MACHINE BUILDING. PROCESS METALLURGY. MATERIALS SCIENCE

МАШИНОБУДУВАННЯ. ТЕХНОЛОГІЯ МЕТАЛІВ. МАТЕРІАЛОЗНАВСТВО

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SYNTHESIS OF NONLINEAR ELASTIC COUPLINGS ON THE BASIS OF MODIFIED KINEMATIC GRAPHS

І.І. Сидоренко, В.О. Курган. **Синтез нелінійних пружних муфт на основі модифікованих кінематичних графів.** Проведено дослідження, пов'язані із аналізом існуючих пружних муфт з лінійним механічним зворотним зв'язком і синтезом, на основі отриманих результатів, пасивних пружних муфт з нелінійним механічним зворотним зв'язком. Аналіз і синтез пристроїв проведено з використанням теорії модифікованих кінематичних графів, згідно з якою сформовано принципи ідеалізованого уявленням структури пасивної пружної муфти з додатковим механічним контуром. Застосований критерій вибору оптимального рішення, що дозволяє виділити одне оптимальне рішення з множини можливих і відтворити відповідну до нього кінематичну схему. Синтезовано нові конструкції пружних муфти з нелінійним механічним зворотним зв'язком.
Ключові слова: пружна муфта, механічний зворотний зв'язок, синтезована конструкція

I.I. Sydorenko, V.O. Kurgan. **Synthesis of nonlinear elastic couplings on the basis of modified kinematic graphs.** In this work the analytical research of existing elastic couplings with linear mechanical feedback and synthesis was carried out taking into account passive elastic couplings with nonlinear mechanical feedback. The analysis and synthesis of devices was carried out using the theory of modified kinematic graphs, according to which the principles of idealized representation of the structure of a passive elastic coupling with an additional mechanical contour are formed. The criterion for choosing the optimal solution was applied, which allows to select one optimal solution from the set of possible ones and reconstruct the corresponding kinematic scheme. New designs of elastic couplings with nonlinear mechanical feedback are synthesized.
Keywords: elastic coupling, mechanical feedback, synthesized design

Introduction. The main feature of any technical system is its reliability. This feature depends on many factors, among which are the following: rational arrangement of the engine, units and components of the TS, correctness and accuracy of their computation, quality of their manufacturing, quality of service, etc. But it is very important to provide the elements of the TS with functionality that determines their effective functional interaction during the operation of the TS.

Recent researches and publications analysis. The capability of elastic couplings with metal elastic elements to work at large oscillation amplitudes determined their widespread use in machine building. It is proved that in systems that implement a nonlinear elastic characteristic with sections of quasinuclear rigidity in the working traverse range, overcoming resonance often happens as a "leap" [1, 2, 3]. In order to provide such conditions mechanical feedback is introduced into the coupling design that facilitates immediate response to load changes and ensures a safe working range of couplings. But already existing elastic couplings with nonlinear mechanical feedback [3, 4] can not fully provide the required elastic characteristic, which is associated with the peculiarities of the coupling design.

Grounded on this, the analysis of existing elastic couplings based on the theory of modified kinematic graphs is considered topical. That will allow formulating principles of the optimal structure of the passive elastic coupling with an additional mechanical contour.

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Purpose of research. The purpose of this research is to: analyze the existing designs of elastic couplings; analyze the functional interaction between their constitutive elements, which defines the basic functionality of couplings and affects main indicators that determine the effectiveness of their application.

Presentation of basic material. Studies related to the application of the method of synthesis of elastic devices based on modified graphs developed by Professor I.I. Sidorenko [5, 6] allowed forming of basic principles of the kinematic synthesis of such systems. According to one of the basic principles, idealized representation of a structure of an ordinary flat passive elastic device with one parameter, which is subject to a controlled change, that can be considered as each of the existing passive elastic couplings with control elements, is a modified kinematic graph that characterizes its zero freeness by the following equation:

$$W = 3\,(p_\Sigma - 1) - 2q_5 - q_4 - q_C \le 0, \quad (1)$$

where p_Σ – total number of points of graph, which corresponds to the number of rigid links of the device;

q_C – number of edges of graph, which is equal to the number of non-kinematic connections (elastic, dissipative, etc.), between the rigid links of the device, while this component does not affect the kinematic characteristics of the device and can be regarded as an excessive kinematic pair.

According to another principle, nature of kinematic control of the properties of a device, that is modeled by a modified kinematic graph, can be evaluated using cyclomatic number index

$$\sigma = q - p + 1 \ge 2, \quad (2)$$

where q – number of graph edges regardless of their type;

p – number of points of graph regardless of their type.

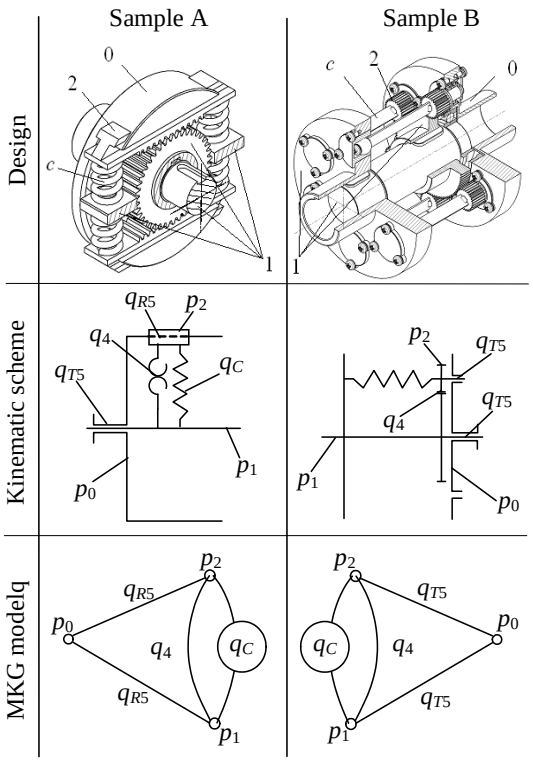

Fig. 1. Modeling of passive elastic couplings with linear mechanical feedback of modified kinematic graph (MKG)

Let us conduct an analysis using two well-known designs of passive elastic couplings with linear mechanical feedback (Fig. 1) by combining their models in the form of modified kinematic graphs, with further determining the differences between the obtained models according to the abovementioned principles. In this case let the couplings be conventionally named (Fig. 1) a sample for analysis A (AA) and a sample for analysis B (AB).

At the initial stage it is advisable to switch from the actual couplings designs, that are subject to analysis, to their kinematic schemes and then to the MKG model on their basis (Fig. 1). The modeling is carried out provided that: the couplings that are considered are flat mechanisms; p_i – is a point of graph out of their total number p_Σ, which defines the i-th element of a coupling with the corresponding numbering; the numbering of points in each graph model starts from the point p_0, which determines the output half-coupling, that is numbered as 0; for the edge of the graph q_5, which defines the functional contact interaction between the two elements of the device and corresponds to the kinematic pair of the 5th class, in order to facilitate more detailed structuring of the model, the possible types of such edge, that correspond to turning and rectilinear pairs (q_{T5} and q_{R5} respectively), are established. It is also established that the coupling has one

elastic element, the presence of which determines an elastic characteristic, which must be changed under control, that causes the presence of q_C=1in the model.

A comparative visual analysis of the obtained models indicates their complete identity and absence of any differences. However, a more detailed analysis requires a number of calculations of indicators, determined by the accepted methodology.

For the obtained samples models, the calculation of the degree of freeness was calculated using the expression (1):

– sample AA

$$W = 3(3 – 1) – 2\cdot2 – 1 – 1 = 0;$$

– sample AB

$$W = 3(3 – 1) – 2\cdot2 – 1 – 1 = 0.$$

According to the accepted method, the results of this calculation indicate that the couplings that are being studied are indeed flat passive elastic devices with one parameter subject to a controlled change.

The calculation of the cyclomatic number using the expression (2) for the obtained samples models was carried out:

– sample AA

$$\sigma = 4 – 3 + 1 = 2;$$

– sample AB

$$\sigma = 4 – 3 + 1 = 2.$$

According to the accepted method the results of this calculation indicate that the models of couplings that are analysed in the form of MKG have two cycles. In the framework of the accepted method, one of the cycles causes the basic functional interaction of the coupling's elements, which determines its efficiency (hereafter basic cycle), and the other one causes the functional interaction of the coupling's elements, that facilitates controlled change of its specific parameter (hereafter control cycle).

In this case, it is appropriate to use the incidence matrix M

$$M = \left(\begin{array}{c|ccc} & p_0 & p_1 & p_2 \\ \hline p_0 & 0 & 0 & 1 \\ \hline p_1 & 0 & 0 & 1 \\ \hline p_2 & 1 & 1 & 0 \end{array}\right), \qquad (3)$$

where 0 or 1 – absence or presence of contact respectively.

Application of incidence matrices for analysed samples

$$M_{\text{AA}} = M_{\text{AA}} = \left(\begin{array}{c|c|c} 0 & 1 & 1 \\ \hline 1 & 0 & 1/1_C \\ \hline 1 & 1/1_C & 0 \end{array}\right) \quad \text{and} \quad M_{\text{AB}} = \left(\begin{array}{c|c|c} 0 & 1 & 1 \\ \hline 1 & 0 & 1/1_C \\ \hline 1 & 1/1_C & 0 \end{array}\right),$$

and their further comparison with the subtraction operation facilitated the zero matrix,

$$M_{\text{AA}} - M_{\text{AB}} = \left(\begin{array}{c|c|c} 0 & 0 & 0 \\ \hline 0 & 0 & 0 \\ \hline 0 & 0 & 0 \end{array}\right),$$

which determines the identity of the matrices.

Based on this the passive elastic couplings with linear mechanical connection that are being considered are also identical in their structure. However, the conducted analysis does not provide any information on constructive differences of the samples being considered. That is why, from the point of

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view of determining the constructive difference, it is proposed to use summation matrices in order to receive a more effective analysis

$$M^{\text{su}} = \left(\begin{array}{c|c|c|c} & p_0 & p_1 & p_2 \\ \hline p_0 & 0 & q_i & q_i \\ \hline p_1 & q_i & 0 & q_i \\ \hline p_2 & q_i & q_i & 0 \end{array}\right), \tag{4}$$

where q_i – weighting coefficient, which determines the class and type of contact and determines the corresponding weighting factor.

For the samples considered, the summation matrices (4) have the following form

$$M_{\text{AA}} = \left(\begin{array}{c|c|c} 0 & q_{T5} & q_{R5} \\ \hline q_{T5} & 0 & q_4 / q_C \\ \hline q_{R5} & q_4 / q_C & 0 \end{array}\right) \quad \text{and} \quad M_{AB} = \left(\begin{array}{c|c|c} 0 & q_{T5} & q_{T5} \\ \hline q_{T5} & 0 & q_4 / q_C \\ \hline q_{T5} & q_4 / q_C & 0 \end{array}\right). \tag{5}$$

Taking into account the type of contact, filling out of the summation matrices is performed under the following conditions: the contact, which is caused by q_{T5}, has a weighting coefficient of 1; the contact, which is caused by q_{R5}, has a weighting coefficient of 1; the contact, which is caused by q_4, has a weighting coefficient of 2.

Application of summation matrices

$$M^{\text{su}}_{\text{AA}} = \left(\begin{array}{c|c|c} 0 & -1 & 1 \\ \hline -1 & 0 & 2 \\ \hline 1 & 2 & 0 \end{array}\right) \quad \text{and} \quad M^{\text{su}}_{\text{AB}} = \left(\begin{array}{c|c|c} 0 & -1 & -1 \\ \hline -1 & 0 & 2 \\ \hline -1 & 2 & 0 \end{array}\right),$$

and their further comparison with the subtraction operation facilitated the nonvacuous matrix

$$M^{\text{su}}_{\text{AA}} - M^{\text{su}}_{\text{AB}} = \left(\begin{array}{c|c|c} 0 & 0 & 2 \\ \hline 0 & 0 & 0 \\ \hline 2 & 0 & 0 \end{array}\right),$$

which determines their difference in constructive design.

It is established that the applied method has no criterion, which allows to estimate what kind of feedback is in the system, linear or nonlinear. So in both analyzed samples the feedback represents an additional mechanical structure in the form of a gear transmission (rack for the sample AA and cylindrical for the sample AB). It is known that these transmissions determine the linear transfer control function and are modeled by the corresponding simple kinematic graph (Fig. 2, *c*).

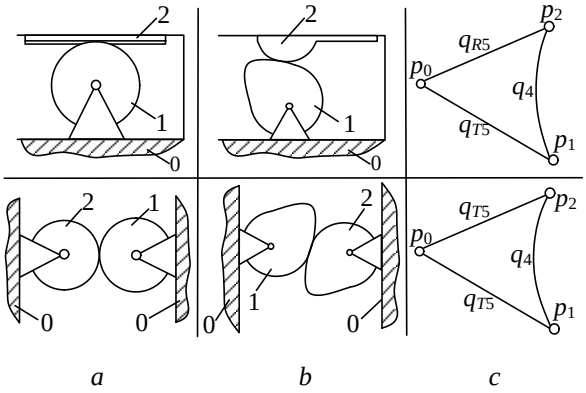

Fig. 2. Modeling of additional mechanical loops by kinematic graphs: with linear control function (a); with nonlinear control function (b); model in the form of kinematic graph (c)

However, the same simple kinematic graph also models mechanisms with variable gear ratios, for example, cam mechanisms, gear mechanisms with variable gear ratios, the Maltese cross mechanism, etc.

In performing analytical studies related to the synthesis of devices, the model of a modified kinematic graph for an elastic coupling with a linear mechanical feedback was used as the basic struc-

ture. In the model, in order to minimize the number of possible solutions, the following conditions apply: $q_4 = 1$ and $q_C = 1$, that is, the synthesized device in its structure has kinematic pairs of the 5th class and one elastic element.

In order to achieve the research goal it is proposed to introduce directed edges into the MKG model, that determine possible movements of the device units, depending on the type of kinematic interaction. And thus to consider the model of the device in the form of oriented graph (orgraph). Then it is possible to identify the existing difference (Fig. 3). The difference lies in the fact that some additional mechanical loops with a nonlinear control function are functional only in one predetermined direction of movement of the contacting units. Based on this and keeping in mind the criteria for the expressions (1) and (2), the structure of the elastic coupling with nonlinear mechanical feedback in the form of an orgraph was synthesized (the synthesized sample CA), on the basis of which the kinematic scheme was reproduced and the constructive execution of the device was carried out (Fig. 4) It should be noted that despite the very similar constructive execution and the same way of implementing management of cumulated torsional rigidity of the structure, the analyzed sample AB (Fig. 1) and the synthesized sample CA (Fig. 4) are represented by various devices in their functionality. This is demonstrated by the analysis of summation matrices with the introduced indicator.

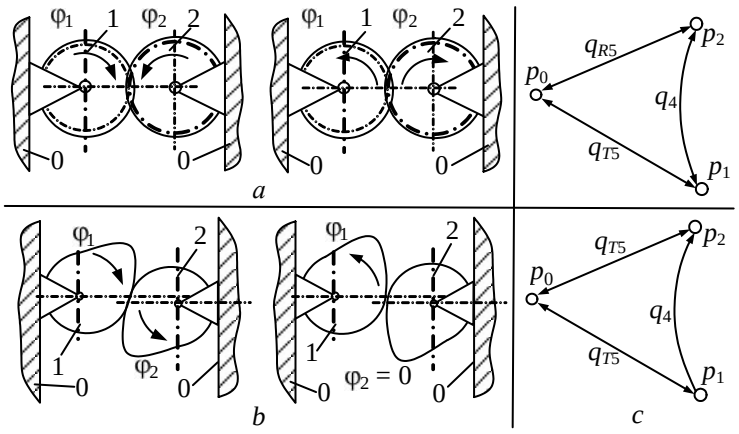

Fig. 3. Modeling of additional mechanical loops by kinematic orgraphs: with linear control function (a); with nonlinear control function (b); corresponding models in the form of kinematic orgraph (c)

$$M_{\text{AB}} = \left(\begin{array}{c|c|c} 0 & q_{T5}\updownarrow & q_{T5}\updownarrow \\ \hline q_{T5}\updownarrow & 0 & q_4\updownarrow/q_C \\ \hline q_{T5}\updownarrow & q_4\updownarrow/q_C & 0 \end{array}\right), \quad M_{\text{CA}} = \left(\begin{array}{c|c|c} 0 & q_{T5}\updownarrow & q_{T5}\updownarrow \\ \hline q_{T5}\updownarrow & 0 & q_4\uparrow/q_C \\ \hline q_{5T}\updownarrow & q_4\uparrow/q_C & 0 \end{array}\right), \tag{6}$$

where $\updownarrow$ – determines contact interaction in both directions,

$\uparrow$ – determines contact interaction in one direction.

However, despite the fact that the proposed approach using an orgraph leads to a positive result, it can not be recognized as an alternative, that gives a clear description of the type of feedback. Thus, a mechanical loop with nonlinear control function in the form of a cam mechanism with kinematic closure will function in the forward and reverse directions of movement of the contacting units. In this case, it can be modeled by an orgraph that is similar to an orgraph, which determines the gear mechanism (Fig. 4).

In this regard it is proposed to use the type of kinematic interaction of additional link as a discrepancy indicator when replacing the kinematic pair of the 4th class of additional structure according to Assur, further taking into account this indicator in the summation matrix (4). Thus, the synthesis of a new device can be based on an extended summation matrix, for example, of a well-known device according to the sample AA in expression (5) while replacing the kinematic pair of the 4th class, which will take the following form

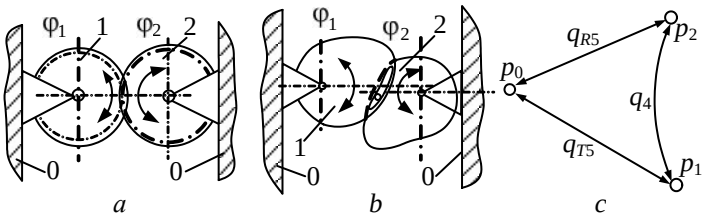

Fig. 4. Modeling of additional mechanical loops by kinematic orgraphs: with linear control function (a); with nonlinear control function (b); corresponding models in the form of kinematic orgraph (c)

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$$M_{\text{AA}} = \left(\begin{array}{c|c|c} 0 & q_{T5} & q_{R5} \\ \hline q_{T5} & 0 & q_4 / q_C \\ \hline q_{R5} & q_4 / q_C & 0 \end{array}\right) \rightarrow \left(\begin{array}{c|c|c|c} 0 & q_{T5} & q_{R5} & 0 \\ \hline q_{T5} & 0 & 0 / q_C & q_{T5} \\ \hline q_{R5} & 0 / q_C & 0 & q_{T5} \\ \hline 0 & q_{T5} & q_{T5} & 0 \end{array}\right).$$

Thus, a device which will be different from the given summation matrices can be considered a new device.

Let us replace the first, according to Assur, contact interaction (edges of the graph) between the elements of the matrix $p_2 - p_3$ (corresponding poles of the graph), which determines the contact of a curved surface with a curved one, with the second contact interaction, specifically the contact of a curved surface with a plane one. In this case, an extended summation matrix will take the following form

$$M_{\text{CB}} = \rightarrow \left(\begin{array}{c|c|c|c} 0 & q_{T5} & q_{R5} & 0 \\ \hline q_{T5} & 0 & 0 / q_C & q_{R5} \\ \hline q_{R5} & 0 / q_C & 0 & q_{T5} \\ \hline 0 & q_{T5} & q_{R5} & 0 \end{array}\right) \rightarrow \left(\begin{array}{c|c|c} 0 & q_{T5} & q_{R5} \\ \hline q_{T5} & 0 & q_4 / q_C \\ \hline q_{R5} & q_4 / q_C & 0 \end{array}\right),$$

and represents the basis for synthesis (Fig. 5).

According to the obtained summation matrix, simple graphs (Fig. 5, *a* and *b*) allow us to recreate the kinematic scheme (Fig. 5, *c*) and constructively implement the new device on it (Fig. 5, *d*).

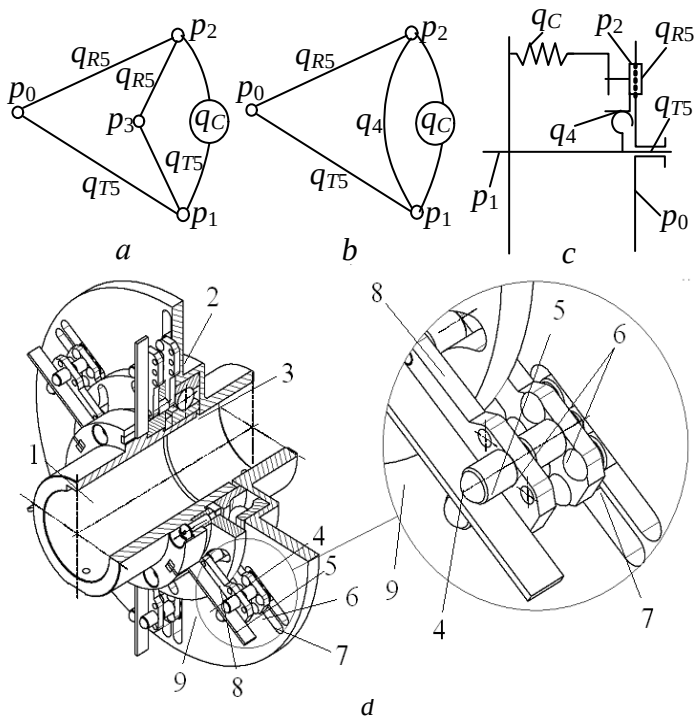

Fig. 5. Synthesis of structure of elastic coupling with nonlinear mechanical feedback and its constructive realization

The coupling contains a driving 1 and a driven 2 half-coupling, a rolling bearing 3 between them to ensure their co-location; elastic connection between the half-couplings 1 and 2 is formed by flat elastic elements 4 which, with one end are radially fixed to the half-coupling 1, and the free end of the contact line, that is perpendicular to the elastic axis of the elastic element, contacts the loading rollers 5 of followers 6, which with their guide rollers 7 contact radial grooves of the half-coupling 2; bearing rollers 8 of the pushers 6 are in contact with the surface of the curvilinear grooves of the disk 9 which is rigidly secured to the half-coupling 1. The follower 6 and the curvilinear groove of the disk 9 formed a cam mechanism with a kinematic closure.

Research of kinematics of the proposed device, carried out during 3D computer simulation by means of CAD, a part of the applied Autodesk Inventor Series software package, confirmed its productivity.

Results. According to the results of the conducted studies, the relevance of further development of the method of synthesis of elastic devices based on the theory of modified kinematic graphs has been established.

Use in the synthesis of structures that cause nonlinear feedback: orgraphs with the formation of corresponding incidence matrices; simple graphs that determine the replacement of a 4th class pair with two 5th class pairs and one link according to Assur. At the same time it is established that nonlinear mechanical connection is determined by the presence of a kinematic 4th class pair, replacement of which according to Assur causes the contact of curved and plain surfaces.

The elements of the incidence matrix of the modified kinematic graph must be given weighting coefficients determined by technological or constructive considerations.

The determined optimal variant of synthesis with the use of modified kinematic graphs should be analyzed and, if necessary, modified, depending on the features of the elastic elements applied in the device, which facilitate the elastic connection.

Taking into account the results of calculations it has been established that a kinematic scheme, in which a nonlinear mechanical feedback is implemented, can be considered as the optimal variant of synthesis.

Conclusions. The considered issues and the solved problems are the basis for application of the presented method of analysis and synthesis of elastic devices for creating new designs that will provide better performance of the technical system in which they work. A nonlinear mechanical feedback, introduced in the device scheme, will allow realizing the target elastic characteristic, which will facilitate controlling the vibrational processes of the technical system.

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