o;\;\; \theta_n = t_n - t_o;\;\; \theta_c = t_c - t_o;\;\; \theta_s = t_s - t_o. \qquad (1)$$

It is proposed to approximate the value of temperature inside the plate depending on temperatures at the surface and in the center using expression of the following form:

$$\theta_x = \theta_c + (\theta_n - \theta_c)\cdot \exp(1 - l/x). \qquad (2)$$

Here l is half the thickness of the plate; x is the coordinate of the point under consideration. The magnitudes of θ_n and θ_c are to be determined.

Write an expression to determine the temperature, mean integral for the thickness of the plate, in the form:

$$\tilde{\theta} = \frac{1}{V}\int_0^l \theta_x S_x \mathrm{d}x, \qquad (3)$$

the coordinate of point of realization of this temperature will be determined from ratio:

$$\tilde{l} = \frac{1}{V}\int_0^l x\cdot \exp(1 - l/x)\mathrm{d}x. \qquad (4)$$

In expressions (3) and (4), V is the volume of the body, S_x is the surface area parallel to the external and passing through the point in question. For the plate, this magnitude equals the area of lateral surface $S_x = S_b$; for the sphere, for example, $S_x = 4\pi x^2$. The values for expressions (3) and (4) are calculated after substituting expression (2) and taking into consideration $V = S_b \cdot l$ for the plate. Integrals (3) and (4) can be determined accurately when using an integral exponential function Ei, pertaining to the specialized category. Their solutions are expressed in the form:

$$\tilde{\theta} = \theta_c + (\theta_n - \theta_c)\cdot k;\;\; \tilde{l} = k\cdot l. \qquad (5)$$

Original approximation dependence (2) is initially approximate in nature. Therefore, the magnitude of coefficient k, while reflecting main features of the mean integral temperature, requires refining for practical application. It can be performed based on existing analytical or numerical solutions. This will be done below. In the ongoing transforms, we shall employ expression (5) in a general form.

With respect to (5), the model of heating a plate in concentrated parameters can be written in the following form:

$$c\cdot\rho\cdot V\frac{d\tilde{\theta}}{d\tau} = S_b\frac{\lambda}{\tilde{l}}(\theta_n - \tilde{\theta}); \qquad (6)$$

$$\alpha\cdot S_b\cdot(\theta_s - \theta_n) = S_b\frac{\lambda}{\tilde{l}}(\theta_n - \tilde{\theta}). \qquad (7)$$

Here, c, ρ, λ are, respectively, heat capacity, density, thermal conductivity of a material of the plate, α is the coefficient of heat transfer from the environment to the plate. The first of these equations reflects the law of preservation of energy. The right side indicates energy that arrives from the surface to the plate body from the difference of temperatures on the surface and inside the plate. The left side describes energy accumulated in the examined body due to its heat capacity. The record of equation (7) assigns a boundary condition of the third kind. This equation expresses the equality of energies transmitted from the environment to the surface of the body and removed from the surface inside the body.

With respect to (5), transform equations (6) and (7) relative to θ_c – temperature in the center of the plate. The result will be:

$$\frac{d\theta_c}{d\tau} = \frac{a}{l^2}\cdot\frac{S_b\cdot l}{V}\cdot\frac{Bi}{1 + k\cdot Bi}\cdot(\theta_s - \theta_c). \qquad (8)$$

Here $a = \lambda/(c\cdot\rho)$ is the coefficient of thermal diffusivity, $Bi = (\alpha\cdot l)/\lambda$ is the Biot criterion. Next, we shall nondimensionalize equation (8) using certain normalizations. Thus, for temperature:

$$\overline{\theta}_c = \frac{\theta_c}{\theta_s},\;\; \overline{\theta}_s = \frac{\theta_s}{\theta_s} = 1. \qquad (9)$$

We shall nondimensionalize time variable according to the method proposed in [16] using a complex included in equation (8) that has a time dimensionality:

$$\frac{a}{l^2}\cdot\frac{S_b\cdot l}{V}\cdot\frac{Bi}{1 + k\cdot Bi}. \qquad (10)$$

Such an approach makes it possible to reduce the number of parameters in the equation to the magnitudes smaller than those set by the π-theorem. The result will be:

$$\frac{d\overline{\theta}_c}{d(\hat{Ho})} = 1 - \overline{\theta}_c \;\text{ or }\; \frac{d\overline{\theta}_c}{d(\hat{Ho})} + \overline{\theta}_c = 1. \qquad (11)$$

Here

$$\hat{Ho} = \frac{a\tau}{l^2}\cdot\frac{S_b\cdot l}{V}\cdot\frac{Bi}{1 + k\cdot Bi} \;\text{ or }\; \hat{Ho} = Fo\cdot\frac{S_b\cdot l}{V}\cdot\frac{Bi}{1 + k\cdot Bi}. \qquad (12)$$

In this expression, Fo is the Fourier number. In the problems on non-stationary heat exchange, it acts as nondimensionalized time and has another name – the number of homochronicity (Greek: *homos* – equal, *chronos* – time). By analogy, we shall denote (12) as extended number of homochronicity. In addition to *Fo*, it includes a geometric complex ($S_b \cdot l/V$) and complex $Bi/(1+k \cdot Bi)$. The Biot criterion is the ratio of thermal resistance of wall l/λ to thermal resistance of heat transfer $1/\alpha$. The Biot criterion determines similarity of temperature fields under geometrical similarity of the heated bodies. Given this, the complex $Bi/(1+k \cdot Bi)$ in (12) by analogy can be named the extended Biot criterion:

$$\hat{B}i = \frac{Bi}{1+k\cdot Bi}. \tag{13}$$

In the theory of automatic control, equation similar to (11) describes the inertial link of the first order. It has solution in the form:

$$\bar{\theta}_c = 1 - \exp(-\hat{H}o). \tag{14}$$

Equation (14) describes a change in the nondimensionalized temperature in the center of the heated infinite plate. Moreover, by carrying out the transforms, similar to those given above, it is possible to obtain an expression to describe a change in temperature at any point in the body under consideration. In addition, by using equation (14), one can describe a change in temperature in the center of other geometric primitives – an infinite cylinder, or a sphere, and, upon appropriate transforms, at any point in these bodies. The changes refer only to geometrical complex K_g. It is simple for computation and it will acquire the following values for an infinite plate, an infinite cylinder, and a sphere, respectively:

$$K_g^{pl} = \frac{S_b\cdot l}{V} = \frac{S_b\cdot l}{S_b\cdot l} = 1; \quad K_g^{cyl} = \frac{S_b\cdot R}{V} = \frac{2\pi RL\cdot R}{\pi R^2 L} = 2;$$

$$K_g^{ball} = \frac{S_b\cdot R}{V} = \frac{4\pi R^2\cdot R}{\frac{4}{3}\pi R^3} = 3. \tag{15}$$

Note that in the second and third expressions in (15) the characteristic dimension is the radius of a cylinder and a sphere, respectively. In the second expression from (15), magnitude L corresponds to the length of a cylinder. Although we consider a cylinder of infinite length ($L=\infty$), this magnitude appears in the numerator and in the denominator in order to correctly reflect the area of its side surface and volume. They will be subsequently reduced.

We shall compare results of approximate calculation based on (14) with similar results of analytical calculations [2]. Comparison makes it possible to refine the magnitude of coefficient k at which approximated results have a minimal deviation from the precise results. In this case, the magnitude itself of these deviations is determined. We shall run a comparison for the magnitude of time of the end of the heating process. We consider the process of heating a body completed at a temperature in the center of the body of $t_c=0.95\cdot t_s$. This matches an error in engineering estimations. In addition, according to the theory of automatic control, a transition process in the first-order inertial link (14) is considered completed at $\hat{H}o=3$. This leads to a deviation in the calculated magnitude of nondimensionalized temperature $\bar{\theta}_c$ from the desired temperature $\bar{\theta}_s=1$ by not more than 5 %. In other words, the moment of nondimensionalized time $\hat{H}o=3$ is matched by $\bar{\theta}_c = 0{,}95\cdot\bar{\theta}_s = 0{,}95$.

Paper [2] misses the magnitude that corresponds to $\hat{H}o$. In it, *Fo* acts as nondimensionalized time. To ensure comparability of results from (12) with respect to (13), we shall express magnitude *Fo*:

$$Fo = \frac{\hat{H}o}{K_g\cdot \hat{B}i}. \tag{16}$$

The magnitude of the Fourier number, employed in [2], will be denoted Fo_1. We shall compare magnitudes of the Fourier numbers, in both cases, at the moment, discussed above, when the heating of a body is over. Note that authors of [2] examine the cooling of a body. But the processes of heating and cooling are symmetrical. Therefore, comparison of results, obtained based on (16), of the magnitudes, given in [2], is valid. It should only be noted that during cooling the moment the process is over is considered to be a point when temperature reaches magnitude $\bar{\theta}_{c1} = 0{,}05\cdot\bar{\theta}_{s1} = 0{,}05$. The comparison was conducted by determining relative error ε of calculating *Fo* based on (16) relative to Fo_1. In the process of comparing, we chose the magnitude of k from (13) that ensures minimum magnitude of relative error ε. The results are given in Table 1. It should be noted that the model was originally built for an infinite plate. Nevertheless, in addition to it, it makes it possible, with respect to (15), to perform calculations for an infinite cylinder and a sphere.

Table 1

Comparison of results of precise [2] and approximate (16) calculations of nondimensionalized time of the end of the process of heating the bodies

Bi	Plate, k=0.42			Cylinder, k=0.39			Sphere, k=0.36		
	Fo_1	Fo	ε, %	Fo_1	Fo	ε, %	Fo_1	Fo	ε, %
0.005	600.3	601.3	0.15	300.1	300.6	0.18	200.1	200.4	0.15
0.01	300.9	301.3	0.1	150.4	150.6	0.15	100.2	100.4	0.17
0.1	31.1	31.3	0.5	15.5	15.6	0.7	10.3	10.4	0.7
1.0	4.20	4.26	1.4	2.0	2.1	3.3	1.31	1.36	3.7
10	1.58	1.56	1.3	0.725	0.735	1.3	0.45	0.46	1.4
100	1.34	1.29	3.6	0.61	0.6	1.9	0.38	0.37	3.0
1,000	1.32	1.26	3.9	0.60	0.59	2.36	0.37	0.36	3.4

It follows from results shown in Table 1 that an error of determining the time of the end of heating the bodies using expression (16) compared to the results of precise analytical solution does not exceed 4 % over the entire examined range of change in *Bi*.

The proposed method has an error relative to the precise analytical solution. At the same time, the error does not exceed the magnitude accepted in engineering calculations (<5 %). Based on the results obtained, the proposed model and the method of calculation can be considered workable.

5. 2. Asymmetrical heating of an infinite plate

We shall consider a problem on the asymmetric heating of a plate. The asymmetry arises from different coefficients of heat transfer. Temperature conditions of the environment are similar to the case of the symmetric heating: t_o is the initial temperature of the environment and the body; t_s is the temperature of the environment after its abrupt change. Body temperature is constant at all its points after the heating and is equal to t_s. In the process of symmetric heating, a minimum of temperature is formed, which is located along the midline of the plate cross-section. Minimum point moves along the mid-line until reaching t_s aligned throughout the entire body of the plate. For the case of asymmetrical heating, the character of motion of the temperature minimum was determined using numerical study. Similar to the previous case, the point of minimum moves along the surface of the plate, but the trajectory of motion is shifted towards the surface with a lower heat transfer coefficient. In the process of heating, a given minimum also shifts along a straight line, parallel to the sides of the plate. Deviations from the described scheme are insignificant and are observed are the starting point of the examined process (Fig. 1, *a*) and can be disregarded for the approximate solution.

Coordinate system for the cross-section of the plate will be constructed in the following way (Fig. 1, *b*):

– vertical reference axis of temperatures passes through their minimum and is parallel to the sides of the cross-section of the plate. The temperature at this point is denoted as t_c – temperature at the center;

– coordinate count of the considered point in the cross-section is performed in either directions from the temperature reference axis (X_1 and X_2);

– distance x_{n1} from the coordinate origin to one of the sides of the plate is unknown and must be determined;

– assuming that thickness of the plate equals $2l$, the distance from coordinate origin to the second surface is determined from relation $x_{n2}=2l-x_{n1}$.

Under such conditions, each part of the plate that corresponds to coordinates X_1 and X_2 can be treated using the approach applied at symmetrical heating. Additional unknown magnitude x_{n1} is determined from the equation of connection, constructed with respect to the equality of temperatures t_c, calculated for each part of the plate.

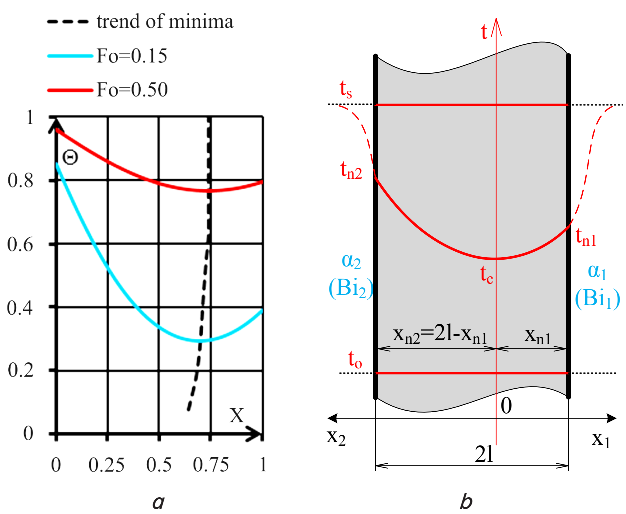

Fig. 1. Temperature profile at asymmetrical heating of a plate: *a* — examples of results of numerical calculation of relative temperature Θ depending on relative coordinate X at different values of Fo; *b* — adopted procedure for analytical calculations

Upon completion of the transforms, similar to those applied when solving a problem on symmetrical heating, we can determine temperatures at its minimum t_c and at any point, including t_{n1}, t_{n2} on the surfaces of the plate. Thus, nondimensionalized temperature at its minimum point is determined based on the relation identical to (14). Structure of the expression for determining the extended homochronicity number $\hat{Ho}$ coincides with the structure of expression (12). It includes: number Fo, geometrical complex K_g, and the extended Biot criterion. The difference is in determining the latter. With respect to (15) $K_g^{pl}=1$, the expression for $\hat{Ho}$ in the examined case takes the following form:

$$\hat{Ho}=Fo\cdot 1\cdot\frac{Bi_1\left(1+\frac{Bi_2}{Bi_1}+2\cdot k\cdot Bi_2\right)^2}{\left(1+k\cdot Bi_2\right)\cdot\left(1+\frac{Bi_2}{Bi_1}+2\cdot k\cdot Bi_2+k\cdot Bi_1+k^2\cdot Bi_1\cdot Bi_2\right)}. \quad (17)$$

Relative coordinate of location of the temperature minimum can be determined from ratio:

$$\frac{x_{n1}}{2l}=\frac{1+k\cdot Bi_2}{1+\frac{Bi_2}{Bi_1}+2\cdot k\cdot Bi_2}. \quad (18)$$

It should be considered that for the case of symmetrical heating, magnitudes Fo and Bi are determined for characteristic dimension l. Whereas for the case of asymmetrical heating, for $2l$.

Estimation of calculation error based on (14) using expressions (17) and (18) was performed by comparing them with the results of numerical calculations. For this purpose, we applied a method of control volumes whose one implementation was described in [11]. Numerical calculation was performed using a one-dimensional grid with 51 computational nodes. We considered an example at $Bi_1=1$ and $Bi_2=10$. The results obtained determine location of the temperature minimum at coordinate $X=0.74$ (Fig. 1, *a*). The coordinate is of relative character as a fraction of the plate's thickness. Its dimension is accepted equal to 1. At numerical calculation, location of the start of a geometric coordinate count (Fig. 1, *a*) differs from the case of the model for analytical study (Fig. 1, *b*). That is why the value for a coordinate of temperature minimum, obtained from numerical calculation, for the variant of analytical study corresponds to magnitude $x_{n1}/(2l)=1-0.74=0.26$. On the other hand, this same magnitude was calculated analytically using expression (18) and turned out to be equal to $x_{n1}/(2l)=0.2632$. These magnitudes coincide in terms of accuracy of numerical calculations. Table 2 gives results of numerical and approximate analytical calculations of nondimensionalized normalized temperature at its minimum point for some values of criterion Fo. In addition, for the calculated values, we determined relative error of analytical calculations relative to numerical calculations.

Table 2

Comparison of results of calculating minimal temperature

Fo	0.05	0.30	0.55	0.80	1.05	1.30
$\Theta_c^{numerical}$	0.084	0.557	0.802	0.912	0.961	0.983
$\Theta_c^{analytical}$	0.115	0.519	0.739	0.858	0.923	0.958
ε, %	3.1	3.8	6.3	5.4	3.8	2.4

The magnitude of error in the results of analytical calculations in some cases exceeds the magnitude generally accepted for engineering calculations (<5 %).

Based on the results obtained, we consider workable the proposed model and the analytical method for taking into consideration the asymmetry of temperature profile at asymmetrical heating of the plate.

5. 3. Nonstationary heat transfer through a flat wall

Solution to a stationary problem on heat transfer is trivial. By assigning temperatures of the environment t_{s1} and t_{s2}, as well as heat transfer coefficients α_1 and α_2 on both sides of the plate, it is possible to determine temperatures on its both surfaces t_{n1} and t_{n2}. A temperature profile inside the plate has a linear dependence and is defined by these temperatures. In the transition from a heat transfer coefficient to the Bi criterion, and from absolute values of temperatures t to their nondimensionalized normalized magnitudes θ, temperatures on the surfaces can be determined from ratios:

$$\theta_{n1} = \theta_{s1} - \frac{\theta_{s1} - \theta_{s2}}{(1 + Bi_1 + Bi_1/Bi_2)};$$

$$\theta_{n2} = \theta_{s2} - \frac{\theta_{s1} - \theta_{s2}}{(1 + Bi_2 + Bi_2/Bi_1)}. \tag{19}$$

Here we accepted direction of the heat flow from the environment with a temperature of θ_{s1} to θ_{s2}. In this case, due to the normalization of θ_{s1}=1, θ_{s2}=0. Geometrical dimension (thickness) of the plate is also considered to be normalized. The coordinate for thickness varies within $X \in [0, 1]$.

Based on numerical calculations [11], we shall consider a change in the temperature profile inside the plate at nonstationary heat transfer in the process of transition from one stationary state to another (Fig. 2, *a–c*). In all cases, the profile (straight line) corresponding to Fo=0 refers to the original stationary state, and for Fo=1.5, (straight line) also models the resulting stationary state. In the examined cases, the reason for a non-stationary process is a stepwise change in criterion Bi from the heating side. An analysis of Fig. 2 can serve a proof for it.

Fig. 2, *a* demonstrates the process from initial state Bi_1=0, Bi_2=100. This corresponds to the identical temperature of the plate at all points and to its equality to the temperature of the environment from a cooling side θs_2=0 (Fig. 2, *a* – on the right). Then the heat transfer coefficient from a heating side (Fig. 2, *a* – on the left) changes to the magnitude that corresponds to Bi_1=2. Development of the process of nonstationary heat transfer is associated with the accumulation of energy in the plate.

Fig. 2, *b* exhibits the process from initial stationary state *Bi*1=1, *Bi*2=100. Then the heat transfer coefficient from a heating side (Fig. 2, *b* – on the left) changes to the magnitude that corresponds to *Bi*1=4. Development of the process of nonstationary heat transfer is associated with the accumulation of energy in the plate.

Fig. 2, *c* shows the process from the initial stationary state *Bi*1=4, *Bi*2=100. Then the heat transfer coefficient from a heating side (Fig. 2, *c* on the left) changes to the magnitude that corresponds to *Bi*1=1. Development of the process of nonstationary heat transfer is associated with the release of energy from the plate.

We shall select at each point in time a longitudinal cross-section (point) in the plate body, in which temperature at the current moment has a maximum deviation from the magnitude of temperature in the final stationary state in this same cross-section. On the temperature profile, it will be matched with the point furthest from the profile (straight line) in the final stationary state. In order to maintain continuity in the terminology with the previously examined cases of heating, we shall name a given point as the "minimum" of temperature. Let us consider its displacement over time. Fig. 2, *a–c* shows it as a dotted line. In all cases, it is possible to separate two regions (Fig. 2, *d*), approximated by sections of the straight line: vertical DK and sloping AK. Schematic (Fig. 2, *d*) is based on Fig. 2, *a*, but all the represented variants in Fig. 2 correspond to it. Under this scheme, the character of displacement of "minimum" in the examined case along section DK corresponds to the displacement of minimum for the case of asymmetrical heating of the plate, considered in chapter 5.2. The sloping section starts (point A) on the heated side of the plate at the point of initial stationary temperature. When analyzing results of numerical calculations, it was assumed that inclination angle of section AK relative to the initial stationary profile (<KAV) is equal to the angle between the initial and resulting stationary temperature profiles (<CBA). The charts (Fig. 2, *a–c*) were complemented with straight line segments (solid black line), constructed based on this assumption. The character of position of these lines allows us to draw a conclusion on the legality of their use as the approximation of trajectory of the temperature "minimum" displacement along a sloping section. Based on the revealed features, we built a model and chose an algorithm for estimating the process of nonstationary heat transfer.

At symmetrical and asymmetrical heating of the plate, its longitudinal cross-section, along which the temperature minimum moves, can be considered a kind of thermally insulated wall. Heat flow from one part to another part of the plate is not transferred through this cross-section. Therefore, the model for each of the parts is composed only of a preservation equation (6) and the boundary condition of the third kind at the surface of the plate (7). For the case of nonstationary heat transfer, the pattern is different. As was the case in chapter 5.2, the coordinate system is connected to the temperature "minimum" (Fig. 1, *b*). Although the temperature "minimum" in section DK also moves along a particular cross-section, the heat flow passes through it. The plate, similarly to the previous cases, is considered to consist of two parts, but, in this case, not "thermally-insulated" from each other. Using the same approach applied in chapter 5.1, a model for each part can be written in a similar form, but considering that the plate is composed of parts. In addition to correcting equations in the form (6), (7), it is required to complement them with a boundary condition of the fourth kind at the border of the composite plate. As a result, the model of the first part, for example, takes the form:

$$c \cdot \rho \cdot V_1 \frac{d\tilde{\theta}_1}{d\tau} = \\ = S_b \cdot \frac{\lambda}{x_{n1} - \bar{x}_1} \cdot (\theta_{n1} - \tilde{\theta}_1) - S_b \cdot \frac{\lambda}{\bar{x}_1} \cdot (\tilde{\theta}_1 - \theta_c), \tag{20}$$

$$S_b \cdot \frac{\lambda}{x_{n1} - \bar{x}_1} \cdot (\theta_{n1} - \tilde{\theta}_1) = \alpha_1 \cdot S_b \cdot (\theta_{s1} - \theta_{n1}), \tag{21}$$

$$S_b \cdot \frac{\lambda}{\bar{x}_1} \cdot (\tilde{\theta}_1 - \theta_c) = S_b \cdot \frac{\lambda}{\bar{x}_2} \cdot (\theta_c - \tilde{\theta}_2). \tag{22}$$

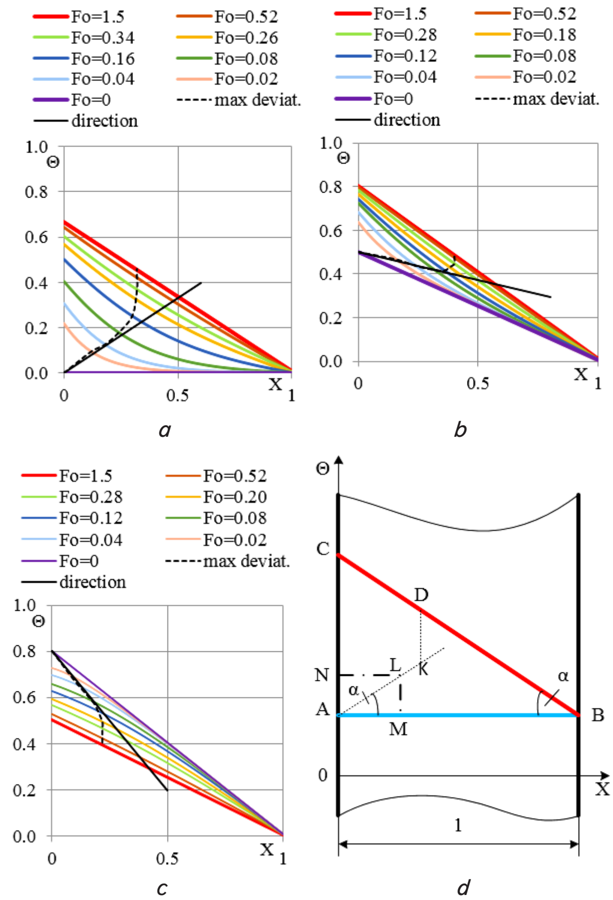

Fig. 2. A change in temperature Θ along plate thickness X at nonstationary heat transfer at different points in time Fo: a – process of energy accumulation from initial state Bi_1=0, Bi_2=100; b – process of energy accumulation from initial state Bi_1=1, Bi_2=100; c – energy release from initial state Bi_1=4, Bi_2=100; d – estimated scheme

Here: $\tilde{\theta}_1$ is the averaged temperature in the examined part of the body; θ_{n1} is temperature at the surface of the plate; θ_c is temperature a current point of its "minimum"; $\tilde{\theta}_2$ is the averaged temperature in the second, conjugate part of the body; θ_{s1} is temperature of the environment, which washes the surface of the examined part of the body; x_{n1} is the distance from the coordinate origin, located in the temperature "minimum" to the surface of the plate (thickness of the part under consideration); $\bar{x}_1$ is the distance from the coordinate origin to the point of application of the averaged temperature in the examined part of the body; $\bar{x}_2$ is the distance from the coordinate origin to the point of application of the averaged temperature in the second part of the body; S_B is the area of the lateral surface of the plate.

We consider an infinite plate (S_b=∞). The magnitude S_b is recorded for the completeness of the model. It is subsequently reduced. All magnitudes of temperatures and dimensions are used in the nondimensionalized and normalized form.

Equation (20), as well as equation (6), reflects the law of energy preservation. The left side of the equality describes the energy, accumulated in the examined part of the body due to its heat capacity. In the right side, the first term marks the energy that arrives from the surface into the plate body due to a temperature difference at the surface and inside the plate. The second term marks the energy that leaves the examined body part through the border that passes through the temperature "minimum". The record of equation (21) assigns the boundary condition of the third kind. It expresses the equality of energies transmitted from the environment to the surface of the body and directed from the surface inside the body. The record of equation (22) assigns the boundary condition of the fourth kind. This equation expresses the equality of energies transmitted from the examined body part to its internal border and directed away from this border to the second part of the body. The model for the second part of the plate from a cooling side is recorded in the similar fashion.

The approach implemented in chapter 5.1, allowed us to obtain a solution in the examined case as well. For example, temperature at its "minimum" point is determined from relation:

$$\theta_c = A_1 \cdot x_{n1} \cdot Bi_1 \cdot \left[1 - \exp(-Ho_1)\right], \qquad (23)$$

where

$$Ho_1 = Fo \cdot \frac{1}{x_{n1}^2} \cdot \frac{1}{A_1 \cdot \left[1 + x_{n1} \cdot Bi_1 \cdot (1-k)\right]}; \qquad (24)$$

$$A_1 = \frac{\left[1 + (1 - x_{n1}) \cdot Bi_2\right]}{x_{n1} \cdot (Bi_1 + Bi_2 + Bi_1 \cdot Bi_2)}. \qquad (25)$$

Here Ho_1 is the extended number of homochronicity (similar to the magnitude determined in chapters 5.1 and 5.2); k is the coefficient that determines location of the point of application of the averaged temperature; Bi_1, Bi_2 are the Biot criteria from the respective sides of the plate.

By applying the same procedure, we can derive expressions for determining other magnitudes employed in a mathematical model of the form (20)–(22). The form of expression (23) is similar to expression (14), used also when solving a problem on asymmetrical heating. Coefficient k is determined using a comparison of results of the numerical and analytical calculations by achieving the minimum of their mismatch.

Let us consider, for example, determining, depending on time, a magnitude and location of temperature "minimum" θ_c:

1. At the initial stage, solution algorithm is based on the same approach as was employed in chapter 5.2. For section D-K (Fig. 2, d), we determine its coordinate. In other words, we determine the location of temperature "minimum" temperature – x_{n1} and, respectively, x_{n2}. Next, knowing the magnitude of angle α (<CBA), we construct a section of straight line AK until its intersection with DK at point K. Path A-K-D is the approximation of trajectory of temperature "minimum" displacement.

2. At the initial moments of the process of nonstationary heat transfer, temperature dependence on time θ(τ) is determined using dependence (23) by solving the inverse problem. By assigning any abscissa x_{n1} (point M), we determine through point L the point corresponding to its ordinates (point N). By using these data, we determine the point of time (Fo) corresponding to them from relation (23).

3. After passing point K, we solve a direct problem in section K-D. Knowing the magnitude of x_{n1} (a coordinate of section K-D), and by assigning some magnitude τ, we determine θ_c from (23).

By applying the algorithm proposed, we performed a series of computations and assessed their adequacy by comparing with the results of numerical study.

We determined positions of vertical plots in the trajectories of motion of temperature "minimum" for three variants (Fig. 2, *a*–*c*). Based on the comparison, among the examined variants of results of numerical and analytical calculations we chose the value of magnitude k=0.67, which ensures their minimum divergence. All subsequent calculations were performed employing a given magnitude. Comparison of results, based on a relative error, with the results of numerical calculations are given in Table 3. The margin of error is defined relative to the thickness of the plate, which is equal to 1.

Table 3

Comparison of results of determining a position of the temperature "minimum" along a vertical section

Calculation technique	Fig. 2, *a*	Fig. 2, *b*	Fig. 2, *c*
x_{n1} analytically	0.32	0.36	0,28
x_{n1} numerically	0.32	0.4	0.22
ε, %	0	4 %	6 %

Next, we determined ordinates of the intersection points of the vertical, calculated analytically, and the sloping sections of trajectories of displacement of the temperature "minimum". Thus, for the variants shown in Fig. 2, *a* and Fig. 2, *b*, which reflect the process of energy accumulation, these magnitudes are equal to, respectively: θ^a=0.212 and θ^b=0.451.

The next step of the algorithm at the sloping section of trajectory (A–K, Fig. 2, *d*) was the selection of abscissa equal to x_{n1}=0.2 for the examined variants of energy accumulation. Ordinates $\theta^a_{0.2}=0.133$ and $\theta^b_{0.2}=0.451$. correspond to them. By using these data, from relations (25) and (24), we determine magnitude A_1 and then Ho_1. Employing the latter magnitude, we derive Fo – nondimensionalized time. In order to estimate error in determining dependence θ=θ(Fo) using such a technique in relation to similar numerical calculations, the latter [11] were performed for the found time points Fo. Results of comparison are given in Table 4. The margin of error is defined relative to the range of change in the nondimensionalized normalized temperature, which is equal to 1.

Table 4

Comparison of results of determining temperatures along a sloping section of the trajectory of their "minimum"

Calculation technique	Fig. 2, *a*	Fig. 2, *b*
θ analytically	0.133	0.451
θ numerically	0.140	0.491
ε, %	0.7 %	4 %

By solving a direct problem (23) for the analytically computed values for the coordinate of position of vertical section K-D (Fig. 2, *d*) of the trajectory of displacement of temperature "minimum" temperature x_{n1} (Table 3) for a certain nondimensionalized time point (Fo) we determined the value of temperature "minimum" θ_c. The same magnitude and at the same value of Fo was determined by numerical calculations [11]. For calculations, we accepted Fo=0.52. Results of their comparison are given in Table 5. Here, similar to previous cases, ε is the relative error.

Table 5

Comparison of results of determining temperatures along a sloped section of the trajectory of their "minimum"

Calculation technique	Fig. 2, *a*	Fig. 2, *b*
θ analytically	0.456	0.516
θ numerically	0.420	0.475
ε, %	3.6 %	4.0 %

The results of calculations given in Tables 3–5 allow us to argue about the applicability of the developed method for determining temperature inside a flat wall in the process of nonstationary heat transfer. An error of calculation does not exceed, or close to permissible, magnitude for engineering calculations, 5 %.

6. Discussion of the study results: special features of the developed method of approximate solution to a problem on nonstationary heat transfer

Heat transfer process is inherently a distributed process. An attempt to use a solution for the examined problem in a concentrated statement is common. The approach that implies dividing an object (a plate) into several layers is also known. One of the special features of the proposed method is dividing the plate in two layers only. The acceptability of such an approach is noted in [11] for numerical calculations. In the case under consideration, it was applied for analytical calculations. Moreover, the previously considered cases involved layer splitting based on various geometrical considerations. In the method in question, this occurs on a physical basis. The border of layers is located at an extremum of temperature when heating (cooling) a plate and its coordinate is calculated. Furthermore, when calculating during heat transfer process at the initial point in time of change in the temperature field, the boundary between layers (temperature "minimum") moves and its coordinate is calculated. It should be noted that in papers [2, 3], in the reported solutions to symmetrical heating of a plate, the border of the examined region was also located at an extremum of the temperature profile. But it was substantiated only by the symmetry of the problem.

The approach to solving a set problem was chosen based on an analysis of results of numerical studies. They served not only for the purpose of identifying the qualitative pattern of the course of analyzed process. Analytical solutions were obtained with an accuracy to a certain coefficient "k", which combines all the unaccounted-for features. Numerical solutions received preliminary have allowed us to determine optimal magnitude for this coefficient.

Errors of all the obtained solutions given in Tables 1–5 do not exceed the permissible limits of engineering accuracy (ε<5 %) or are close to them. This is significantly less than the possible errors, considered previously, in determining the source data (heat transfer coefficients), which may reach 50 %. Although the proposed method is approximate, it makes it possible to obtain analytical solutions, acceptable from an engineering point of view, in all of the cases examined, in the absence of their precise forms.

In order to create effective control systems, it is required to have tools for the calculation of temperature fields and accumulating properties of the elements of heat power equipment under non-stationary conditions of heat exchange. At the design stage, when setting up control systems, results of analytical studies are important. The availability of results of analytical studies becomes even more relevant when using fuel of variable composition, characterized by a sharp increase in the number of transition processes. This explains the relevance and usefulness of the present research.

7. Conclusions

1. Using a problem on symmetrical heating of a plate as an example, which has an exact analytical solution in a distributed statement, we estimated error in the proposed approximate analytical solution in concentrated statement. The acceptability of the obtained result within the precision of engineering calculations is shown. A special feature of the proposed method is the possibility of its application as an integral part when solving the problems on nonstationary heat transfer.

2. By using a numerical study, we revealed the character of displacement of the minimum of temperature profile for thickness of the plate for the case of its asymmetric heating. This made it possible, when employing the proposed approximate analytical method, to obtain a solution to the problem on asymmetric heating of the body. A special feature of the solution is the devised approach to determining the minimum of temperature for thickness of the plate. Such an approach might be used as another constituent part for solving a problem on nonstationary heat transfer.

3. A nonstationary heat transfer through a flat wall was considered as a transition process between two stationary states, described by temperature profiles in the form of straight lines: initial and resulting. Numerical study allowed us to identify a characteristic point in the varying temperature profile. Its current position is determined by the maximum distance from a given point to the resulting stationary profile. Numerical study also enabled us to determine the trajectory of motion of this point, which we named, for the continuity of terminology, the “minimum” of temperature. Based on these data and applying the developed method, we demonstrated the possibility of approximate analytical solution to the problem on nonstationary heat transfer through a flat wall.

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